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RESEARCH ARTICLE

ON EQUIVALENCE OF A-SET AND AB- SET DUE TO DONTCHEV AND TONG

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Abstract

Erdal Ekici , Takashi Noiri (Decomposition of continuity, α-continuity and AB-continuity, Chaos, Solitons and Fractals 41(2009) 2055 − 2061) have introduced the interesting concepts of A-set and AB-set for defining generalized versions of continuity viz. A-continuity and AB-continuity. In this paper, it has been noticed that the conditions applied to derive A-set and AB-set with respect to a given topology emerge the same family of the sets and hence the concepts of A-continuity and AB-continuity coincide.

1. INTRODUCTION

The idea of C-sets, η-sets, A-sets, AB-sets, AC-sets, α-AB-sets has been initiated by Noiri et al. [10] and Ekici et al. [6]. By using the concept of these sets they have defined [6] four generalized concepts of continuity viz. A-continuity, AB-continuity, AC-continuity and α - AB-continuity.

In the present paper, it has been studied that the concepts of A-set and AB-set are equivalent. In fact, we have established the **necessary and sufficient condition** for the set to be a semi-regular set which in turns establishes the equivalence of two families of the sets viz. A-sets and AB-sets.

Hence, in view of our assertion, it follows directly that the two different definitions of A-continuity and AB-continuity introduced in [6] turned equivalent.

2. PREREQUISITES

Throughout this paper (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed. F_X and F_Y denote collection of closed sets corresponding to the topologies on X and Y respectively. For a subset A ⊂ X, the closure and the interior of A are denoted by cl(A) and int(A) respectively. A function f : X → Y denotes a single valued function of a topological space (X, τ) into topological space (Y, σ).

We recall the following definitions, which are required for our study.

DEFINITION 2.1 A subset S of a space (X, τ) is called **Semi-open** [8] if S ⊆ cl(int(S)). The complement of Semi-open set is called **Semi-closed set**. The collection of Semi-open sets is denoted by SO(X).

EXAMPLE 2.1 Consider a set X = {a, b, c} with the topology τ = {X, φ, {a, b}, {a}, {b}}, F_X = {φ, X, {c}, {b, c}, {a, c}}. Let A = {a, c} be a subset of X.

Then, int{a, c} = {a}, cl(int{a, c}) = cl{a} = {a, c}⇒{a, c} ⊆ cl(int{a, c}). Hence, A is **semi-open** and complement of {a, c} is {b} which is **semi-closed.**

DEFINITION 2.2 A subset S of a space (X, τ) is called **semi-regular** if it is both semi-open and semi-closed. The collection of all semi-regular sets in X is denoted by Sr(X) (cf.[6]).

EXAMPLE 2.2 Consider a topological space X = {a, b, c} with the topology τ = {X, φ, {a}, {b}, {a, b}}, F_X = {φ, X, {b, c},{a, c},{c}}. Let A = {b, c} be a subset of X, then int{b, c} = {b} and cl(int{b, c}) = cl{b}= {b, c} ⇒ {b, c} ⊆ cl(int{b, c}) = {b, c}.

Hence, A is a semi-open set. In order to establish that A is semi-closed, it is enough if we show that complement of A = {a} is semi-open. Since, {a} ∈ τ , and every open set is semi-open, hence, {a} is semi-open. Therefore, A is **semi-regular set**.

REMARK 2.1 Collection of semi-regular sets of X does not form a topology on X.

EXAMPLE 2.3 Consider a topological space X = {a, b, c} with the topology τ = {X, φ, {a}, {b}, {a, b}}, F_X = {φ, X, {b, c}, {a, c}, {c}}. It is easy to verify that the collection of semi-regular sets of X viz. Sr(X) = {X, φ, {a}, {b}, {a, c}, {b, c}} precisely. It may be noted that the collection of Sr(X) is not a topology on X.

The following concepts of AB-set and A-set have been introduced in [6]. We focus our study on these two different classes of sets.

DEFINITION 2.3 A subset H of a space (X, τ) is called

- An **AB-set** [11] if H (∈ AB-set) = {A ∩ B, where A is open and B is semi-regular}.
- An **A-set** [5] if H (∈ A-set) = {A ∩ B : A ∈ τ, B = cl(int(B))}.

For a topological space X, we denote F_A = {A-set in X}, F_{AB} = {AB-set in X}.

EXAMPLE 2.4 Consider a topological space X = {a, b, c} with the topology τ = {X, φ, {a}}, F_X = {φ, X, {b, c}}. It is easy to see that there are only two open sets say X and φ which are semi-regular. Given A = {a} ∈ τ is open in X, then, {a} ∩ X = {a} ∈ $\mathbf{F_{AB}}$.

EXAMPLE 2.5 Consider a topological space X = {a, b, c} with the topology τ = {X, φ, {a}, {b}, {a, b}}, F_X = {φ, X, {b, c}, {a, c}, {c}}. Consider A = X ∈ τ open in X and B = {a, c}⊂ X. Then, int{a, c} = {a} and cl(int{a, c}) = cl{a} = {a, c}, B = {a, c} = cl(int{a, c}) = {a, c}. Now we see that A ∩ B = X ∩ {a, c} = {a, c} ∈ $\mathbf{F_A}$.

DEFINITION 2.4 A function f : (X, τ) → (Y, σ) is called **A-continuous** if $f^{-1}(G) \in F_A$ for each G ∈ σ (cf [6]).

EXAMPLE 2.6 Consider the topological spaces X = {a, b, c, d}, Y = {1, 2, 3, 4} with their corresponding topologies τ = {X, φ, {b}, {c}, {b, c}, {a, b}, {a, b, c}}, σ = {Y, φ, {2, 3}, {1, 2, 4}, {2}} respectively. F_A = {X, φ, {b}, {c}, {b, c}, {a, b},{a, b, c}, {c, d}, {a, b, d}}. Define a function f : X → Y as

$$f(x) = \begin{cases} 1, & \text{for} \quad x = a \\ 2, & \text{for} \quad x = b \\ 3, & \text{for} \quad x = c \\ 4, & \text{for} \quad x = d \end{cases} \tag{2.1}$$

In view of (2.1), for G ∈ σ, we get

$$f^{-1}(G) = \begin{cases} X, & \text{for} \quad G = Y \\ \phi & \text{for} \quad G = \phi \\ \{b\}, & \text{for} \quad G = \{2\} \\ \{b, c\} & \text{for} \quad G = \{2,3\} \\ \{a, b, d\} & \text{for} \quad G = \{1,2,4\} \end{cases} \tag{2.2}$$

Hence, we notice that $f^{-1}(G) \in F_A$ and conclude that f is **A-continuous** (when we appeal to Definition 2.4), but not continuous.

DEFINITION 2.5 A function f : (X, τ) → (Y, σ) is called **AB-continuous** if $f^{-1}(G) \in F_{AB}$ for each G ∈ σ. (cf [6]).

EXAMPLE 2.7 Consider the topological spaces X = {a, b, c, d}, Y = {1, 2, 3, 4} with their corresponding topologies τ = {X, φ, {b}, {d}, {b, d}}, σ = {Y, φ, {3, 4}, {1, 3, 4}} respectively. F_{AB} = {X, φ, {b}, {d}, {b, d}, {b, c}, {a, d}, {a, b},{c, d}, {a, b, c}, {a, c, d}}. Define a function f : X → Y as

$$f(x) = \begin{cases} 1, & \text{for} \quad x = a \\ 2, & \text{for} \quad x = b \\ 3, & \text{for} \quad x = c \\ 4, & \text{for} \quad x = d \end{cases} \tag{2.3}$$

In view of (2.3), for G ∈ σ, we get

$$f^{-1}(G) = \begin{cases} X, & \text{for} \quad G = Y \\ \phi & \text{for} \quad G = \phi \\ \{c, d\} & \text{for} \quad G = \{3,4\} \\ \{a, c, d\} & \text{for} \quad G = \{1,3,4\} \end{cases} \tag{2.4}$$

Hence, we notice that $f^{-1}(G) \in F_{AB}$ and conclude that f is **AB-continuous** (when we appeal to Definition 2.5), but not continuous.

REMARK 2.2 Collection of AB-sets, A-sets, do not form a topology on X.

The following result play a crucial role in establishing the equivalence of the classes F_{AB} and F_A.

THEOREM 2.1 Let A be a subset of the topological space X. Then $x \in \overline{A}$ if and only if every open set U containing x intersects A (cf.[9], Theorem 17.5, page 96).

3. ASSERTION DUE TO EKICI AND NOIRI

Using the concepts of AB-set and A-set, Ekici et al. [6] have concluded the following assertions:

- A-set ⇒ AB-set. (cf.[5], [11])
- A-continuity ⇒ AB-continuity.

4. MAIN RESULTS

Equivalence of the concepts of A-sets and AB-sets is a direct consequence of the following Lemma which is the main objective of this paper.

LEMMA 4.1 Let (X, τ) be the topological space then B be semi-regular with respect to the topology τ on X if and only if B = cl(int(B)).

PROOF OF THE LEMMA Referring the definitions of A-sets and AB-sets from [6], we consider

$$F_A = \{A \cap B : A \in \tau, B = cl(int(B))\} \tag{4.1}$$

$$F_{AB} = \{A \cap B : A \in \tau, B \text{ is semi - regular}\} \tag{4.2}$$

We have to show that, B is semi-regular ⇔ B = cl(int(B)). It is enough if we show that (4.1) ≅(4.2).

We first show that (4.2) ⇒ (4.1).

B is semi-regular ⇒ B is both semi-open and semi-closed (cf. [6]).

⇒ B, CB are both semi-open.
(Throughout our further discussions C denotes the complement of a set).

Since B is semi-open, by applying the definition 2.1, we have

$$B \subseteq cl(int(B)) \tag{4.3}$$

Similarly, since CB is semi-open, we have CB ⊆ cl(int(CB)) which is same as

$$B \supseteq C[cl(int(CB))] \tag{4.4}$$

In view of (4.4), it is enough if we show the following :

$$C[cl(int(CB))] \supseteq cl(int(B)) \tag{4.5}$$

Let $x \in cl(int(B))$. Then every neighborhood V_x of x intersects int(B) (cf. Th. 2.1, see also [9]))

$\Rightarrow V_x \cap int(B) \neq \varphi \Rightarrow y \in V_x \cap int(B) \Rightarrow y \in V_x$ and $y \in int(B)$.

$$\text{Since } y \in int(B) \text{ then } y \in cl(int(B)) \tag{4.6}$$

Since, $y \in int(B)$, $\exists$ a neighborhood V_y of y such that $V_y \subseteq B$

$\Rightarrow V_y \cap CB = \varphi \Rightarrow V_y \cap int(CB) = \varphi \Rightarrow y \notin cl(int(CB))$

It is clear that

$$y \in C[cl(int(CB))] \tag{4.7}$$

Referring conclusions (4.6) and (4.7), we obtain

$$cl(int(B)) \subseteq C[cl(int(CB))] \subseteq B \quad \Rightarrow cl(int(B)) \subseteq B \qquad \text{(cf. [4.4])} \tag{4.8}$$

Combining (4.3) and (4.8), we get $B = cl(int(B))$.

We now establish (4.1) ⇒ (4.2).

Given $B = cl(int(B))$, this shows that $B \subseteq cl(int(B))$, this implies B is semi-open. Also, $B \supseteq cl(int(B))$ and it is already proved that $cl(int(B)) \subseteq C[cl(int(CB))]$. Hence

$$B \supseteq C[cl(int(CB))] \tag{4.9}$$

Taking the complement in (4.9), we get

$$CB \subseteq cl(int(CB)) \tag{4.10}$$

In view of (4.10), CB is semi-open, therefore B is semi-closed. Hence B is both semi-open and semi-closed. This shows that B is semi-regular.

Thus, (4.1) ⇒ (4.2). We therefore conclude that (4.1) ≅ (4.2).

THEOREM 4.1 Let (X, τ) be the topological space. Consider the collection of A(X) (cf. [6]) and AB(X) sets (cf.[6]) defined as follows :

$$A(X) = \{A \cap B : A \in \tau, B = cl(int(B))\} \cong F_A \text{ (say)}$$

$$AB(X) = \{A \cap B : A \in \tau, B \text{ is semi-regular }\} \cong F_{AB} \text{ (say)}$$

Then, $F_A \cong F_{AB}$.

PROOF OF THE THEOREM It is a direct consequence of Lemma 4.1.

CONCLUSION By establishing the necessary and sufficient condition for the semi-regularity of sets, several results depending on the concept of A-sets and AB-sets may be presented in the concise form.

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