ISSN NO. 2320-5407

Journal homepage: http://www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH

RESEARCH ARTICLE

Equations of Motion of the Galactic Orbits due to Breakdown of Integrated Systems (BIS) when both the Primaries are Axes Symmetric about the Collinear Liberation Points

Tuhin Dutta[1], Madan Mohan Bajaj[2], Deepak Kumar[3], Sobinder Singh[4], Rajendra Agarwal[5], M.C. Jain[6]

1. Assistant Professor, Department of Physics, ARSD College, University of Delhi, Delhi, India E-Mail: tuhindutta38@gmail.com.

2. Retd. Senior Professor of Physics, Department of Physics & Astrophysics, University of Delhi, Delhi, India E-Mail: madanmohanbajaj@rediffmail.com

3. Research Scholar of Physics, Department of Physics, Singhania University, Rajasthan, India E-Mail: deepakm256@gmail.com

4. Assistant Professor, Department of Applied Sciences, Maharaja Surajmal Institute of Technology, GGSIP University, Delhi, India E-Mail: sobinder_singh@yahoo.co.in

5. Dean & Professor of Physics, Gurukul Kangri Vishwavidyalaya, Haridwar, Uttarakhand, India

6. Head & Senior Professor, *Department of Physics, R.K. P.G. College, Shamli, CCS University, Meerut, India.*

Manuscript Info

Manuscript History:

Received: 10 May 2013
Final Accepted: 25 May 2013
Published Online: June 2013

Key words:
Axis Symmetric Body,
Collinear Points,
Restricted Three Bodies,
Periodic Orbits.

Abstract

BIS signifies breakdown of integrated system. In our present paper we will study the BIS effect in the galactic .Three dimensional periodic orbits around the collinear liberation points $L_i (i = 1, 2, 3)$ in the restricted three body problem has been studied when both the primaries (earth-moon) are axis symmetric bodies with one of their axes as axis of symmetry and equatorial planes coinciding with the plane of motion by taking different values of semi-axes of the axis symmetric bodies (earth-moon). With the help of predictor corrector method, we have computed the initial conditions by taking different values of the semi-axes of the axis symmetric bodies. With these initial conditions, we have drawn three dimensional periodic orbits in the different cases.

Introduction

We studied earlier the three dimensional periodic orbits around the collinear liberation points in the restricted three body problem by assuming the bigger primary as an axis symmetric rigid body with its equatorial plane coinciding with the plane of motion. In this paper, we wish to generalize the earlier by taking both the primaries as axis symmetric bodies viz. the earth-moon system. We continue the investigation using the value for $\mu = 0.01215$ for the mass parameter of the problem. The numerical study presented here is based on the same method and techniques as the investigations mentioned in our earlier papers.

2. Equations of Motion

By adopting the notations and terminology of Szebehely and taking the distance between the primaries unity and the sum of the masses of the primaries as one, unit of time is so chosen so as to make gravitational constant G=1. Equations of motion of m_3 in dimensionless variables and Cartesian form can be written as

$$\ddot{x} - 2n\dot{y} = \Omega_x,\ \ddot{y} + 2n\dot{x} = \Omega_Y, \ddot{z} = \Omega_z, \tag{1}$$

where

$$\Omega = \frac{1}{2}n^2(x^2+y^2) - \frac{3(1-\mu)}{2r_1^5}(\sigma_1-\sigma_2)y^2 + \frac{(1-\mu)}{2r_1^3}(2\sigma_1-\sigma_2) + \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} - \frac{3(1-\mu)}{2r_1^5}\sigma_1 z^2 + \frac{\mu}{2r_2^3}(2\sigma_1'-\sigma_2') - \frac{3\mu}{2r_2^5}(\sigma_1'-\sigma_2')y^2 - \frac{3\mu}{2r_2^5}\sigma_1' z^2,$$

$$\mu = m_2, \quad 1-\mu = m_1,$$

$$\sigma_1 = \frac{(a_1^2-a_3^2)}{5R^2}, \quad \sigma_2 = \frac{a_2^2-a_3^2}{5R^2}, \quad \sigma_1,\sigma_2 < 1,$$
$$\sigma_1' = \frac{(a_1'^2-a_3'^2)}{5R^2}, \quad \sigma_2' = \frac{a_2'^2-a_3'^2}{5R^2}, \quad \sigma_1',\sigma_2' < 1,$$

R=Dimensional distance between the primaries.

G =Gravitational Constant,

$a_1, a_2, a_3, a_1', a_2', a_3'$ are the lengths of the semi axes of the axis symmetric bodies of masses m_1 and m_2 .The mean motion n of the primaries is given by

$$n^2 = 1 + \frac{3}{2}(2\sigma_1-\sigma_2) + \frac{3}{2}(2\sigma_1'-\sigma_2').$$

3. Collinear Liberation Points at (L_1,L_2,L_3)

The libration points are given by the solution of

$$\Omega_x = 0, \qquad \Omega_y = 0, \qquad \Omega_z = 0.$$

Since Ω_X >0 in each of the open intervals $(-\infty, \mu-1), (\mu-1, \mu), (\mu, \infty)$ the function Ω is strictly increasing in each of them.

$$\textit{Also } \Omega_X \to -\infty \textit{ as } x \to -\infty, (\mu-1)+0 \textit{ or } \mu+0,$$
$$\textit{and } \Omega_X \to \infty \textit{ as } x \to \infty, (\mu-1)-0 \textit{ or } \mu-0.$$

Therefore, there exists one and only one value of x in each of the above intervals. Further $\Omega(\mu-2) < 0,\ \Omega(0) \le 0$ *and* $\Omega(\mu+1) > 0.$ Therefore, there are only three real roots, one lying in each of the intervals $(\mu-2, \mu-1)$, $(\mu-1, \mu)$ and $(\mu, \mu+1)$. Thus there are three collinear libration points. The first collinear point is located left of the primary of mass m_1, the second is between the primaries, and the third collinear libration point is to the right of the primary of mass m_2.

4. Motion around the Collinear Liberation Points

Let L be any of the collinear liberation points $L_j, j=1,2,3,$. If a new coordinate system is defined with L as origin and x, y *and* z its axes, parallel to Ox , Oy and Oz respectively, the transformation between the two systems is given by the relations:

$$x \to x_L + x, \quad y \to y, \quad z \to z \qquad (2)$$

Then the Equations of motion are transformed through (2) in the L_{xyz} coordinate system as,

$$\ddot{x} - 2n\dot{y} = \Omega_x(x_L+x, y, z),$$
$$\ddot{y} + 2n\dot{x} = \Omega_y(x_L+x, y, z), \qquad (3)$$
$$\ddot{z} = \Omega_z(x_L+x, y, z)$$

Putting values of various derivatives in equation (3), we get

$$\ddot{x} - 2n\dot{y} = A_1x + A_2x^2 + A_3y^2 + A_4z^2,$$

$$\ddot{y} - 2n\dot{x} = B_1y + B_2xy,$$

$$\ddot{z} = C_1z + C_2xz. \qquad (4)$$

where

$$A_1 = n^2 + \frac{2(1-\mu)}{|x_L-\mu|^3} + \frac{2\mu}{|x_L+1-\mu|^3} + \frac{6(1-\mu)(2\sigma_1-\sigma_2)}{|x_L-\mu|^5} + \frac{6\mu(2\sigma_1'-\sigma_2')}{|x_L-\mu+1|^5},$$

$$A_2 = -3\Big[\frac{(1-\mu)(x_L-\mu)}{|x_L-\mu|^5} + \frac{5(1-\mu)(2\sigma_1-\sigma_2)(x_L-\mu)}{|x_L-\mu|^7} + \frac{\mu(x_L+1-\mu)}{|x_L+1-\mu|^5} + \frac{5\mu(2\sigma_1'-\sigma_2')(x_L-\mu+1)}{|x_L-\mu+1|^7}\Big],$$

$$A_3 = \frac{3}{2}\Big[\frac{(1-\mu)(x_L-\mu)}{|x_L-\mu|^5} + \frac{\mu(x_L+1-\mu)}{|x_L+1-\mu|^5} + \frac{5(1-\mu)(2\sigma_1-\sigma_2)(x_L-\mu)}{2|x_L-\mu|^7} + \frac{5(1-\mu)(\sigma_1-\sigma_2)(x_L-\mu)}{|x_L-\mu|^7} + \frac{5\mu(2\sigma_1'-\sigma_2')(x_L-\mu+1)}{2|x_L-\mu+1|^7} + \frac{5\mu(\sigma_1'-\sigma_2')(x_L-\mu+1)}{|x_L-\mu+1|^7}\Big],$$

$$A_4 = \frac{3}{2}\Big[\frac{(1-\mu)(x_L-\mu)}{|x_L-\mu|^5} + \frac{\mu(x_L+1-\mu)}{|x_L+1-\mu|^5} + \frac{5(1-\mu)(2\sigma_1-\sigma_2)(x_L-\mu)}{2|x_L-\mu|^7} + \frac{5(1-\mu)\sigma_1(x_L-\mu)}{|x_L-\mu|^7} + \frac{5\mu(2\sigma_1'-\sigma_2')(x_L-\mu+1)}{2|x_L-\mu+1|^7} + \frac{5\mu\sigma_1'(x_L-\mu+1)}{|x_L-\mu+1|^7}\Big],$$

$$B_1 = n^2 - \frac{(1-\mu)}{|x_L-\mu|^3} - \frac{\mu}{|x_L+1-\mu|^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2|x_L-\mu|^5} - \frac{3(1-\mu)(\sigma_1-\sigma_2)}{|x_L-\mu|^5} - \frac{3\mu(2\sigma_1'-\sigma_2')}{2|x_L+1-\mu|^5} - \frac{3\mu(\sigma_1'-\sigma_2')}{|x_L+1-\mu|^5}$$

$$B_2 = \frac{3(1-\mu)(x_L-\mu)}{|x_L-\mu|^5} + \frac{3\mu(x_L+1-\mu)}{|x_L+1-\mu|^5} + \frac{15(1-\mu)(2\sigma_1-\sigma_2)(x_L-\mu)}{2|x_L-\mu|^7} + \frac{15(1-\mu)(\sigma_1-\sigma_2)(x_L-\mu)}{|x_L-\mu|^7} + \frac{15\mu(2\sigma_1'-\sigma_2')(x_L-\mu+1)}{2|x_L-\mu+1|^7} + \frac{15\mu(\sigma_1'-\sigma_2')(x_L-\mu+1)}{|x_L-\mu+1|^7},$$

$$C_1 = \frac{-(1-\mu)}{|x_L-\mu|^3} - \frac{\mu}{|x_L+1-\mu|^3} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2|x_L-\mu|^5} - \frac{3(1-\mu)\sigma_1}{|x_L-\mu|^5} - \frac{3\mu(2\sigma_1'-\sigma_2')}{2|x_L-\mu+|1|^5} - \frac{3\mu\sigma_1'}{|x_L-\mu+1|^5},$$

$$C_2 = \frac{3(1-\mu)(x_L-\mu)}{|x_L-\mu|^5} + \frac{3\mu(x_L+1-\mu)}{|x_L+1-\mu|^5} + \frac{15(1-\mu)(2\sigma_1-\sigma_2)(x_L-\mu)}{2|x_L-\mu|^7} + \frac{15(1-\mu)\sigma_1(x_L-\mu)}{|x_L-\mu|^7} + \frac{15\mu(2\sigma_1'-\sigma_2')(x_L-\mu+1)}{2|x_L-\mu+1|^7} + \frac{15\mu\sigma_1'(x_L-\mu+1)}{|x_L-\mu+1|^7}$$

5. Second Order Approximation of Periodic Solution

We search for periodic solutions in the form of second order expansions in powers of a parameter ε :(0 to 0.009).

$$x(\tau) = x_1(\tau)\varepsilon + x_2(\tau)\varepsilon^2,$$
$$y(\tau) = y_1(\tau)\varepsilon + y_2(\tau)\varepsilon^2, \qquad (5)$$
$$z(\tau) = z_1(\tau)\varepsilon + z_2(\tau)\varepsilon^2$$

In order to erase any secular term in future analysis and retaining terms of powers in ε not greater than two and denoting by dot(.) the τ-derivatives and ignoring the terms $\sigma_i\varepsilon^2, \quad i = 1, 2,$ .we have to solve the system:

$$F(D)\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad (6)$$

where

$$F(D) = \begin{pmatrix} D^2 - A_1 & -2nD & 0 \\ 2nD & D^2 - B_1 & 0 \\ 0 & 0 & D^2 - C_1 \end{pmatrix}$$

The general solution of the equation (6) is given by

$$x_1(\tau)=\sum_{i=1}^{4} c_i Exp(\lambda_i \tau), \qquad (7a)$$

$$y_1(\tau)=\sum_{i=1}^{4} d_i Exp(\lambda_i \tau), \qquad (7b)$$

$$z_1(\tau)=\sum_{j=5}^{6} e_j Exp(\lambda_j \tau) \qquad (7c)$$

Where $\lambda_i, i=1,2,3,4$ are the characteristic roots of the system given by the Equation (7a) and (7b) and $\lambda_j,\ j=5,6$ are given by equation (7c). Periodic orbits can be obtained if at least one pair of Imaginary roots exists. By a suitable choice of the coefficients of the exponential of (7), we may have a special periodic solution which contains only the frequency ω. Equation (7) admit the periodic solution,

$$x_1(\tau)=A\cos\omega\tau+B\sin\omega\tau,$$
$$y_1(\tau)=A^*\cos\omega\tau+B^*\sin\omega\tau,$$
$$z_1(\tau)=A'\cos\omega\tau+B'\sin\omega\tau,$$

where the coefficients A, B, A*, B*, A' and B' are given by

$$A^*=\frac{2nB\omega}{(B_1+\omega^2)}.$$

$$B^*=\frac{-2nA\omega}{B_1+\omega^2}.$$

$$B'=\frac{\varepsilon}{\omega}$$

Without any loss of generality, we put $y_1(0)=0$, $z_1(0)=0$ then $A^*=0$, $A'=0$ and consequently, *B=0*. This means that $\dot{x}_1(0)=0$, $\dot{y}_1(0)\neq 0$ and $\dot{z}_1(0)\neq 0$ too. Finally, the above solution becomes,

$$x_1(\tau)=A\cos\omega\tau,$$
$$y_1(\tau)=B^*\sin\omega\tau, \qquad (8)$$
$$z_1(\tau)=B'\cos\omega\tau$$

The second order system is given by

$$F(D)\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}=\begin{pmatrix} g_1(\tau) \\ g_2(\tau) \\ g_3(\tau) \end{pmatrix}, \qquad (9)$$

where

$$g_1(\tau)=K_o+K_1\cos 2\omega\tau,$$
$$g_2(\tau)=\Lambda_1\sin 2\omega\tau,$$
$$g_3(\tau)=\Lambda_2\sin 2\omega\tau,$$

And

$$K_o=\frac{1}{2}[A_2A^2+A_3B^{*2}+A_4B'^2],$$
$$K_1=\frac{1}{2}[A_2A^2-A_3B^{*2}-A_4B'^2],$$
$$\Lambda_1=\frac{1}{2}[B_2AB^*],$$
$$\Lambda_2=\frac{1}{2}[C_2AB'].$$

Periodic Solution of equation (9) is given by

$$x_2(\tau)=M_o+M_1\cos 2\omega\tau,$$
$$y_2(\tau)=N_1\sin 2\omega\tau,$$
$$z_2(\tau)=N_2\sin 2\omega\tau. \qquad (10)$$

where

$$M_o=\frac{-K_o}{A_1},$$
$$M_1=\frac{1}{\psi}(-4K_1\omega^2+4\Lambda_1 n\omega-B_1K_1),$$
$$N_1=\frac{1}{\psi}(-4\Lambda_1\omega^2+4k_1 n\omega-A_1\Lambda_1),$$
$$N_2=\frac{\Lambda_2}{(-4\omega^2-C_1)},$$
$$\psi=16\omega^4-4(4n^2-A_1-B_1)\omega^2+A_1B.$$

Finally, a second order approximation of periodic solutions around the collinear libration points, as a function of parameter, ε is obtained as

$$x(t,\varepsilon) = A\cos\omega t\,\varepsilon + [M_o + M_1\cos 2\omega t]\varepsilon^2,$$
$$y(t,\varepsilon) = B^*\sin\omega t\,\varepsilon + N_1\sin 2\omega t\varepsilon^2,$$
$$z(t,\varepsilon) = B'\sin\omega t\,\varepsilon + N_2\sin 2\omega t\varepsilon^2.$$

Period of this solution is, $T = \dfrac{2\pi}{\omega}$.

6. NUMERICAL RESULTS

$(\mu = 0.01215)$ Using the Formulae (4) we find, a first approximation, which are close to $L_j,\ j = 1,2,3$. Then, by a linear predictor-corrector algorithm based on numerical integration of the equations of motion we can draw three dimensional periodic orbits for different values of σ_1 and σ_2.

Table 1 gives the parameter of earth and moon

Table 1

Cases	1	2	3	4	5
a_1	6400	6400	6400	6400	6400
a_2	6400	6390	6380	6370	6360
a_3	6400	6380	6360	6340	6320
a'_1	1750	1750	1750	1750	1750
a'_2	1750	1740	1730	1720	1710
a'_3	1750	1730	1710	1690	1670

a_1, a_2, a_3 and $a'_1, a'_2\ a'_3$ are the semi-axes of the earth and the moon respectively

Table 2

Collinear liberation points

$Earth - Moon\,case\,\mu = 0.01215, R = 384400km$.

Case	1	2	3	4	5
σ_1	0	$9.42\text{x}10^{-8}$	$1.873\text{x}10^{-7}$	$2.794\ \text{x}10^{-7}$	$3.703\text{x}10^{-7}$
σ_2	0	$4.697\text{x}10^{-8}$	$9.312\text{x}10^{-8}$	$1.385\ \text{x}10^{-7}$	$1.83\text{x}10^{-7}$
σ'_1	0	$3.46\text{x}10^{-7}$	$6.908\text{x}10^{-7}$	$1.0345\text{x}10^{-6}$	$1.377\text{x}10^{-6}$
σ'_2	0	$1.728\text{x}10^{-7}$	$3.449\text{x}10^{-7}$	$5.161\text{x}10^{-7}$	$6.865\text{x}10^{-7}$
L_1	-1.15567991	-1.1556813	-1.1556828	-1.1556843	1.1556857
L_2	-.836918007	-.83691163	-.83691470	-.83691306	-.83691142
L_3	1.005062401	1.00506213	1.00506186	1.00506161	1.00506136

7. CONCLUSION

With the help of predictor method, we have computed the initial conditions by taking different values of the semi-axes of the axes-symmetric bodies. With these initial conditions, we can draw actual three dimensional periodic orbits.

8. ACKNOWLEDGEMENT

We are grateful to the Vice Chancellors of University of Delhi, Singhania University and IP University for providing adequate laboratory and library facility in the premises.

9. REFERENCES.

1. **V. Szebehely**; *Theory of Orbits, (Academic, New York:) (1967)*
2. **Goudas.C.L.**,*(1963, Icarus,Volume-2 ,pp-1-18).*
3. **Papadakis.K.E**.,*1992, (Astrophysics and Space Science,Volume- 191,pp- 223-229).*
4. **Markellos.V.V**.,*1993, (Celest.Mech and Dyn. Astron,Volume-9, pp-365-350.)*
5. **Olle Merce,Pacha R.Joan** *,1999, (Astron. Astrophy,Volume-351,pp-1149-1164)*
6. **Perdios. E.A,2001**, *(Astrophysics and Space Science;Volume-278, pp. 405-407)*
7. **Perdios.E.A., S.S. Kanavos ,V.V. Markellos,***2004, (Astrophysic and Space Science, Volume-262,pp-75-87.)*
8. **Kalvouridis.T,Mavraganis. A, Pangalos.C, 2004,***(Astrophysics and Space Science, pp-255-256*