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*RESEARCH ARTICLE*

# LINEAR PRIME LABELING OF SOME DIRECT CYCLE RELATED GRAPHS.

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## *Abstract*

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Linear prime labeling of a graph is the labeling of the vertices with {0,1,2---,p-1} and the direct edges with twice the value of the terminal vertex plus value of the initial vertex. The greatest common incidence number of a vertex (**gcin**) of in degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the **gcin** of each vertex of in degree greater than one is one, then the graph admits linear prime labeling. Here we investigated some direct cycle related graphs for linear prime labeling.



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## Introduction:-

All graphs in this paper are finite and direct. The direction of the edge is from $v_i$ to $v_j$ iff $f(v_i) < f(v_j)$. The symbols V(G) and E(G) denote the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2],[3] and [4] . Some basic concepts are taken from Frank Harary [2]. In this paper we investigated linear prime labeling of some direct cycle related graphs.

**Definition:**
Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of in degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

**Definition**
A Graph is said to be a di graph if each edge of G has a direction**.**

**Definition**
In-degree of a vertex in a digraph is the number of edges incident at that vertex.

## Main Results:-

**Definition**
Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection

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f : V(G) → {0,1,2,---, p-1} by f($v_i$) = i-1, for every i from 1 to p and define a 1-1 mapping $f_{lpl}^*$ : E(G) → set of natural numbers N by $f_{lpl}^*(v_i v_j) = f(v_i) + 2f(v_j)$ for every direct edge $v_i v_j$. The induced function $f_{lpl}^*$ is said to admit linear prime labeling, if for each vertex of in degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

**Definition**
A direct graph which admits linear prime labeling is called linear prime graph.

**Theorem**
Cycle $C_n$ (n > 2) admits linear prime labeling.

**Proof:**
Let G = $C_n$ and let $v_1, v_2$,---,$v_n$ are the vertices of G.
Here |V(G)| = n and |E(G)| = n.
Define a function f : V → {0,1,2,---,n-1} by

$$f(v_i) = i-1,\ i = 1,2,---,n$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f_{lpl}^*$ is defined as follows

$f_{lpl}^*(v_{2i-1}v_{2i+1})$ = 6i-2, i = 1,2,---,$\frac{n-2}{2}$, if n is even.
i = 1,2,---,$\frac{n-1}{2}$, if n is odd.

$f_{lpl}^*(v_{2i}v_{2i+2})$ = 6i+1, i = 1,2,---,$\frac{n-2}{2}$, if n is even.
i = 1,2,---,$\frac{n-3}{2}$, if n is odd.

$f_{lpl}^*(v_1 v_2)$ = 2.
$f_{lpl}^*(v_{n-1}v_n)$ = 3n-4.
Clearly $f_{lpl}^*$ is an injection.
***gcin*** of ($v_n$) = gcd of {$f_{lpl}^*(v_{n-2}v_n)$, $f_{lpl}^*(v_{n-1}v_n)$}
= gcd of {3n-5, 3n-4}
= 1.
So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence $C_n$ , admits linear prime labeling.

**Example (a) G = $C_6$.**

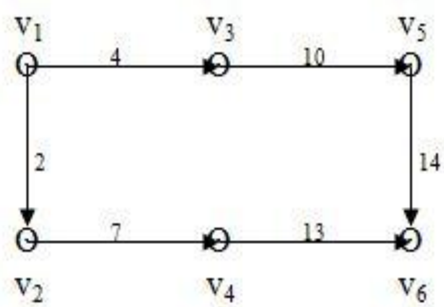


**Fig 2.1a:-**

**Example (b) G = $C_7$.**

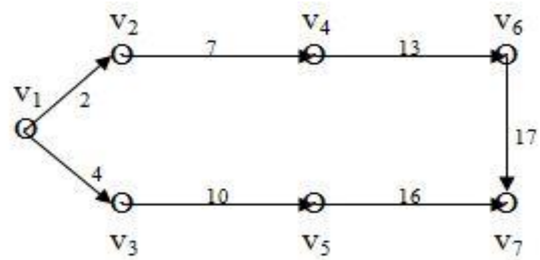


**Fig2.1b**:-

**Theorem**

Direct graph of two copies of cycle $C_n(n > 2)$ sharing a common edge admits linear prime labeling.

**Proof:**

Let G be the graph and let $v_1, v_2, ---, v_{2n-2}$ are the vertices of G.

Here $|V(G)| = 2n-2$ and $|E(G)| = 2n-1$.

Define a function $f : V \rightarrow \{0,1,2,---,2n-3\}$ by

$$f(v_i) = i-1 \text{ , } i = 1,2,---,2n-2$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{lpl}^*$ is defined as follows

$f_{lpl}^*(v_{2i-1}v_{2i+1}) = 6i-2$, $i = 1,2,---,n-2$

$f_{lpl}^*(v_{2i}v_{2i+2}) = 6i+1$, $i = 1,2,---,n-2$

$f_{lpl}^*(v_{n-1}v_n) = 3n-4$.

$f_{lpl}^*(v_{2n-3}v_{2n-2}) = 6n-10$.

Clearly $f_{lpl}^*$ is an injection.

***gcin*** of $(v_n)$ = gcd of $\{f_{lpl}^*(v_{n-2}v_n), f_{lpl}^*(v_{n-1}v_n)\}$
= gcd of $\{3n-5, 3n-4\}$ = 1.

***gcin*** of $(v_{2n-2})$ = gcd of $\{f_{lpl}^*(v_{2n-4}v_{2n-2}), f_{lpl}^*(v_{2n-3}v_{2n-2})\}$
= gcd of $\{6n-11, 6n-10\}$ = 1.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence G , admits linear prime labeling.

**Example (a) G be the direct graph of two copies of cycle $C_6$ sharing a common edge.**

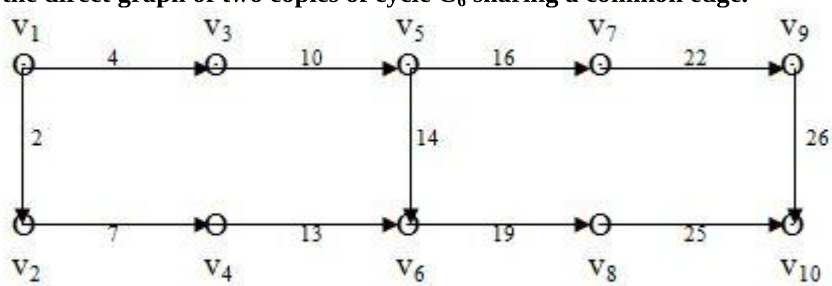


**Fig 2.2a:-**

**Example (b) G be the direct graph of two copies of cycle $C_7$ sharing a common edge.**

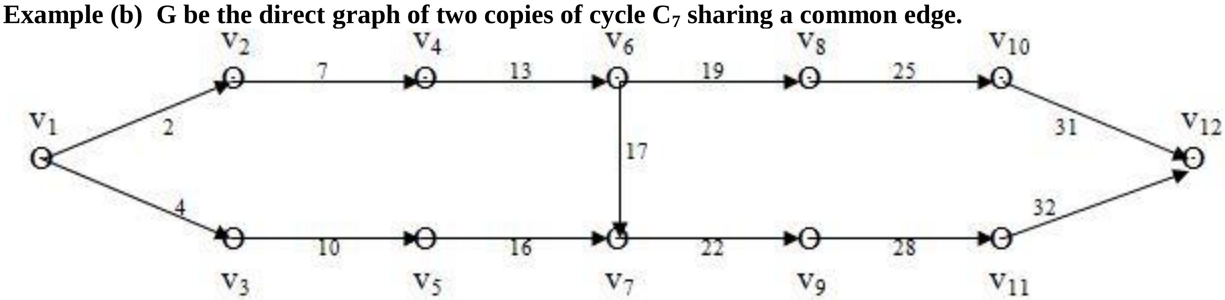


**Fig 2.2b:-**

**Theorem**

Direct graph of two copies of cycle $C_n(n > 2)$ sharing a common vertex admits linear prime labeling.

**Proof:**

Let G be the graph and let $v_1, v_2, ---, v_{2n-1}$ are the vertices of G.

Here $|V(G)| = 2n-1$ and $|E(G)| = 2n$.

Define a function $f : V \rightarrow \{0,1,2,---,2n-2\}$ by

$$f(v_i) = i-1 \text{ , } i = 1,2,---,2n-1.$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{lpl}^*$ is defined as follows

$f_{lpl}^*(v_1v_2)$ = 2.
$f_{lpl}^*(v_{2i-1}v_{2i+1})$ = 6i-2, i = 1,2,---,n-1,if n is odd.
i = 1,2,---,$\frac{n-2}{2}$, if n is even.
$f_{lpl}^*(v_{2i}v_{2i+2})$ = 6i+1, i = 1,2,---,n-2, if n is even
i = 1,2,---,$\frac{n-3}{2}$, if n is odd.
$f_{lpl}^*(v_{n-1}v_n)$ = 3n-4.
$f_{lpl}^*(v_nv_{n+1})$ = 3n-1.
$f_{lpl}^*(v_{n+2i-1}v_{n+2i+1})$ = 3n+6i-2, i = 1,2,---,$\frac{n-3}{2}$,if n is odd.
i = 1,2,---,$\frac{n-2}{2}$, if n is even.
$f_{lpl}^*(v_{2n-2}v_{2n-1})$ = 6n-7.
Clearly $f_{lpl}^*$ is an injection.
***gcin*** of ($v_n$) = gcd of $\{f_{lpl}^*(v_{n-2}v_n), f_{lpl}^*(v_{n-1}v_n)\}$
= gcd of {3n-5, 3n-4} = 1.
***gcin*** of ($v_{2n-1}$) = gcd of $\{f_{lpl}^*(v_{2n-3}v_{2n-1}), f_{lpl}^*(v_{2n-2}v_{2n-1})\}$
= gcd of {6n-8, 6n-7} = 1.
So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence G , admits linear prime labeling.

**Example (a) G be the direct graph of two copies of cycle $C_6$ sharing a common vertex**

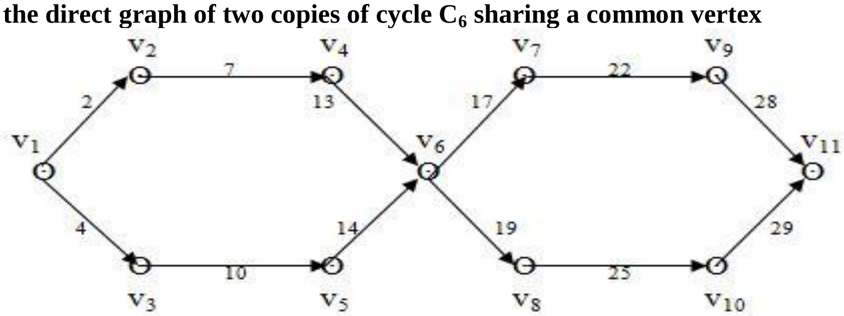


**Fig 2.3a:-**

**Example (b) G be the direct graph of two copies of cycle $C_5$ sharing a common vertex.**

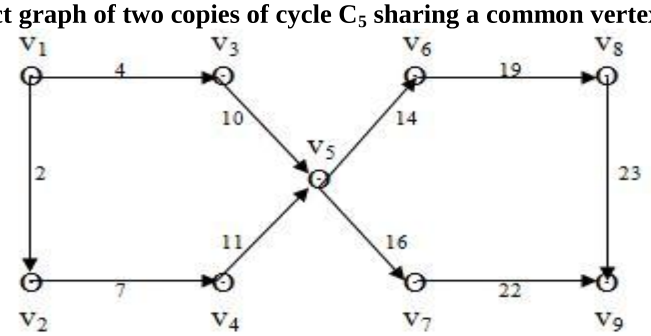


**Fig 2.3b:-**

**Theorem**

Let G be the graph obtained by joining two copies of cycle $C_n(n > 2)$ by an edge. Direct graph of G admits linear prime labeling.

**Proof:**

Let G be the graph and let $v_1, v_2, ---, v_{2n}$ are the vertices of G.
Here |V(G)| = 2n and |E(G)| = 2n+1.
Define a function f : V → {0,1,2,---,2n-1} by
$f(v_i)$ = i-1 , i = 1,2,---,2n.
Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{lpl}^{*}$ is defined as follows

$f_{lpl}^{*}(v_1v_2)$ = 2.

$f_{lpl}^{*}(v_{2i-1}v_{2i+1})$ = 6i-2, i = 1,2,---,$\frac{n-1}{2}$,if n is odd.
i = 1,2,---,$\frac{n-2}{2}$, if n is even.

$f_{lpl}^{*}(v_{2i}v_{2i+2})$ = 6i+1, i = 1,2,---,$\frac{n-3}{2}$, if n is odd
i = 1,2,---,$\frac{n-2}{2}$, if n is even.

$f_{lpl}^{*}(v_{n-1}v_n)$ = 3n-4.

$f_{lpl}^{*}(v_nv_{n+1})$ = 3n-1.

$f_{lpl}^{*}(v_{n+1}v_{n+2})$ = 3n+2.

$f_{lpl}^{*}(v_{n+2i-1}v_{n+2i+1})$ = 3n+6i-2, i = 1,2,---,$\frac{n-1}{2}$,if n is odd.
i = 1,2,---,$\frac{n-2}{2}$, if n is even.

$f_{lpl}^{*}(v_{n+2i}v_{n+2i+2})$ = 3n+6i+1, i = 1,2,---,$\frac{n-3}{2}$,if n is odd.
i = 1,2,---,$\frac{n-2}{2}$, if n is even.

$f_{lpl}^{*}(v_{2n-1}v_{2n})$ = 6n-4.

Clearly $f_{lpl}^{*}$ is an injection.

***gcin*** of ($v_n$) = gcd of {$f_{lpl}^{*}(v_{n-2}v_n)$, $f_{lpl}^{*}(v_{n-1}v_n)$}
= gcd of {3n-5, 3n-4}
= 1.

***gcin*** of ($v_{2n}$) = gcd of {$f_{lpl}^{*}(v_{2n-2}v_{2n})$, $f_{lpl}^{*}(v_{2n-1}v_{2n})$}
= gcd of {6n-5, 6n-4}
= 1.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence G , admits linear prime labeling.

**Example (a) G be the direct graph of two copies of cycle $C_7$ joined by an edge.**

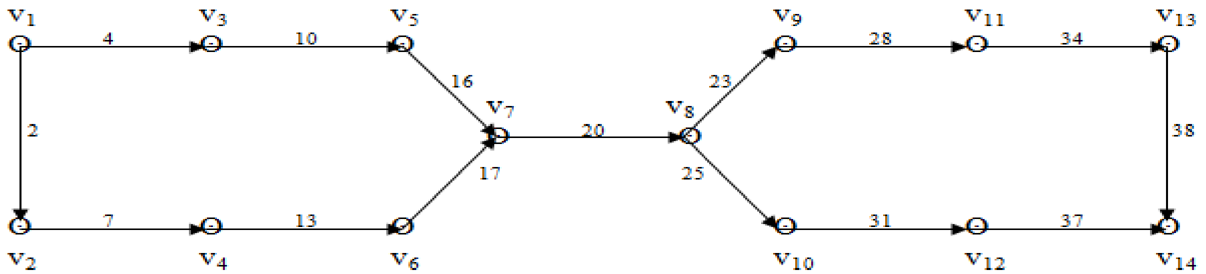


**Fig 2.4a:-**

**Example (b) G be the direct graph of two copies of cycle $C_6$ joined by an edge.**

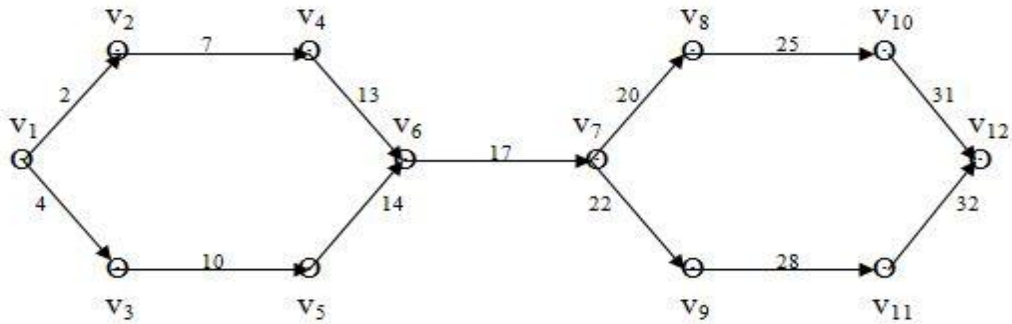


**Fig 2.4b:-**

**Theorem**

Let G be the graph obtained by joining one end vertex of a path $P_m$ to any vertex of a cycle $C_n(n > 2)$. Direct graph of G admits linear prime labeling.

**Proof:**

Let G be the graph and let $v_1, v_2, ---, v_{n+m-1}$ are the vertices of G.
Here $|V(G)| = n+m-1$ and $|E(G)| = n+m-1$.
Define a function $f : V \rightarrow \{0,1,2,---,n+m-2\}$ by

$$f(v_i) = i-1 , \quad i = 1,2,---,n+m-1.$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f_{lpl}^{*}$ is defined as follows

$f_{lpl}^{*}(v_1 v_2) = 2.$

$f_{lpl}^{*}(v_{2i-1} v_{2i+1}) = 6i-2,$ $\quad i = 1,2,---,\frac{n-1}{2}$, if n is odd.
$\quad i = 1,2,---,\frac{n-2}{2}$, if n is even.

$f_{lpl}^{*}(v_{2i} v_{2i+2}) = 6i+1,$ $\quad i = 1,2,---,\frac{n-3}{2}$, if n is odd
$\quad i = 1,2,---,\frac{n-2}{2}$, if n is even.

$f_{lpl}^{*}(v_{n-1} v_n) = 3n-4.$

$f_{lpl}^{*}(v_{n+i-1} v_{n+i}) = 3n+3i-4,$ $\quad i = 1,2,---,m-1.$

Clearly $f_{lpl}^{*}$ is an injection.

***gcin*** of $(v_n)$ $= \text{gcd of } \{f_{lpl}^{*}(v_{n-2} v_n), f_{lpl}^{*}(v_{n-1} v_n)\}$
$= \text{gcd of } \{3n-5, 3n-4\}$
$= 1.$

So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence G , admits linear prime labeling.

**Example (a) G be the direct graph of a path $P_4$ joined to a cycle $C_6$ .**

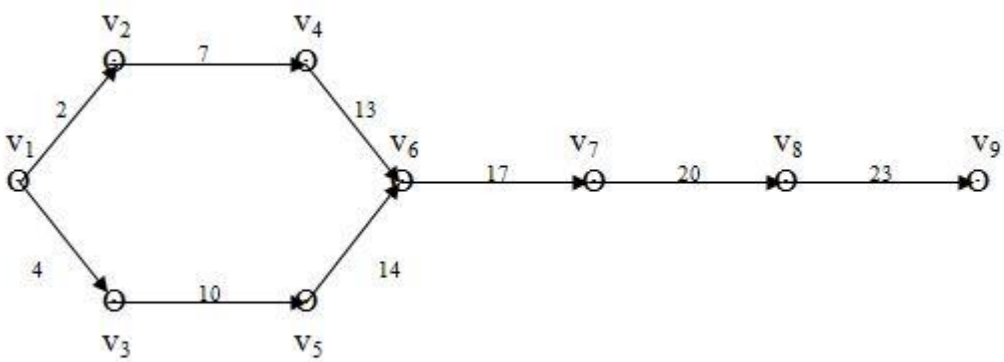


**Fig 2.5a:-**

**Example (b) G be the direct graph of a path $P_4$ joined to a cycle $C_5$ .**

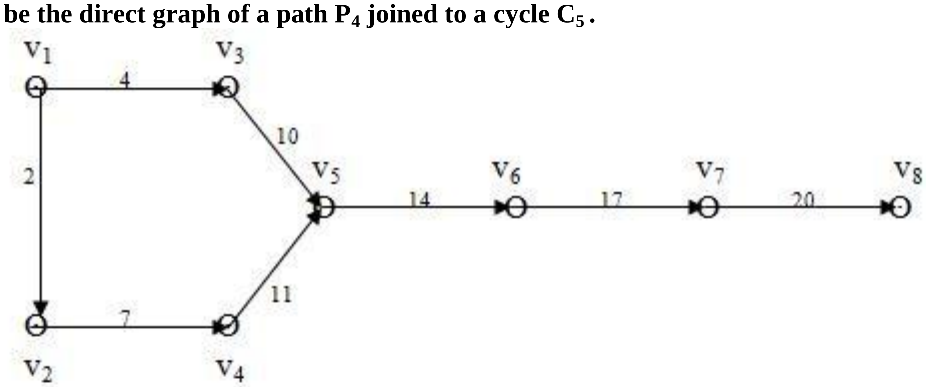


**Fig 2.5b:-**

**Theorem**

Let G be the graph obtained by joining two copies of a path $P_m$ to two consecutive vertices of a cycle $C_n(n > 2)$. Direct graph of G admits linear prime labeling.

**Proof:**

Let G be the graph and let $v_1, v_2, ---, v_{n+2m-2}$ are the vertices of G.
Here $|V(G)| = n+2m-2$ and $|E(G)| = n+2m-2$.
Define a function $f : V \rightarrow \{0,1,2,---,n+2m-3\}$ by

$$f(v_i) = i-1 , i = 1,2,---,n+2m-2.$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f_{lpl}^*$ is defined as follows

$f_{lpl}^*(v_1 v_2) = 2.$

$f_{lpl}^*(v_{2i-1} v_{2i+1}) = 6i-2,$ $i = 1,2,---,\frac{n-1}{2}+ m-1$ if n is odd.
$i = 1,2,---,\frac{n-2}{2} + m-1$ if n is even.

$f_{lpl}^*(v_{2i} v_{2i+2}) = 6i+1,$ $i = 1,2,---,\frac{n-3}{2}+ m-1$ if n is odd.
$i = 1,2,---,\frac{n-2}{2} + m-1$ if n is even.

$f_{lpl}^*(v_{n-1} v_n) = 3n-4.$

Clearly $f_{lpl}^*$ is an injection.

***gcin*** of $(v_n)$ = gcd of $\{f_{lpl}^*(v_{n-2} v_n), f_{lpl}^*(v_{n-1} v_n)\}$
= gcd of {3n-5, 3n-4}
= 1.

So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence G , admits linear prime labeling.

**Example (a) G be the direct graph of two copies of path $P_4$ joined to two consecutive vertices of cycle $C_6$.**

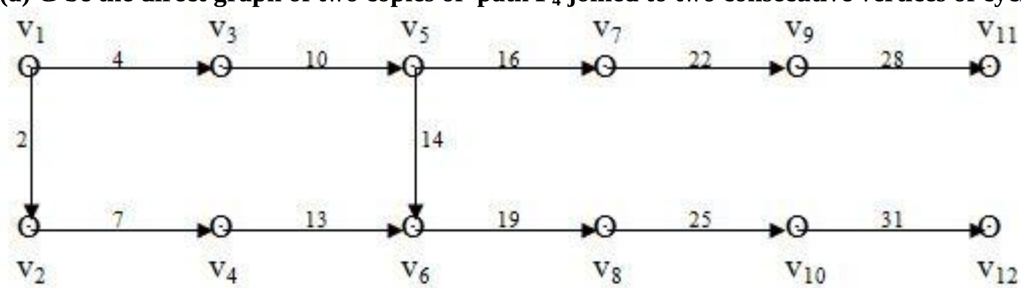


**Fig 2.6a:-**

**Example (b) G be the direct graph of two copies of path $P_4$ joined to two consecutive vertices of cycle $C_5$.**

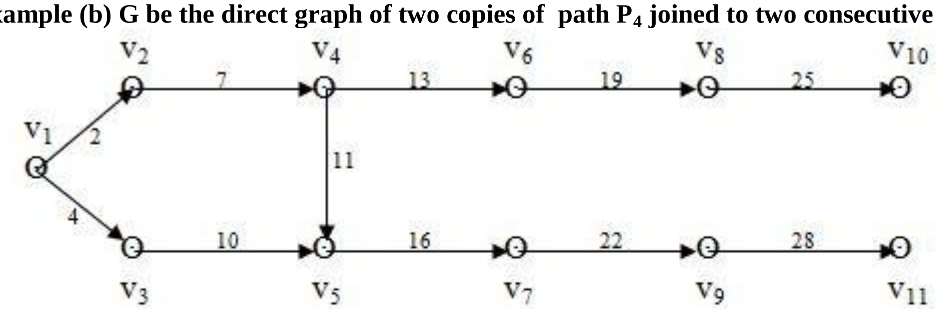


**Fig 2.6b:-**

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