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RESEARCH ARTICLE

ON STABILITY OF ZERO SOLUTION OF SOME TYPES OF FIFTH ORDER.

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Abstract

This paper is illustrates the sufficient conditions of theuniformly asymptotically stableand the bounded of the zero solution of fifth order nonlinear differential equation with a variable delay $\tau(t)$as given by the following:

$$\begin{aligned}x^{(5)} + \psi(\dddot{x})x^{(4)} + f\left(x(t-\tau(t)), \dot{x}(t-\tau(t)), \cdots, x^{(4)}(t-\tau(t))\right)\dddot{x}(t) \\ + a\ddot{x}(t) \\ + g\left(x(t-\tau(t)), \dot{x}(t-\tau(t)), \cdots, x^{(4)}(t-\tau(t))\right)\dot{x}(t) \\ + h\left(x(t-\tau(t))\right) + k\left(\dot{x}(t-\tau(t))\right) \\ = P\left(t, x(t), x(t-\tau(t)), \dot{x}(t), \dot{x}(t-\tau(t)), \cdots, x^{(4)}(t), x^{(4)}(t \right. \\ \left. -\tau(t))\right) \cdots (1)\end{aligned}$$

Where $x \in \mathbb{R}$, $\tau(t)$is variables delay, a is positive constant, ψ, f, g, h, k and p are continuous functions in $\mathbb{R}, \mathbb{R}^5, \mathbb{R}^5, \mathbb{R}, \mathbb{R}$, and $\mathbb{R}^+ \times \mathbb{R}^{10}$, respectively, and the dots in(1) denote differentiation respect with to $t, t \in \mathbb{R}^+$.

Introduction:-

In 2002, Tunç [2] studied the asymptotic stability of the zero solution of equation (2) with $p \equiv 0$ and the boundedness of solution of equation (2)with$p \not\equiv 0$, this equation of the form:

$$x^{(5)} + \varphi(x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)})x^{(4)} + \psi(x, \dot{x}, \ddot{x}, \dddot{x}) + h(x, \dot{x}, \ddot{x}) + g(x, \dot{x}) + f(x) = p(t, x, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)}) \cdots (2)$$

In this paper we studied the uniformly asymptotically stableand the bounded of the zero solutions for the differential equation of fifth order with variable delayof the form:

$$\begin{aligned}x^{(5)} + \psi(\dddot{x})x^{(4)} + f\left(x(t-\tau(t)), \dot{x}(t-\tau(t)), \cdots, x^{(4)}(t-\tau(t))\right)\dddot{x}(t) + a\ddot{x}(t) \\ + g\left(x(t-\tau(t)), \dot{x}(t-\tau(t)), \cdots, x^{(4)}(t-\tau(t))\right)\dot{x}(t) + h\left(x(t-\tau(t))\right) + k\left(\dot{x}(t-\tau(t))\right) \\ = P\left(t, x(t), x(t-\tau(t)), \dot{x}(t), \dot{x}(t-\tau(t)), \cdots, x^{(4)}(t), x^{(4)}(t-\tau(t))\right)\end{aligned}$$

Where $x \in \mathbb{R}$, $\tau(t)$is variables delay, a is positive constant,ψ, f, g, h, k and p are continuous functions in$\mathbb{R}, \mathbb{R}^5, \mathbb{R}^5, \mathbb{R}, \mathbb{R}$,and $\mathbb{R}^+ \times \mathbb{R}^{10}$,respectively, and $t \in \mathbb{R}^+$.

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Rewriting equation (1),in systems form as follows:

$$\begin{aligned} x^{(5)} = \dot{u} = P\left(t, x(t), x\big(t-\tau(t)\big), y(t), y\big(t-\tau(t)\big), \cdots, u(t), u\big(t-\tau(t)\big)\right) - \psi(w)u \\ - f\left(x\big(t-\tau(t)\big), y\big(t-\tau(t)\big), \cdots, u\big(t-\tau(t)\big)\right) w - az \\ - g\left(x\big(t-\tau(t)\big), y\big(t-\tau(t)\big), \cdots, u\big(t-\tau(t)\big)\right) y - h(x) - k(y) + \int_{t-\tau(t)}^{t} \dot{h}\big(x(s)\big)y(s)ds \\ + \int_{t-\tau(t)}^{t} \dot{k}\big(y(s)\big)z(s)ds \cdots (3) \end{aligned}$$

Which were obtained by taking $\dot{x} = y, \ddot{x} = z, \dddot{x} = w, x^{(4)} = u$ in equations (1).
Theuniformly asymptotically stableand the bounded of the zero solutions for the differential equation of fifth order with variable delay has not been discussed by any authors and this paper is considered the first to discussed this topic, to the best of our knowledge.

Preliminaries:-
In order to guarantee the existence of the solution we assume that:

Remark 2.1:-
Consider the general autonomous delay differential system with finite delay:

$$\dot{x} = F(x_t), x_t = x(t+\theta) - r \le \theta \le 0, t \ge 0$$

Where $r \ge 0$, $F: G \to \mathbb{R}^n$ is a continuous and maps closed and bounded sets into bounded sets where G is an open subset of $C = C([-r,0], \mathbb{R}^n)$ with $\|\phi\| = \max_{-r\le s\le 0}|\phi(s)|$, $\phi \in C$. If $x: [-r, A) \to \mathbb{R}^n$ is continuous,$0 < A \le \infty$, then for each t in $[0, A)$,x_t in C is defined by:

$$x_t(s) = x(t+s) - r \le s \le 0, t \ge 0$$

It follows from these conditions on F that each initial value problem:

$$\dot{x} = F(x_t), \quad x_o = \phi \in G,$$

has a unique solution defined on some interval $[0, A), 0 < A \le \infty$. This solution will be denoted by $x(\phi)(.)$ so that $x_o(\phi) = \phi$.

Main Result:-
We will give the important theorem to give theuniformly asymptotically stable and the bounded of the zero solution of delay differential equation with fifth order.
For the first case when $p\left(t, x(t), x\big(t-\tau(t)\big), \cdots, u(t), u\big(t-\tau(t)\big)\right) \equiv 0$, we shall prove the following theorem

Theorem 3.1:-
Besidesthe a summiting of ψ, f, g, h and $k$that appear in equation (1), assume that there exist a positive constants$g, m, g_1, h_1, d, e, d_1, e_1, c_1, f, b_1, c, m_1$ and hsuch that following the conditions are hold:

i- $\dot{h}(x) < g$, $\dot{k}(y) < g_1$

ii- $f\left(x\big(t-\tau(t)\big), y\big(t-\tau(t)\big), \cdots, u\big(t-\tau(t)\big)\right) > h_1$, where $h_1 > 1$

$$g\left(x\big(t-\tau(t)\big), y\big(t-\tau(t)\big), \cdots, u\big(t-\tau(t)\big)\right) > e$$

iii- $\frac{h(x)}{x} > e_1$, $x \ne 0$,$\frac{k(y)}{y} > f$, $y \ne 0$,$e_1{}^2 \le \min\left\{\frac{(f+a)e_1}{64}, \frac{he_1}{64}, \frac{ae_1}{64}\right\}$,$a^2 \le \frac{ha}{16}$,$(f+h_1)^2 \le \frac{a(f+a)}{64}$, $1 \le \frac{(f+a)}{64}$,$1 \le \frac{h}{64}$,$e \ge e_1$ and $1 \le \frac{a}{64}$.

iv- $\psi(w) > h$,$0 \le \tau(t) \le \beta$, $\acute{\tau}(t) \le B_\circ$,where $0 < B_\circ < 1$.

v- $(e-e_1)^2$
$\le \min\left\{\frac{1}{4}(e-e_1+f)(a-h_1-f), \frac{1}{64}(e-e_1+f)(h_1-a)\right\}$,
$(h+e+f)^2 \le \frac{1}{4}(e-e_1+f)(h-1)$,
$(h_1-1)^2 \le \frac{1}{4}(h_1-a)(h-1)$,
$(h-1)^2 \le \frac{1}{64}(a-h_1-f)(h-1)$,

And $h_1^{\,2} \le \frac{1}{4}(a-h_1-f)(h_1-a)$
And

$$\beta < \min\left\{\frac{7(e-e_1+f)}{16\left(\frac{2g}{(1-B\circ)}+\frac{g}{2}+\frac{g_1}{2}\right)},\frac{7(a-h_1-f)}{16\left(\frac{2g_1}{(1-B\circ)}+\frac{g}{2}+\frac{g_1}{2}\right)},\quad \frac{7(h-1)}{16\left(\frac{g}{2}+\frac{g_1}{2}\right)},\frac{7(h_1-a)}{16\left(\frac{g}{2}+\frac{g_1}{2}\right)}\right\}$$

Where $\lambda_1 = \frac{2g}{(1-B\circ)}$, $\lambda_2 = \frac{2g_1}{(1-B\circ)}$
Then the zero solution of (1) is uniformly asymptotically stable.

Proof: see [3].

The next theorem give the boundedness of zero solution, the case when $p\Big(t,x(t),x\big(t-\tau(t)\big),\cdots,u(t),u\big(t-\tau(t)\big)\Big) \not\equiv 0$, we shall prove the following theorem

Theorem 3.2:
Besides of the assumption of theorem (3.1) and

$$\left|p\Big(t,x(t),x\big(t-\tau(t)\big),\cdots,u(t),u\big(t-\tau(t)\big)\Big)\right| \le q(t)$$

Such that $q \in L^1(0,\infty)$
Then there exists a positive constant M such that the solution $x(t)$ of equation (1) is defined by the initial functions:

$$x(t) = \phi(t), \dot{x}(t) = \dot{\phi}(t), \ddot{x}(t) = \ddot{\phi}(t), \dddot{x}(t) = \dddot{\phi}(t), x^{(4)}(t) = \phi^{(4)}(t)$$

Satisfies the inequalities:

$$|x(t)| \le \sqrt[2]{M}, |\dot{x}(t)| \le \sqrt[2]{M}, |\ddot{x}(t)| \le \sqrt[2]{M}, |\dddot{x}(t)| \le \sqrt[2]{M}, \left|x^{(4)}(t)\right| \le \sqrt[2]{M}$$

For all $t \ge t_\circ \ge 0$, where $\phi \in C^4([t_\circ - \tau, t_\circ], \mathbb{R})$

Proof:-
Using Lyapunov functional which is defined on theorem (3.1) we get:

$$\dot{V} \le -D_7y^2 - D_8z^2 - D_9u^2 - D_{10}w^2 + (y+w+z+u)\, p\Big(t,x(t),x\big(t-\tau(t)\big),\cdots,u(t),u\big(t-\tau(t)\big)\Big)$$
$$\dot{V} \le (y+w+z+u)p\Big(t,x(t),x\big(t-\tau(t)\big),\cdots,u(t),u\big(t-\tau(t)\big)\Big)$$
$$\dot{V} \le |y+w+z+u| \times \left|p\Big(t,x(t),x\big(t-\tau(t)\big),\cdots,u(t),u\big(t-\tau(t)\big)\Big)\right|$$

Where

$$\left|p\Big(t,x(t),x\big(t-\tau(t)\big),\cdots,u(t),u\big(t-\tau(t)\big)\Big)\right| \le q(t)$$

Now we use:

$$|y| \le 1 + y^2, |w| \le 1 + w^2$$
$$|z| \le 1 + z^2$$
$$|u| \le 1 + u^2$$

$\dot{V} \le (4 + y^2 + z^2 + u^2 + w^2)q(t)$
$y^2 + z^2 + u^2 + w^2 \le y^2 + z^2 + u^2 + w^2 + x^2 \le D_6^{\,-1}V$

$$\dot{V} \le \big(4 + D_6^{\,-1}V\big)q(t)$$

Integratingthe both sides of the above equation from 0 to t

$$V(x,y,z,w,u) \le V(x_\circ,y_\circ,z_\circ,w_\circ,u_\circ) + 4\int_0^t q(s)ds + D_6^{\,-1}\int_0^t V(x_s,y_s,z_s,w_s,u_s)q(s)ds$$

By using Grönwall inequality [1] we get:

$$\le (V(x_\circ,y_\circ,z_\circ,w_\circ,u_\circ) + 4A)\exp\big(D_6^{\,-1}A\big)$$
$$y^2 + z^2 + u^2 + w^2 + x^2 \le D_6^{\,-1}V \le D_6^{\,-1}K = M$$

Where $K = (V(x_\circ,y_\circ,z_\circ,w_\circ,u_\circ) + 4A)\exp\big(D_6^{\,-1}A\big)$

$$|x(t)| \le M, |y(t)| \le M, |w(t)| \le M, |z(t)| \le M, |u(t)| \le M$$

That is:

$$|x(t)| \le \sqrt[2]{M}, |\dot{x}(t)| \le \sqrt[2]{M}, |\ddot{x}(t)| \le \sqrt[2]{M}, |\dddot{x}(t)| \le \sqrt[2]{M}, \left|x^{(4)}(t)\right| \le \sqrt[2]{M}$$

For $t \ge t_\circ \ge 0$, the end of theorem.

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