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RESEARCH ARTICLE

LOCALLY-ROTATIONALLY SYMMETRIC BIANCHI TYPE-V COSMOLOGICAL MODEL IN PRESENCE OF PERFECT FLUID AND MAGNETIC FLUX WITH VARIABLE MAGNETIC PERMEABILITY.

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Abstract

In this paper, we have investigated the nature of locally-rotationally symmetric Bianchi type-V cosmological model in presence of perfect fluid and magnetic flux in general relativity. We have considered the magnetic permeability as variable quantity. The magnetic field is due to an electric current produced along x-axis, F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get the deterministic solution, it has been assumed that $\rho = -p$. We also discuss the physical and geometrical properties of model.

Introduction:-

Cosmology is the study of origin, current state and the future of Universe. The Bianchi cosmologies which are spatially homogeneous and anisotropic play an important role in the theoretical cosmology and have been much studied since 1960s. The simplest models of the expanding Universe are those which are spatially homogeneous and isotropic at each instant of time. The LRS Bianchi type models have great important to describe the simplified large scale behavior of Universe. The study of Bianchi type-V cosmological model is an interesting as these model contains isotropic special cases and permit arbitrarily small anisotropy levels at some instant of cosmic time. Also the Bianchi type-V cosmological models are more complicated than the simplest Bianchi type model. Space time models of Bianchi type-I and V Universe are generalization of FRW models and it will be interesting to construct the cosmological model which will be of class I.

Bianchi type V cosmological models have been studied by Farnsworth [1], Maartens and Nel [2], Wainwraight et al. [3], Collins [4], Coley and Dunn [5], Coley and Hoogen [6], Meena and Bali [7] and Pradhan and Rai [8], amongst others. Roy and Singh [9] have investigated Bianchi type-V cosmological models with viscosity and heat flow. The occurrence of magnetic fields on galactic scale is well established fact today and it's important for a variety of astrophysical phenomenon is generally acknowledged by several authors Harrison [10], Asseo and Sol [11] and Kim et al. [12] have pointed out the importance of magnetic field in the different context.

In recent years, the solutions of Einstein field equations for homogeneous and anisotropic Bianchi type models have been studied by several authors, Raj Bali [13] has investigated Bianchi type-V magnetized Universe with variable magnetic permeability. Yadav et al [14] have studied the geometrical and physical properties of Bianchi type V cosmological model in presence of cosmic string. Singh [15] has discussed a law of variation for the mean of Hubble parameter in homogeneous LRS Bianchi type-V cosmological model that yields a constant value of deceleration parameter, he obtained two types of exact singular and non singular solutions of Einstein's field equations with constant deceleration. Roy and Prasad [16] investigated Bianchi type-V universe which are LRS and are of embedding class one filled with perfect fluid with heat conduction and radiation.

Many of eminent authors like, Pradhan [17] has obtained Bianchi type-VI_0 massive string cosmological model with bulk viscous fluid as a source of matter in the presence of magnetic field , Saha [18] has investigated Bianchi type-I string cosmological model in presence of magnetic field and he has obtained explicit analytic solutions.

In this paper, We have investigated LRS Bianchi type-V model with perfect fluid in presence of magnetic field with variable magnetic permeability, To get the deterministic model ,we have assume that F_{23} is the only non vanishing component of F_{ij}. The physical implication of model is also discussed.

Metric and Field Equations:-

We consider locally-rotationally symmetric (LRS) Bianchi type-V space time metric

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2x}B^2(t)(dy^2 + dz^2), \tag{2.1}$$

in which $A(t), B(t)$ are cosmic scale functions.

The Einstein field equations are

$$R_{ij} - \frac{1}{2} g_{ij} R = T_{ij} \tag{2.2}$$

Where T_{ij} is energy momentum tensor,

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} + E_j^i \tag{2.3}$$

The electromagnetic field E_j^i is,

$$E_j^i = \bar{\mu}\left[|h|^2\left(u_j u^i - \frac{1}{2}\delta_j^i\right) - h_j h^i\right], \tag{2.4}$$

where $\bar{\mu}$ is called the magnetic permeability, and h_j is magnetic flux vector defined by,

$$h_j = \frac{1}{\bar{\mu}} * F_{ij} u^i, \tag{2.5}$$

in which $*F_{ij}$ is dual electromagnetic field tensor defined as,

$$*F_{ij} = \frac{\sqrt{-g}}{2}\epsilon_{ij\alpha\beta} F^{\alpha\beta}, \tag{2.6}$$

Here $F^{\alpha\beta}$ is electromagnetic field tensor and $\epsilon_{ij\alpha\beta}$ is the totally antisymmetric Levi- Civita tensor with $\epsilon_{ij\alpha\beta} = +1$

The co-moving co-ordinates are taken as, $u^0 = 1, u^1 = u^2 = u^3 = 0$, and choose the incident magnetic field in the direction of x-axis; so that the magnetic flux vector has only one nontrivial component, viz. $h_1 \neq 0$. In view of aforementioned assumption from (5), one obtain $F_{12} = F_{13} = 0$. Also, we assume that the conductivity of the fluid is infinite. This leads to,

$$F_{01} = F_{02} = F_{03} = 0. \tag{2.7}$$

We have only one non-trivial component of F_{ij} i.e. F_{23}, then the first set of Maxwell equation $F_{\mu\nu;\beta} + F_{\nu\beta;\mu} + F_{\beta\mu;\nu} = 0$, one finds,

$$F_{23.} = I, \quad I = \text{Constant} \tag{2.8}$$

Using (2.6),(2.8) in (2.5), We obtain,

$$h_1 = \frac{AI}{\bar{\mu}\, e^{4x} B^2} \tag{2.9}$$

Finally, we obtain the electromagnetic field tensor E_j^i as,

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{I^2}{B^4 e^{4x} \bar{\mu}} \tag{2.10}$$

here we assumed that magnetic permeability is variable quantity and considered as
$e^{-4x} = \bar{\mu}$
Thus $\bar{\mu} \to 0$ as $x \to \infty$ and $\bar{\mu} = 1$ when $x \to 0$

Equation (2.10) takes the following form,

$$E_0^0 = E_1^1 = -E_2^2 = -E_3^3 = \frac{I^2}{B^4} \tag{2.11}$$

And $T'_{ij} = (\rho + p)u_i u_j + p g_{ij}$ is energy momentum tensor of perfect fluid where ρ is energy density and p is thermodynamic pressure

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \left(p - \frac{I^2}{2B^4}\right) \tag{2.12}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\ddot{A}}{A} - \frac{1}{A^2} = p + \frac{I^2}{2B^4} \tag{2.13}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} = -\rho - \frac{I^2}{2B^4} \tag{2.14}$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \tag{2.15}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{2.16}$$

Where $H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{B}}{B}$, are directional Hubble parameter in the direction of x y and z-axis respectively.
And scalar expansion $\theta = u^i_{;i} = (\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B})$
From equation (2.16) we obtained the relation

$$H = \frac{1}{3}(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}) \tag{2.17}$$

Using equation. (2.15), we get $A = KB$ (2.18)

That's give $\frac{\dot{A}}{A} = \frac{\dot{B}}{B}$ and $\frac{\ddot{B}}{B} = \frac{\ddot{A}}{A}$ (2.19)

Solutions of Field Equations:-

The L.H.S. of equations (2.12) and (2.13) are identical so we assume the another condition,

$$\rho = -p\,, i.e.\ p = -\rho \tag{3.1}$$

Using equation (2.14) And subtracting (2.13) from (2.14) we obtain

$$2\frac{\dot{B}^2}{B^2} - \frac{\ddot{B}}{B} - 2\frac{1}{(BK)^2} + \frac{I^2}{B^4} = 0$$

Solving above equation yields,

$$B^2 = \frac{(t+k_1)^2}{K^2} + \frac{I^2}{4K^2} \tag{3.2}$$

Let $T = (t + k_1), \frac{1}{k^2} = N, \frac{I^2}{4K^2} = M$, Therefore $B^2 = NT^2 + M$

Hence, $\frac{\dot{B}}{B} = \frac{NT}{(T^2N+M)}$, $\frac{\ddot{B}}{B} = \frac{NM}{(T^2N+M)^2}$

Now from equation. (2.12) ,

$$p = \frac{4NMK^2 + 2K^2(TN)^2 - 2(T^2N+M) + I^2K^2}{2(T^2N+M)^2k^2} \tag{3.3}$$

Using equation (2.12) in (3.1), we have

$$\rho = \frac{-4NMK^2 - 2K^2(TN)^2 + 2(T^2N+M) - I^2K^2}{2(T^2N+M)^2k^2} \tag{3.4}$$

As $B^2 = NT^2 + M$
From And $A = KB$, therefore $A^2 = K^2\,(NT^2 + M)$, (3.5)

Therefore line element in equation (2.1) is

$$ds^2 = -dt^2 + K^2\,(NT^2 + M)dx^2 + e^{2x}(NT^2 + M)(dy^2 + dz^2) \tag{3.6}$$

And $\theta = 3\left(\frac{A}{A} + 2\frac{\dot{B}}{B}\right) = 3\frac{\dot{B}}{B} = 3NT/(2(NT^2 + M)\ )$, (3.7)

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{\dot{B}}{B} = \frac{TN}{T^2N+M} \quad (3.8)$$

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2 \text{ ,i.e. } \sigma = 0 \quad (3.9)$$

Conclusion:-

We have studied radiating LRS Bianchi type–V cosmological model with perfect fluid and magnetic flux, here we assumed that magnetic permeability as a variable quantity. As T increases model expands, as T decreases model shrink. In the presence of electromagnetic field it expands and p increases, simultaneously decreases ρ. Also in the absence of electromagnetic field it reduces. The cosmic time T increases, the expansion of the universe (θ) diminishes and as T increases, Hubble parameter H decreases. From this, It is clear that Hubble parameter (H) and the scalar expansion (θ) are inversely proportional to T. since $\sigma = 0$ represents that, the model (3.6) is isotropic in nature.

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