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RESEARCH ARTICLE

REGULAR MILDLY GENERALIZED CLOSED AND REGULAR MILDLY GENERALIZED OPEN MAPS IN TOPOLOGICAL SPACES.

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Abstract

In this article, we introduce a new class of RMG-closed and RMG-open maps in topological spaces and study some of their properties as well inter relationship with other closed and open maps.

Introduction:-
Different types of closed and open mappings were studied by various researchers in general topology. In 1982 Malghan [6] introduced and investigated some properties of generalized closed maps. El-Deeb et. al[3], M. Sheik John [15], N. Nagaveni[9], I Arockiarani[1] and Benchalli et. al[2] have introduced and studied pre-closed and pre-open maps, w-closed and w-open maps, wg-closed, rwg-closed and wg-open, rwg-open maps, rg-closed and rg-open maps and rw-closed, rw-open maps respectively. But in this article we introduce new class of weaker forms of closed and open maps i.e. RMG-closed maps and RMG-open maps and also stronger form of RMG-closed and RMG-open maps called RMG*-closed and RMG*-open maps. Here we discuss the properties of all newly formed maps and relationship with existed maps in topological spaces.

Preliminaries:-
Throughout this paper (X,τ), (Y,σ) and (Z,γ) (or simply X, Y and Z) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. We denote the closure, RMG-closure, interior, RMG-interior of A by cl(A), RMG-cl(A), int(A) and RMG-int(A) respectively and neighbourhood of an element in any topological space is denoted as nbd of x. $X-A$ or A^c denotes the complement of A in X.
Now we recall the following known definitions and results that are used in our work;

Definition 2.1 A subset A of a topological space X is called
(i) Regular open [16], if A=int(cl(A)) and regular closed if A=cl(int(A)).
(ii) Pre-open [8], if A⊆int(cl(A)) and pre-closed if cl(int(A))⊆A.
(iii) α-open [10], if A⊆int(cl(int(A))) andα-closed if cl(int(cl(A)))⊆A.

Definition 2.2 A subset A of a topological space X is called

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(i) Generalized closed (briefly g-closed) [4] if cl(A) ⊆U whenever A⊆U and U is open in X.
(ii) Generalized α-closed (briefly gα-closed) [7] if α-cl(A) ⊆U whenever A⊆U and U is α-open in X.
(iii) Weakly generalized closed (briefly wg-closed) [9] if cl(int(A)) ⊆U whenever A⊆U and U is open in X.
(iv) Strongly generalized closed (briefly g*-closed) [15] if cl(A) ⊆U whenever A⊆U and U is g-open in X.
(v) Weakly closed (briefly w-closed) [15] if cl(A) ⊆U whenever A⊆U and U is semi-open in X.
(vi) Mildly generalized closed (briefly mildly g-closed) [12] if cl(int(A)) ⊆U whenever A⊆U and U is g-open in X.
(vii) Regular weakly generalized closed (briefly rwg-closed) [9] if cl(int(A)) ⊆U whenever A⊆U and U is regular open in X.
(viii) Regular weakly closed (briefly rw-closed)[2] if cl(A)⊆U whenever A⊆U and U is regular semi open in X.
(ix) Regular generalized closed (briefly rg-closed) [11] if cl(A)⊆U whenever A⊆U and U is regular open set in X.
The complements of above all closed sets are their respective open sets in the same topological space X.

Definition 2.3: A subset A of a space X is said to be Regular Mildly Generalized closed (briefly RMG-closed) set [17], if cl(int(A))⊆U whenever A⊆U and U is rg-open set in X.
Definition 2.4: A subset A of X is called Regular Mildly Generalized open (briefly RMG-open) set [18], if X-A is RMG-closed set in X.
Definition 2.5: For a subset A of a space X, RMG-cl(A)= ∩{F:A⊆F and F is RMG-closed set in X}is called RMG-closure of A [18].
Definition 2.6: Let A is a subset of X. A point x ∈ A is said to be RMG-interior point of A, if A is a RMG-nhd of x. The set of all RMG-interior [18] of A and is denoted by RMG-int(A).
Definition 2.7: For a subset A of X, RMG-closure[18] of A is defined as RMG-cl(A) to be the intersection of all RMG-closed sets containing A.
Definition 2.8: Let X be any topological space and let x∈X. A subset N is said to be RMG-nbd[18] of x, if and only if there exists a RMG-open set G such that x∈G⊆**N**.
Definition 2.9: A subset N of X is a RMG-nbd[18] of A⊆X in topological space (X, τ), if there exists a RMG-open set G such that A⊆X⊆**N**.
Definition 2.10: A function f: X→Y is said to be RMG-continuous function [19], if f^{-1}(V) is RMG-closed set of X for every closed set V of Y.
Definition 2.11 A function f: X→Y is called RMG-irresolute [19], if f^{-1}(V) is RMG-closed set in X for every RMG-closed subset V of Y.
Definition 2.12A function f: X→Yis said to be
(i) rg-irresolute [11], if f^{-1}(V) is rg-open set in X for every rg-open set V of Y.
(ii) strongly RMG-continuous [19], if f^{-1}(V) is open set in X for every RMG-open set V of Y.

Definition 2.13 A function f: X→Y is called
(i) regular closed [5] if f(F) is closed in Y for every regular closed set F of X.
(ii) g-closed [6] if f(F) is g-closed in Y for every closed set F of X.
(iii) w-closed [15] if f(F) is w-closed in Y for every closed set F of X.
(iv) pre-closed [3] if f(F) is pre-closed in Y for every closed set F of X.
(v) wg^*(=mildly-g)-closed [13] if f(F) wg^*-closed in Y for every closed set F of X.
(vi) wg-closed [9] if f(F) is wg-closed in Y for every closed set F of X.
(vii) rwg-closed [9] if f(F) is rwg-closed in Y for every closed set F of X.
(viii) rw-closed [2] if f(F) is rw-closed in Y for every closed set F of X.
(ix) rg-closed [1] if f(F) is rg-closed in Y for every closed set F of X.
(x) g^*-closed [14] if f(F) is g^*-closed in Y for every closed set F of X.

Definition 2.14: A function f: X→Y is called
(i) regular open[5] if f(F) is open in Y for every regular open set F of X.
(ii) g-open [6] if f(F) is g-open in Y for every open set F of X.
(iii) w-open [15] if f(F) is w-open in Y for every open set F of X.
(iv) pre-open [3] if f(F) is pre-open in Y for every open set F of X.
(v) wg^*-closed[13] if f(F) wg^*-open in Y for every open set F of X.
(vi) wg-open [9] if f(F) is wg-open in Y for every open set F of X.
(vii) rwg-open [9] if f(F) is rwg-open in Y for every open set F of X.
(viii) rw-open [2] if f(F) is rw-open in Y for every open set F of X.

(ix) rg-open [1] if f(F) is rg-open in Y for every open set F of X.
(x) g*-open [14] if f(F) is g*-open in Y for every open set F of X.

Lemma 2.15: Let X be any topological space, in which
(i) Every closed (resp. w-closed, gα-closed, pre-closed) set is RMG-closed set in X [17].
(ii) Every RMG-closed set is mildly-g-closed (resp. wg-closed, rwg-closed) set in X [17].

Definition 2.16: A topological space (X, τ) is called
(i) T_{RMG}-space [19] if every RMG-closed set is closed.
(ii) $T_{½}$-space [6] if every g-closed set is closed.

Definition 2.17: A function f: X→Y is called
(i) closed if f(F) is closed in Y for every closed set F of X.
(ii) gα-closed if f(F) is gα-close in Y for every closed set F of X.

Definition 2.18: A function f: X→Y is called
(i) open if f(F) is open in Y for every open set F of X.
(ii) gα-open if f(F) is gα-open in Y for every open set F of X.

Regula Mildly Generalized Closed Maps In Topological Spaces:-

Definition 3.1: A map f:X→Y is said to be Regular Mildly Generalized closed (briefly, RMG-closed) map, if the image of every closed set in X is RMG-closed in Y.

Theorem 3.2: Every closed map is RMG-closed map but not conversely.

Proof: Let f: X→Yis closed map. Let F be any closed set in X. Then f(X) is closed but every closed set is RMG-closed set [17]. Hence f is RMG-closed map.

Example 3.3: Let X={p, q, r} with topology τ= {X, ∅, {p}, {p, q}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=c, f(q)=d and f(r)=d. Then f is RMG-closed map but not closed, since the image of closed set {r} in X is {a}, which is not closed in Y.

Theorem 3.4: Every w-closed map is RMG-closed map but not conversely.

Proof: Let f:X→Yis w-closed map. Let F be any closed set in X. Then f(X) is w-closed but every w-closed set is RMG-closed set [17]. Hence f is RMG-closed map.

Example 3.5: Let X={m, n, o} with topology τ= {X, ∅, {m, n}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c},{a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(m)=a, f(n)=c and f(o)=b. Then f is RMG-closed map but not w-closed, since the image of closed set {o} in X is {b}, which is not w-closed in Y.

Theorem 3.6: Every gα-closed map is RMG-closed map but not conversely.

Proof: Let f:X→Yis gα-closed map. Let F be any closed set in X. Then f(X) is gα-closed but every gα-closed set is RMG-closed set [17]. Hence f is RMG-closed map.

Example 3.7: Let X={p, q} with topology τ= {X, ∅, {p}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=b and f(q)=a. Then f is RMG-closed map but not gα-closed, since the image of closed set {q} in X is {a}, which is not gα-closed in Y.

Theorem 3.8: Every pre-closed map is RMG-closed map but not conversely.

Proof: Let f:X→Yis pre-closed map. Let F be any closed set in X. Then f(X) is pre-closed but every pre-closed set is RMG-closed set [17]. Hence f is RMG-closed map.

Example 3.9: Let X={x, y. z} with topology τ= {X, ∅, {x},{x, z}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(x)=p, f(y)=b, and f(z)=a. Then f is RMG-closed map but not pre-closed, since the image of closed set {y} in X is {b}, which is not pre-closed in Y.

Theorem 3.10: Every RMG-closed map is mildly- g-closed map but not conversely.

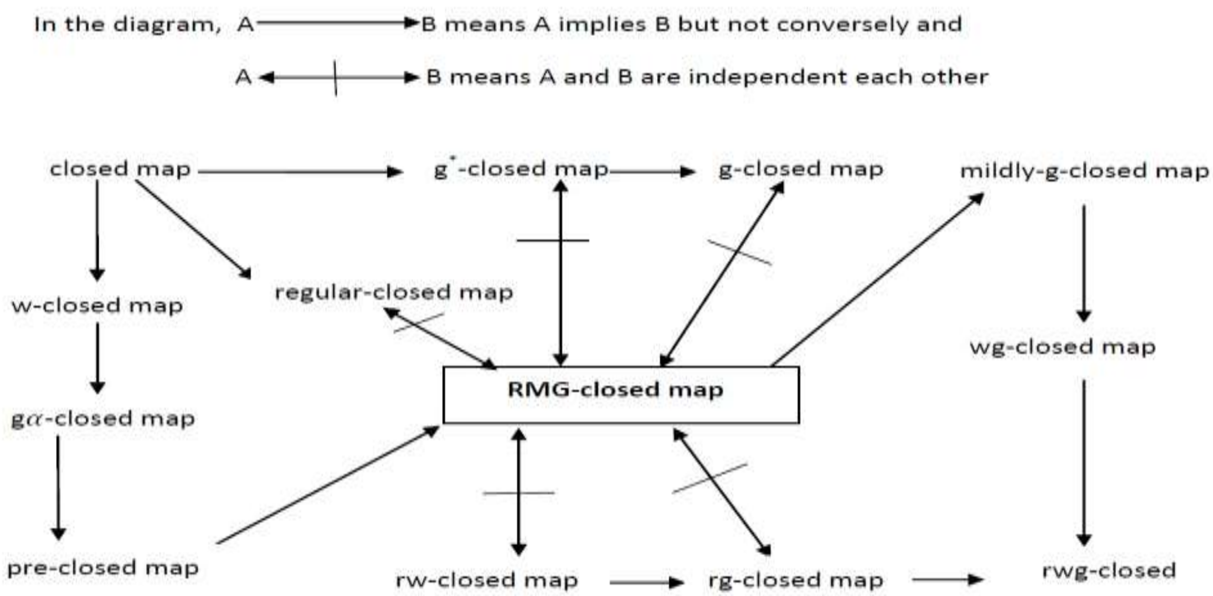
Proof: Let f:X→Yis RMG-closed map. Let F be any closed set in X. Then f(X) is RMG-closed but every RMG-closed set is mildly-g-closed set[17]. Hence f is mildly-g-closed map.

Example 3.11: Let X={a, b, c} with topology τ= {X, ∅, {b, c}} and Y = {a, b, c, d} with topology σ= {∅, {a}, {b}, {a, b}, {a, b, c}, Y}. Let a map f: (X, τ) →(Y, σ) be defined as f(a)=a, f(b)=c and f(c)=d. Then f is mildly-g-closed map but not RMG-closed, since the image of closed set {a} in X is {a}, which is not RMG-closed in Y.

Theorem 3.12: Every RMG-closed map is wg-closed map but not conversely.

Proof: Let f:X→Yis RMG-closed map. Let F be any closed set in X. Then f(X) is RMG-closed but every RMG-closed set is wg-closed set[17]. Hence f is wg-closed map.

Example 3.13: Let X={p, q, r} with topology τ= {X, ∅, {p}, {q, r}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=b, f(q)=d and f(r)=a. Then f is wg-closed map but not RMG-closed, since the image of closed set {q, r} in X is {a, d}, which is not RMG-closed in Y.
Theorem 3.14: Every RMG-closed map is rwg-closed map but not conversely.
Proof: Let f:X→Yis RMG-closed map. Let F be any closed set in X. Then f(X) is RMG-closed but every RMG-closed set is rwg-closed set[17]. Hence f is rwg-closed map.
Example 3.15: Let X={x, y, z} with topology τ= {X, ∅, {x}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(x)=d, f(y)=a and f(z)=b. Then f is rwg-closed map but not RMG-closed, since the image of closed set {y, z} in X is {a, b}, which is not RMG-closed in Y.
Remark 3.16: The regular closed map and RMG-closed maps are independent. This can be seen from following example.
Example 3.17: Let X={a, b, c} with topology τ= {X, ∅, {a, b}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(a)=c, f(b)=a and f(c)=b. Then f is regular closed map but not RMG-closed, since the image of closed set {c} in X is {b}, which is not RMG-closed in Y.
Example 3.18: let X= {a, b, c} with topology τ= {X, ∅, {a}, {b}, {a, b}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be a defined by f(a)=a, f(b)=c, and f(c)=d. Then f is RMG-closed map but not regular closed map, since the image of regular closed set {a, c} in X is {b, d}, which is not closed set in Y.
Remark 3.19: The following example show that g-closed maps and RMG-closed maps are independent.
Example 3.20: Let X={p, q, r} with topology τ= {X, ∅, {p, q}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=d, f(q)=b and f(r)=c. Then f is RMG-closed map but not g-closed, since the image of closed set {r} in X is {c}, which is not g-closed in Y.
Example 3.21: Let X={p, q, r} with topology τ= {X, ∅, {p}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined by f(p)=c, f(q)=b and f(r)=d. Then f is g-closed map but not RMG-closed, since the image of closed set {q, r} in X is {b, d}, which is not RMG-closed in Y.
Remark 3.22: The following example show that g^*-closed maps and RMG-closed maps are independent.
Example 3.23: Let X={p, q} with topology τ= {X, ∅, {p}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=b, and f(q)=c. Then f is RMG-closed map but not g^*-closed, as the image of closed set {q} in X is {c}, which is not g^*-closed in Y.
Example 3.24: Let X={x, y, z} with topology τ= {X, ∅, {x}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(x)=b, f(y)=a and f(z)=d. Then f is g^*-closed map but not RMG-closed, since the image of closed set {y, z} in X is {a, d}, which is not RMG-closed in Y.
Remark 3.25: The following example show that rw-closed maps and RMG-closed maps are independent.
Example 3.26: Let X={a, b, c} with topology τ= {X, ∅, {a}, {a, c}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined by f(a)=b, f(b)=c and f(c)=d. Then f is RMG-closed map but not rw-closed, as the image of closed set {b} in X is {c}, which is not rw-closed in Y.
Example 3.27: Let X={a, b, c} with topology τ= {X, ∅, {a}, {a, c}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b},{a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined by f(a)=b, f(b)=c and f(c)=a. Then f is rw-closed map but not RMG-closed, as the image of closed set {b} in X is {c}, which is not rw-closed in Y.
Remark 3.28: The following example show that rg-closed maps and RMG-closed maps are independent.
Example 3.29: Let X={x, y, z} with topology τ= {X, ∅, {y, z}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined by f(x)=b, f(y)=a and f(z)=d. Then f is RMG-closed map but not rg-closed, as the image of closed set {x} in X is {b}, which is not rg-closed in Y.
Example 3.30: Let X={x, y, z} with topology τ= {X, ∅, {y}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined by f(x)=b, f(y)=d and f(z)=a. Then f is rg-closed map but not RMG-closed, as the image of closed set {x, z} in X is {a, b}, which is not RMG-closed in Y.
Remark 3.31: From the above discussion and known results we have the following implications.



Remark 3.32: The composition of two RMG-closed maps need not be RMG-closed map in general. This can be shown by the following example.

Example 3.33: Let X={p, q, r} with topology τ= {X, ∅, {p}, {p, r}}, Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}} and Z ={x, y, z, w} with topology η={Z, ∅, {x}, {y}, {x, y}, {y, z}, {x, y, z}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=c, f(q)=b and f(r)=d and g:(Y, σ) →(Z, η) defined by g(a)=z, g(b)=x, g(c)=y and g(d)=w. Then f and g are two RMG-closed maps but their composition g∘f: (X, τ) →(Z, η)is not RMG-closed map because F={q} is closed in X, but(g∘f)(F)=g(f({q}))=g({b})={x}, which is not RMG-closed in Z.

Theorem 3.34: If f: X→Y is closed map and g: Y→Z is RMG-closed map, then the g∘f: X→Z is RMG-closed map.

Proof: Let F be any closed set in X. Since f is closed map, f(F) is closed set in Y. Since g is RMG-closed map, g(f(F))=(g∘f)(F) is RMG-closed set in Z. Hence g∘f is RMG-closed map.

Remark 3.35: If f: X→Y is RMG-closed map and g: Y→Z is closed map, then the composition need not be RMG-closed map. This can be seen from following example.

Example 3.36: Let X={a, b, c} with topology τ= {X, ∅, {a, b}}, Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}} and Z ={a, b, c, d} with topology η={Z, ∅, {a}, {b}, {a, b}, {a, b. c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(a)=c, f(b)=d and f(c)=b and g:(Y, σ) →(Z, η) defined by g(a)=c, g(b)=b, g(c)=c and g(d)=d. Then f is RMG-closed map and g is a closed map but their composition g∘f: (X, τ) → (Z, η) is not RMG-closed map because F={c} is closed in X, but (g∘f)(F)=g(f({c}))=g({b})={b}, which is not RMG-closed in Z.

Theorem 3.37: If f: X→Y and g: Y→Z are two RMG-closed maps and Y be a T_{RMG}-space then g∘f: X→Z is RMG-closed map.

Proof: Let A be a closed set of X. Since f is RMG-closed map, f(A) is RMG-closed in Y. Then by hypothesis, f(A) is closed. Since g is RMG-closed, g(f(A)) is RMG-closed in Z and g(f(A))=(g∘f)(A). Therefore g∘f is RMG-closed map.

Theorem 3.38: If f: X→Y is g-closed map and g: Y→Z is RMG-closed maps and Y be $T_{1/2}$-space then g∘f: X→Z is RMG-closed map.

Proof: Let A be a closed set of X. Since f is g-closed, f(A) is g-closed in Y. Since Y is $T_{1/2}$, f(A) is closed in Y. Since g is RMG-closed, g(f(A)) is RMG-closed in Z and g(f(A))=(g∘f)(A). Therefore g∘f is RMG-closed map.

Theorem 3.39: Composition of closed maps is RMG-closed map.

Proof: Proof is straight forward and fact that every closed set is RMG-closed set.

Theorem 3.40: A map f:X→Y is said to be RMG-closed map if and only if for each subset A of Y and for each open set U containing f^{-1}(A), there is a RMG-open set V of Y such that A⊆V and f^{-1}(V)⊆U.

Proof: Suppose f is RMG-closed map. Let A is a subset of Y and U is a open set of X such that f^{-1}(A)⊆U. Now X-U is a closed set in X. Since f is RMG-closed map, f(X-U) is a RMG-closed set in Y i.e. V=Y-f(X-U), V=Y-f(X-U) is RMG-open set of Y. Note that f^{-1}(A)⊆U implies that A⊆V and f^{-1}(V)=X-f^{-1}(f(X-U))=X-(X-U)=U i.e.f^{-1}(V)⊆U.

Conversely, suppose that F is a closed set in X. Then f^{-1}(f(X-F))⊆X-F and X-F is open in X. By the hypothesis, there exists a RMG-open set V in Y such that Y-f(F)⊆V and f^{-1}(V)⊆X-F. Therefore, F⊆X-f^{-1}(V). Hence Y-V⊆f(F)⊆f(X-f^{-1}(V))⊆Y-V which implies f(F)⊆V. Since Y-V is RMG-closed, f(F) is RMG-closed. Therefore f(F) is RMG-closed in Y. Hence f is RMG-closed map.

Theorem 3.41: If f:X→Y is g-closed map and Y is a $T_{1/2}$-space, then f:X→Y is RMG-closed map.

Proof: Let F be a closed set in X. Since f is g-closed map, f(F) is g-closed set in Y. As Y is a $T_{1/2}$-space, we have f(F) is closed in Y. As every closed set is RMG-closed, f(F) is a RMG-closed in Y. Thus f is a RMG-closed map.

Theorem 3.42: If f: X→Y is RMG-closed map, then RMG-cl(f(A))⊆f(cl(A)) for every subset A of X.

Proof: Suppose that f is RMG-closed and A⊆X. Then cl(A) is closed in X and so f(cl(A)) is RMG-closed in Y. We have f(A)⊆f(cl(A)) and by Theorem 6.2[18], RMG-cl(f(A))⊆RMG-cl(f(cl(A)))…(i). Since f(cl(A)) is RMG-closed set in Y, RMG-cl(f(cl(A)))=f(cl(A)) …(ii), by Theorem 6.3[18]. From (i) and (ii), RMG-cl(f(A))⊆f(cl(A)) for every subset A of X.

Corollary 3.43: If f: X→Y is a RMG-closed then the image f(A) of closed set A in X is τ_{RMG}-closed in Y.

Proof: Let A be a closed set in X. Since f is RMG-closed, by Theorem 3.42, RMG-cl(f(A))⊆f(cl(A))…(i). Also cl(A)=A as A is a closed set and so f(cl(A))=f(A)…(ii). From (i) and (ii), RMG-cl(f(A))⊆f(A). We know that f(A)⊆RMG-cl(A) and so RMG-cl(f(A))=f(A) . Therefore f(A) is τ_{RMG}-closed in Y.

Theorem 3.44: Let X and Y are two topological spaces where 'RMG-cl(A)=pcl(A) for every subset A of Y' and f:X→Y be map, then the following are equivalent;

(i) f is RMG-closed map.
(ii) RMG-cl(f(A))⊆f(cl(A)) for every subset A of X.

Proof: (i)⇒(ii) follows from the Theorem 3.42.

(ii)⇒(i), let A be any closed set of X then A=cl(A) and so RMG-cl(f(A))⊆f(cl(A))=f(A), by hypothesis. We have f(A)⊆RMG-cl(A) by Theorem 6.2[18]. Therefore f(A)=RMG-cl(f(A)). Also f(A)=RMG-cl(f(A))=pcl(f(A)), by hypothesis. i.e. f(A)=pcl(f(A)) and so f(A) is pre-closed set in Y. Thus f(A) is RMG-closed set in Y. Hence f is RMG-closed map.

Theorem 3.45: Let $f_i: (X_i, \tau_i) \rightarrow (X_{i+1}, \tau_{i+1})$ be a map, then following are true;

(i) If f_1,f_2,f_3,…,f_n are closed maps then their compositions $f_n \circ f_{n-1} \circ f_{n-2} \circ \ldots \circ f_1$ is RMG-closed map.
(ii) If f_1,f_2,f_3,…,f_{n-1} are closed maps and f_n is a RMG-closed map then the compositions $f_n \circ f_{n-1} \circ f_{n-2} \circ \ldots \circ f_1$ is RMG-closed map.

Proof: (i) The proof follows from the Theorem 3.39 and fact that every closed set is RMG-closed set.

(ii) The proof follows from the Theorem 3.34.

Theorem 3.46: If f: X→Y and g: Y→Z are two mappings such that their composition g∘f: X→Z is RMG-closed map, then the following statements are true;

(i) If f is continuous and surjective, then g is RMG-closed map.
(ii) If g is RMG-irresolute and injective, then f is RMG-closed map.
(iii) If f is g-continuous, surjective and X is a $T_{1/2}$-space, then g is RMG-closed map.
(iv) If g is strongly RMG-continuous and injective, then f is RMG-closed map.

Proof: (i) Let A be a closed set of X. Since f is continuous, f^{-1}(A) is closed in X and since g∘f is RMG-closed, (g∘f)(f^{-1}(A)) is RMG-closed in Z. i.e. g(A) is RMG-closed in Z, since f is surjective. Therefore g is RMG-closed map.

(ii) Let B be a closed set of X. Since g∘f is RMG-closed, (g∘f)(B) is RMG-closed in Z. Since g is RMG-irresolute, g^{-1}((g∘f)(B)) is RMG-closed set in Y implies that f(B) is RMG-closed in Y, since f is injective. Therefore f is RMG-closed map.

(iii) Let C be a closed set of Y. Since f is g-continuous, f^{-1}(C) is g-closed set in X. Since X is a $T_{1/2}$-space, f^{-1}(C) is RMG-closed set in X. Since g∘f is RMG-closed, (g∘f)(f^{-1}(C)) is RMG-closed in Z implies g(C) is RMG-closed in Z, since f is surjective. Therefore g is RMG-closed map.

(iv) Let D be a closed set of X. Since (g∘f)(D) is RMG-closed in Z. Since g is strongly RMG-continuous, g^{-1}((g∘f)(D)) is closed set in Y implies f(D) is closed set in Y, since g is injective. Therefore f is closed map.

Theorem 3.47: If f: X→Y is rg-irresolute, RMG-closed and A is a RMG-closed subset of X, then f(A) is a RMG-closed set in Y.

Proof: Let f(A)⊆G, where G is a rg open in Y. Since f is rg-irresolute, f^{-1}(G) is rg-open in X by definition 2.12 and A⊆ f^{-1}(G). Since A is a RMG-closed set in X, cl(int(A))⊆ f^{-1}(G)[19]. Since f is RMG-closed, f(cl(int(A))) is RMG-closed set contained in rg-open set G implies that cl(int(f(cl(int(A)))))⊆f(cl(int(A))⊆G and so cl(int(f(A)))⊆G. Hence f(A) is RMG-closed set in Y.

Corollary 3.48: If f: X→Y be a RMG-closed map and g: Y→Z be RMG-closed and rg-irresolute map, then their composition $g \circ f: X \to Z$ is RMG-closed map.
Proof: Let A be a closed set of X. Since f is a RMG-closed map, f(A) is RMG-closed in Y. Since g is both RMG-closed and rg-irresolute, g(f(A)) is RMG-closed in Z by Theorem 3.47. Also g(f(A))=(g∘f)(A). Therefore g∘f is RMG-closed map.
Theorem 3.49: If f: X→Y is an open, continuous, RMG-closed, surjection and cl(int(F))=F for every RMG-closed set in Y, where X is regular, then Y is regular.
Proof: Let U be an open set in Y and p∈U. Since f is surjection, there exists a point x∈X such that f(x)=p. Since X is regular and f is continuous, there is an open set V in X such that x∈V⊆cl(int(V)) $\subseteq f^{-1}$(U). Here p∈f(V)⊆f(cl(int(V)))⊆U…(i). Since f is RMG-closed, f(cl(int(V))) is a RMG-closed set contained in the open set U. By hypothesis cl(int(f(cl(int(A)))))=f(cl(int(V))) and cl(int(f(V)))⊆cl(int(f(cl(int(V)))))…(ii). From (i) and (ii), p∈f(V)⊆f(cl(int(V)))⊆U and f(V) is open, since f is open. Hence Y is regular.
Theorem 3.50: If f: X→Y is RMG-closed and A is closed set of X, then its restriction f_A:(A, τ_A) →Y is RMG-closed map.
Proof: Let F be a closed set of A. Then F=A∩E for some closed set E of X and so F is closed set of X. Since f is RMG-closed, f(F) is RMG-closed set in Y. But f(F)=f_A(F). Therefore f_A:(A, τ_A) →Y is RMG-closed map.
Now we define the new class of stronger form of RMG-closed maps is called RMG*-closed maps in topological spaces.
Definition 3.51: A map f: X→Y is said to be **RMG*-closed maps**, if the image of every RMG-closed set of X is RMG-closed set in Y.
Theorem 3.52: If f: X→Y is RMG*-closed map, then which is RMG-closed map, but not conversely.
Proof: Let F be a closed set in X and the fact that every closed set is RMG-closed set. Hence F is RMG-closed set in X. Since f: X→Y be a RMG*-closed map, f(F) is RMG-closed set in Y. Therefore f is RMG-closed map.
Example 3.53: Let X= {a, b, c, d} with topology τ= {X, ∅, {a}, {b, c}, {a, b, c}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(a)=c, f(b)=a, f(c)=d and f(d)=b. Then f is RMG-closed map but not RMG*-closed, since the image of RMG-closed set {b, d} in X is {a, b}, which is not RMG-closed in Y.
Theorem 3.54: If f: X→Y and g: Y→Z are two RMG*-closed maps, then their composition g∘f: X→Z is RMG*-closed map.
Proof: Let F be a RMG-closed set in X. Since f is RMG*-closed map, f(F) is RMG-closed set in Y. Since g is RMG*-closed map, g(f(F)) is RMG-closed set in Z. Hence g∘f is RMG*-closed map.
Theorem 3.56: If f: X→Y is irresolute and RMG-closed map then f is RMG*-closed map.
Theorem 3.57: If f: X→Y be a closed map and g: Y→Z be RMG*-closed, then their composition g∘f: X→Z is RMG-closed map.
Proof: Let F be a closed set in X. Then f(F) is closed in Y. The fact that every closed set is RMG-closed set implies that f(F) is RMG-closed set in Y. Since g is RMG*-closed map, g(f(F))=(g∘f)(F) is RMG-closed set in Z. Hence g∘f is RMG-closed map.
Theorem 3.58: If f: X→Y be a RMG-closed map and g: Y→Z be RMG*-closed, then their composition g∘f: X→Z is RMG-closed map.
Proof: Let F be a closed set in X. Since f is RMG-closed map, f(F) is RMG-closed set in Y. Since g is RMG*-closed map, g(f(F))= (g∘f)(F) is RMG-closed set in Z. Hence g∘f is RMG-closed map.

Regular Mildly Generalized Open Maps In Topological Spaces:-
Definition 4.1: A map f: X→Y is said to be Regular Mildly Generalized open (briefly, RMG-open) map, if the image of every open set in X is RMG-open in Y.
From the definition 4.1 we have following results;
Theorem 4.2: (i) Every open map is RMG-open map, but not conversely.
(ii) Every w-open map is RMG-open map, but not conversely.
(iii) Every gα-open map is RMG-open map, but not conversely.
(iv) Every pre-open map is RMG-open map, but not conversely.
(v) Every RMG-open map is mildly-g-open map, but not conversely
(vi) Every RMG-open map is wg-open map, but not conversely.
(vii) Every RMG-open map is rwg-open map, but not conversely.
Proof: Proofs follow from Definition 4.1 and fact that lemma 2.14.

Example 4.3: Let X={s, t, r} with topology τ= {X, ∅, {s}, {s, r}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(s)=b, f(t)=c and f(r)=a. Then f is RMG-open map but not open, since the image of open set {s} in X is {b}, which is not open in Y.
Example 4.4: Let X={p, q, } with topology τ= {X ∅, {p}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=c, and f(q)=a. Then f is RMG-open map but not w-open, since the image of open set {p} in X is {c}, which is not w-open in Y.
Example 4.5:Let X={x, y, z} with topology τ= {X, ∅, {z}, {x, z}} and Y= {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(x)=b, f(y)=d and f(z)=c. Then f is RMG-open map but not gα-open, since the image of open set {z} in X is {c}, which is not gα-open in Y.
Example 4.6: Let X={m, n, o} with topology τ= {X, ∅, {m, n}} and Y= {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(m)=a, f(n)=c and f(o)=b. Then f is RMG-open map but not pre-open, since the image of open set {m, n} in X is {a, c}, which is not pre-open in Y.
Example 4.7: Let X={p, q, r} with topology τ= {X, ∅, {p}, {q}, {p, q}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=b, f(q)=c and f(r)=a. Then f is mildly-g-open map but not RMG-open, since the image of open set {p, q} in X is {b, c}, which is not RMG-open in Y.
Example 4.8: Let X={x, y, z} with topology τ= {X, ∅, {x, y}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b}, {a, b}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(x)=a, f(y)= c and f(z)=b. Then f is wg-open map but not RMG-open, since the image of open set {x, y} in X is {a, c}, which is not RMG-open in Y.
Example 4.9: Let X={m, n, o} with topology τ= {X, ∅, {m}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(m)=d, f(n)=a and f(o)=b. Then f is rwg-open map but not RMG-open, since the image of open set {m} in X is {d}, which is not RMG-open in Y
Theorem 4.10: If f: X→Y is RMG-open, then f(int(A))⊆RMG-int(f(A)) for every subset A of X.
Proof: Let f: X→Yis an open map and A is any subset of X. Then int(A) is open in X and so f(int(A)) is RMG-open set in Y. We have f(int(A))⊆f(A). Therefore by the Theorem 5.8 [18], f(int(A))⊆RMG-int(f(A)).
Theorem 4.11: A map f: X→Y be RMG-open if and only if for any subset S of Y and any closed set of X containing f^{-1}(S), there exists a RMG-closed set T of Y containing S such that f^{-1}(T)⊆F.
Proof: Suppose f: X→Yis RMG-open map. Let S⊆Y and F be a closed set of X such that f^{-1}(S)⊆F. Now X-F is an open set in X. Since f is RMG-open map, f(X-F) is RMG-open set in Y. Then T=Y-f(X-F) is a RMG-closed set in Y. Note that f^{-1}(S)⊆F implies S⊆T and f^{-1}(T)=X-f^{-1}(X-F)⊆X-(X-F)=F. i.e. f^{-1}(T)⊆F.
Conversely, suppose U be an open set of X. Then (Y-f(U))⊆X-U is a closed set in X. By hypothesis, there exists a RMG-closed set T of Y such that Y-f(U)⊆T and f^{-1}(T)⊆X-U and so U⊆X-f^{-1}(T). Hence Y-T⊆f(U) ⊆Y-f(f^{-1}(T)⊆Y-T which implies f(U)=Y-T. Since Y-T is a RMG-open, f(U) is RMG-open in Y and therefore f is RMG-open map.
Theorem 4.12: If f: X→Y is RMG-open, then f^{-1}(RMG-cl(A)) ⊆cl(f^{-1}(A)) for each subset A of Y.
Proof: Let f: X→Yis a RMG-open map and A be any subset of Y. Then f^{-1}(A)⊆cl(f^{-1}(A)) and cl(f^{-1}(A)) is closed set in X. Then by above Theorem 4.11, there exists a RMG-closed set B of Y such that A⊆B and f^{-1}(B)⊆cl(f^{-1}(A)). Now RMG-cl(A)⊆RMG-cl(B)=B, by Theorem 6.2(ii) and 6.3[18], as B is RMG-closed set of Y. Therefore f^{-1}(RMG-cl(A))⊆ f^{-1}(B) and so f^{-1}(RMG-cl(A))⊆ f^{-1}(B)⊆ cl(f^{-1}(A)). Thus f^{-1}(RMG-cl(A))⊆cl(f^{-1}(A)) for each subset of A of Y.
Theorem 4.13: If f: X→Y is RMG-open, then for each neighbourhood U of x in X, there exists a RMG-neighbourhood **N** of f(x) in Y such that **N**⊆f(U).
Proof: Let f: X→Yis a RMG-open map. Let x∈X and U be an arbitrary neighbourhood of x in X. Then there exists an open set G in X such that x∈G⊆U. Now f(x)∈f(G)⊆f(U) and f(G) is RMG-open set in Y, as f is RMG-open map. By Theorem 4.10 [18] f(G) is RMG-neighbourhood of each of its points. By taking f(G)=**N**, **N**is a RMG-nbd of f(x) in Y such that **N**⊆f(U).
Theorem 4.14: For any bijection map f: X→Y, the following statements are equivalent;
(i) f^{-1}: Y→X is RMG-continuous.
(ii) F is RMG-open map.
(iii) F is RMG-closed map.
Proof: (i)⟹(ii), let U be an open set of X. By assumption, $(f^{-1})^{-1}$(U)=f(U) is RMG-open in Y and so f is RMG-open.
(ii)⟹(iii), let F be a closed set of X, X-F is open set in X. By assumption, f(X-F) is RMG-open in Y i.e. f(X-F) is RMG-open set in Y, since every open set is RMG-open Corollary 3.6(3)[18] and therefore f(F) is RMG-closed in Y. Hence f is RMG-closed map.

(iii)⟹(i), let F be a closed set of X. By the assumption, f(F) is RMG-closed in Y. But f(F)=$(f^{-1})^{-1}$(F) and therefore f^{-1} is continuous.

Remark 4.15: The composition of two RMG-open maps need not be a RMG-open map.

Now we define the new class of stronger form of RMG-open maps is called RMG*-open maps in topological spaces.

Definition 4.16: A map f: X→Y is said to be **RMG*-open map**, if the image f(A) is RMG-open set in Y for every RMG-open set A in X.

Remark 4.17: Since every open set is a RMG-open set, we have every RMG*-open map is RMG-open map. The converse is not true generally as seen from the following example.

Example 4.18: Let X={p, q, r} with topology τ= {X, ∅, {q}, {q, r}} and Y = {a, b, c, d} with topology σ= {Y, ∅, {a}, {b, c}, {a, b, c}}. Let a map f: (X, τ) →(Y, σ) be defined as f(p)=b, f(q)=a and f(r)=d. Then f is RMG-open map but not RMG*-open, since the image of RMG-open set {q, r} in X is {a, d}, which is not RMG-open in Y. Hence f is not RMG*-open map.

Theorem 4.19: If f: X→Y and g: Y→Z be two RMG*-open maps, then their composition g∘f: X→Z is RMG*-open map.

Proof: Proof is similar to the Theorem 3.54.

Theorem 4.20: For any bijective map f: X→Y, the following statements are equivalent;

(i) f^{-1}: Y→X is RMG-irresolute map.
(ii) f is RMG*-open map.
(iii) f is RMG*-closed map.

Proof: Proof is similar to the Theorem 4.14.

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