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RESEARCH ARTICLE

Interaction of electromagnetic and zero-mass scalar fields with viscous fluid

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Abstract

Considering the Robertson-Walker metric the problem of cosmological viscous fluid model in presence of electromagnetic field and zero-mass scalar field are discussed subject to various physical conditions. It has been shown that an electromagnetic field survived in the Robertson-Walker universe in presence of viscous fluid provided a scalar field is a function of radial co-ordinate. Exact solutions have been obtained by considering the cases of dust distribution, stiff fluid distribution and disordered distribution of radiation and their results are also discussed.

Introduction

Several authors have already studied the cosmological problems involving viscous fluid interacting with massive scalar field. The solutions obtained in general relativity have been conveniently used for the study of present state of the universe as well as the initial stages of the formation of the universe. Some of them are Charanch [1,2], Manihar [5,6,7,8] and Rao et al [12]. It has been shown by Das and Agarwal [4] that the electromagnetic field cannot survive in presence of perfect fluid in the Friedmann universe. Mehra and Bohra [9] have studied the cosmological problems involving electromagnetic field in presence of perfect fluid by considering a modified form of the Robertson-Walker metric.

The cosmological problems involving scalar fields and electromagnetic fields in presence of gravitational fields have been studied by Charach and Malin [3] thereby showing that the matter fields have become a cause of evolution in a manner quite different from those of the vacuum models. It is further observed that the modified Robertson-Walker metric suggested by Mehra and Bohra [9] is not always necessary for the survival of electromagnetic field due to the presence of scalar field. Tarachand and Ibotombi [11] have also studied the cosmological problems involving viscous fluid in presence of electromagnetic field and zero-mass scalar field.
Here we studied the problem of cosmological viscous fluid model in presence of electromagnetic field and zero-mass scalar field subject to various physical conditions by considering Robertson-Walker metric. It has been shown that an electromagnetic field survived in the Robertson-Walker universe in presence of viscous fluid provided a scalar field is a function of radial co-ordinate. Exact solutions have been obtained by considering the cases of dust distribution, stiff fluid distribution and disordered distribution of radiation and their results are also discussed.

1 Mathematical Formulation

The Maxwell field equations for viscous fluid in the presence of zero-mass scalar field are

$$G^i_j = R^i_j - \frac{1}{2} g^i_j R + \wedge g^i_j = -8\pi (T^i_j + E^i_j + S^i_j), \qquad (1)$$

where T^i_j, E^i_j and S^i_j are the energy-momentum tensors of viscous fluid, electromagnetic field and zero-mass scalar field respectively given by

$$T^i_j = (\rho + p - \zeta\Theta) u^i u_i - (p - \zeta\Theta) g^i_j + 2\eta\sigma^i_j , \quad (2)$$

$$E^i_j = \frac{1}{4\pi}\left[-F^{i\alpha} F_{j\alpha} + \frac{1}{4}\delta^i_j F^{\alpha\beta} F_{\alpha\beta}\right] \quad (3)$$

and

$$S^i_j = \frac{1}{4\pi}\left\{V^{'i} V_{,j} - \frac{1}{2}\delta^i_j V_{,s} V^{'s}\right\}. \quad (4)$$

Here ρ, p, u^i, Θ, σ_{ij} , η and ζ are respectively the density, pressure, four-velocity vector, expansion factor, shear tensor, and shear and bulk viscosity co-efficients.

The scalar field V(r,t) satisfies the wave equation

$$g^{ij} V_{;ij} = 0, \quad (5)$$

where for notation comma and semi-colon followed by index denote partial and covariant differentiation.

The metric taken for the present problem is

$$ds^2 = dt^2 - R^2(t)\left[(1 - Kr^2)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2\right], \quad (6)$$

wheret is the cosmic time ; R(t), the radius of the universe ; and K, the curvature index which takes up the values +1, 0, and -1.

The electromagnetic field tensor F_{ij} is defined in terms of a four-potential A_i (r,t) as

$$F_{ij} = \varphi_{i,j} - \varphi_{j,i} = A_{i,j} - A_{j,i}. \quad (7)$$

Since we are using comoving co-ordinates defined by $u^1 = u^2 = u^3 = 0$ and $u^4 = 1$ and because of a spherically-symmetry, the only surviving component of the electromagnetic field is the radial electric field F^{41} given by

$$\frac{\partial}{\partial r}\left[F_{14} (-g)^{\frac{1}{2}} g^{11} g^{44}\right] = -4\pi J^4 (-g)^{\frac{1}{2}} ,$$

or

$$\frac{\partial}{\partial r}\left\{\frac{R^3 r^2 F^{41}}{\sqrt{1-Kr^2}}\right\} = -4\pi\left\{\frac{r^2 R^3 J^4}{\sqrt{1-Kr^2}}\right\}. \quad (8)$$

From equation (8), we obtain

$$F^{41} = -\frac{\{(1-Kr^2)^{\frac{1}{2}} Q(r,t)\}}{R^3 r^2} , \quad (9)$$

where Q(r, t) is the charge given by

$$Q(r,t) = 4\pi\int_0^r J^4 (r^2 R^3)(1 - Kr^2)^{-\frac{1}{2}} \, dr. \quad (10)$$

The non-vanishing components of the electromagnetic energy-momentum tensor E^i_j are

$$E^1_1 = -E^2_2 = -E^3_3 = E^4_4 = -\frac{1}{8\pi} g^{11} g^{44} F^2_{14} . \qquad (11)$$

2 Field Equations and their Solutions

The Einstein's field equations (1) for the line element (6) reduce to

$$G^1_1 = -(2R\ddot{R} + \dot{R}^2 + K)R^{-2} + \wedge = = -8\pi\left[-p + \Theta\zeta + E^1_1 - \frac{1}{8\pi}\left\{\dot{V}^2 + (1 - Kr^2)V'^2 R^{-2}\right\}\right], \qquad (12)$$

$$G^2_2 = G^3_3 = -(2R\ddot{R} + \dot{R}^2 + K)R^{-2} + \wedge = = -8\pi\left[-p + \Theta\zeta - E^1_1 + \frac{1}{8\pi}\left\{(1 - Kr^2)V'^2 R^{-2} - \dot{V}^2\right\}\right] \qquad (13)$$

$$G^4_4 = -3(K + \dot{R}^2)R^{-2} + \wedge = -8\pi\left[\rho + E^1_1 + \frac{1}{8\pi}\left\{(1 - Kr^2)V'^2 R^{-2} + \dot{V}^2\right\}\right] \qquad (14)$$

and

$$G^1_4 = 0 = V'\dot{V}, \qquad (15)$$

where the dots and dashes denote the usual partial derivatives with respect to t and r respectively.

Equation (5) reduces to

$$(1 - Kr^2)V''R^{-2} + (2 - 3Kr^2)r^{-1}R^{-2}V' - \frac{3\dot{R}}{R}\dot{V} - \ddot{V} = 0. \qquad (16)$$

From equations (12) and (13), we obtain

$$8\pi E^1_1 = (1 - Kr^2)\, V'^2\, R^{-2}, \qquad (17)$$

or

$$F_{14} = V' .$$

From equations (15) and (17), we obtain

$$\dot{V} = 0 . \qquad (18)$$

Using equation (18) in (16), we get

$$V'' + \frac{(2 - 3Kr^2)}{r(1 - Kr^2)} V' = 0. \qquad (19)$$

Integrating (19), we obtain

$$V = d - c_o (1 - Kr^2)^{1/2} r^{-1}, \qquad (20)$$

where d and c_o are arbitrary constants.

From equations (12), (13) and (18), we obtain

$$-8\pi p + 24\pi\zeta \frac{\dot{R}}{R} = \frac{2\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{K}{R^2} - \wedge . \qquad (21)$$

From equations (14), (18) and (20), we obtain

$$8\pi\rho = \frac{3(K + \dot{R}^2)}{R^2} - \frac{2c_0^2}{r^4 R^2} - \wedge . \qquad (22)$$

Case 1.

Charged dust viscous fluid in presence of zero-mass scalar field with $\wedge = 0$:

Equation (21) becomes

$$2R\ddot{R} + \dot{R}^2 - 24\pi\zeta R\dot{R} = -K. \qquad (23)$$

Case 1(a)

We assume that R = at, where a is positive non-zero constant.

From equation (23), we obtain

$$\zeta = \frac{a^2 + K}{24\pi\, a^2 t}. \qquad (24)$$

Then we have

$$\rho = \frac{3}{8\pi}\left[\frac{a^2 + K}{a^2} - \frac{2c_o^2}{3a^2 r^4}\right] t^{-2}, \qquad (25)$$

$$Q = c_o\, a\, t, \qquad (26)$$

$$\Theta = \frac{3}{t} \qquad (27)$$

and

$$V = d - c_0 (1 - K r^2)^{1/2}\, r^{-1}. \qquad (28)$$

Case 1(b)

We assume that R = A $e^{\alpha t}$, where A and α are constants.

From equation (23), we obtain

$$\zeta = \frac{3A^2\alpha^2 e^{2\alpha t} + K}{24\pi\alpha A^2 e^{2\alpha t}}. \qquad (29)$$

Then we have

$$\rho = \frac{3}{8\pi}\left[\alpha^2 + \left\{K - \frac{2}{3}c_o^2\, r^{-4}\right\} A^{-2} e^{-2\alpha t}\right], \qquad (30)$$

$$Q = c_0 A e^{\alpha t} \qquad (31)$$

and

$\Theta = 3\alpha$. (32)

The scalar field V remains the same as given by (28).

Case 2.

Charged dust viscous fluid in presence of zero-mass scalar field with $\wedge \neq 0$, K = 0 :

Equation (21) becomes

$$2R\ddot{R} + \dot{R}^2 - 24\pi\zeta R\dot{R} = \wedge R^2 . \qquad (33)$$

Case 2(a)

We assume that R = at, where a is positive non-zero constant.

From equation (33), we obtain

$$\zeta = \frac{1 - \wedge t^2}{24\pi t} . \qquad (34)$$

Then we have

$$\rho = \frac{3}{8\pi}\left[\left\{1 - \frac{2}{3} c_0^2 a^{-2} r^{-4}\right\} t^{-2} - \frac{\wedge}{3}\right] \qquad (35)$$

and

$$V = d - \frac{c_0}{r} . \qquad (36)$$

The charge Q and the expansion factor Θ remain the same as given by (26) and (27) respectively.

Case 2(b)

We assume that R = A $e^{\alpha t}$, where A and α are constants.

From equation (33), we obtain

$$\zeta = \frac{3\alpha^2 - \wedge}{24\pi\alpha} . \qquad (37)$$

Then we have

$$\rho = \frac{1}{8\pi}\left[3\alpha^2 - \wedge - 2c_0^2 r^{-4} A^{-2} e^{-2\alpha t}\right]. \qquad (38)$$

The charge Q, the expansion factor Θ and the scalar field V in this case remain the same as given by (31), (32) and (36) respectively.

Case 3

Charged dust viscous fluid in presence of zero-mass scalar field with $\wedge \neq 0$, $K \neq 0$:

Equation (21) becomes

$$2R\ddot{R} + \dot{R}^2 - \wedge R^2 - 24\pi\zeta R\dot{R} = -K. \qquad (39)$$

Case 3(a)

We assume that R = a t, where a is positive non-zero constant.

From equation (39), we obtain

$$\zeta = \frac{K + a^2 \ (1 - \wedge t^2)}{24\pi a^2 \ t}. \qquad (40)$$

Then we have

$$\rho = \frac{1}{8\pi}\left[\frac{1}{a^2 t^2}\left\{3(K + a^2) - 2c_0^2 r^{-4}\right\} - \wedge\right]. \qquad (41)$$

The charge Q, the expansion factor Θ and the scalar field V in this case remain the same as given by (26), (27) and (28) respectively.

Case 3 (b)

We assume that R = A $e^{\alpha t}$, where A and α are constants.

From equation (39), we obtain

$$\zeta = \frac{K}{24\pi\alpha A^2} e^{-2\alpha t} , \qquad (42)$$

where

$$3\alpha^2 = \wedge . \qquad (43)$$

Then we have

$$\rho = \frac{1}{8\pi}\left(\frac{3K - 2c_0^2 r^{-4}}{A^2 e^{2\alpha t}}\right). \qquad (44)$$

The charge Q, the expansion factor Θ and the scalar field V remain the same as given by (31), (32) and (28) respectively.

Case 4

Charged stiff viscous fluid in presence of zero-mass scalar field.

From Equation (21) and (22), we obtain

$$12\pi\zeta R\dot{R} = 2K + 2\dot{R}^2 + R\ddot{R} - \wedge R^2 - c_0^2 \ r^{-4} . \qquad (45)$$

Case 4(a)

We assume that R = a t, where a is positive non-zero constant.

From equation (45), we obtain

$$\zeta = \frac{1}{12\pi a^2 t}\left\{2(K + a^2) - \wedge a^2 t^2 - c_0^2 r^{-4}\right\}. \tag{46}$$

Then we have

$$p = \rho = \frac{1}{8\pi}\left[\frac{1}{a^2 t^2}\left\{3(K + a^2) - 2c_0^2 r^{-4}\right\} - \wedge\right]. \tag{47}$$

The charge Q, the expansion factor Θ and the scalar field V remain the same as given by (26), (27) and (28) respectively.

Case 4(b)

We assume that R = A $e^{\alpha t}$, where A and α are constants.

From equation (45), we obtain

$$\zeta = \frac{1}{12\pi}\left(\alpha - \frac{c_0^2 r^{-4} - 2K}{A^2 \alpha e^{2\alpha t}}\right), \tag{48}$$

where

$$2\alpha^2 = \wedge . \tag{49}$$

Then we have

$$p = \rho = \frac{1}{8\pi}\left(\alpha^2 - \frac{2c_0^2 r^{-4} - 3K}{A^2 e^{2\alpha t}}\right). \tag{50}$$

The charge Q, the expansion factor Θ and the scalar field V in this case remain the same as given by (31), (32) and (28) respectively.

Case 5

Charged disordered distribution of radiation of viscous fluid in presence of zero-mass scalar field.

From equations (21) and (22), we obtain

$$36\pi\zeta R\dot{R} = 3R\ddot{R} + 3\dot{R}^2 + 3K - 2\wedge R^2 - \frac{c_0^2}{r^4}. \tag{51}$$

Case 5(a)

We assume that R = at, where a is a positive non-zero constant.

From equation (51), we obtain

$$\zeta = \frac{1}{36\pi a^2 t}\left\{3(a^2 + K) - 2\wedge a^2 t^2 - \frac{c_0^2}{r^4}\right\}. \tag{52}$$

Then we have

$$p = \frac{1}{8\pi}\left\{\left(1 + \frac{K}{a^2} - \frac{2c_0^2}{3r^4 a^2}\right)t^{-2} - \frac{\wedge}{3}\right\} \tag{53}$$

and

$$\rho = \frac{3}{8\pi}\left\{\left(1 + \frac{K}{a^2} - \frac{2c_0^2}{3r^4 a^2}\right)t^{-2} - \frac{\wedge}{3}\right\}. \qquad (54)$$

The charge Q, the expansion factor Θ and the scalar field V remain the same as given by (26), (27) and (28) respectively.

Case 5 (b)

We assume that $R = A e^{\alpha t}$, where A and α are constants.

From equation (51), we obtain

$$\zeta = \frac{1}{36\pi\alpha A^2}\left(3K - \frac{c_0^2}{r^4}\right)e^{-2\alpha t}, \qquad (55)$$

where

$$3\alpha^2 = \wedge. \qquad (56)$$

Then we have

$$p = \frac{1}{24\pi A^2}\left(3K - \frac{2c_0^2}{r^4}\right)e^{-2\alpha t} \qquad (57)$$

and

$$\rho = \frac{1}{8\pi A^2}\left(3K - \frac{2c_0^2}{r^4}\right)e^{-2\alpha t}. \qquad (58)$$

The charge Q, the expansion factor Θ and the scalar field V remain the same as given by (31), (32) and (28) respectively.

3. Results and Discussions

In case 1(a), the bulk viscosity ζ is found to be inversely proportional to time t and it tends to zero as $t \to \infty$ and the mass density ρ varies inversely as the square of the time co-ordinate t at a fixed point. However, the reality condition for the matter distribution is given by

$$3(a^2 + K)r^4 > 2c_0^2.$$

The expansion factor Θ varies inversely as time co-ordinate t and it tends to zero as $t \to \infty$ thereby showing that the viscous fluid will cease to expand when time t is infinitely large. We also observe that when the radial co-ordinate r tends to infinity, the mass density ρ is found to vary inversely as the square of the amount of charge Q. For the flat model of the universe, the scalar field V reduces to an arbitrary constant as $r \to \infty$ while it becomes physically unrealistic for a closed model of the universe as $r \to \infty$ and it becomes a constant for the open model of the universe as $r \to \infty$. Now we observe further that for the open model of the universe, the scalar field V disappears when $d = c_0$ as $r \to \infty$ and it cannot interact with charged viscous fluid.

In case 1(b), the bulk viscosity ζ becomes a constant for a flat model of the universe while it decreases as time t increases for the closed model of the universe and it increases as time t increases for the open model of the universe. The solution for the mass density ρ is physically realistic provided

$$r^4\left(K + A^2\alpha^2 e^{2\alpha t}\right) > \frac{2}{3}c_0^2 \; .$$

The scalar expansion has a constant value corresponding to each model of the universe.

In case 2(a), the solution for the mass density ρ is physically realistic provided

$$1 > \frac{2c_0^2}{3a^2 r^4} + \frac{\wedge}{3}t^2 \; ,$$

which shows that the solution remains valid for a very small finite range of time. The radius of the universe R(t), the charge Q increase with time and vanish at t = 0, while the expansion factor Θ decreases with time.

In case 2(b), the corresponding mass density ρ tends to a constant as $t \to\infty$. The solution is physically realistic for all values of time t provided

$$A^2\,(3\alpha^2 - \wedge)\; r^4\, e^{2\alpha t} > 2c_0^2 \; .$$

In case 3(a), it has been observed that the bulk viscosity ζ decreases as t increases and at a certain stage it becomes negative as t tends to infinity, since the negative term increases at a faster rate as t increases provided $\wedge > 0$. But it tends to ∞ as $t \to\infty$ provided $\wedge < 0$. The mass density ρ decreases as time t increases and tends to $-\frac{\wedge}{8\pi}$ as $t \to\infty$. For a physically realistic solution the cosmological constant $\wedge$ should be less than zero at the time when $t \to\infty$. So the role of $\wedge$ is very important.

In case 3(b), it is observed that the mass density ρ is found to be less than zero corresponding to the value of the curvature index K = - 1. Therefore, we do not get physically realistic solution for the open model of the universe. We obtain physically realistic solution corresponding to the closed model of the universe. The bulk viscosity ζ is found to be an exponentially decreasing function of time and it tends to zero as $t \to\infty$. The reality condition obtained for the mass density ρ to be positive is that

$$r^4 > \frac{2}{3}c_0^2$$

andρ tends to zero as $t \to\infty$. The solution remains physically realistic even when the radial co-ordinate $r \to\infty$.

In case 4, we consider the problem of charged stiff viscous fluid in presence of zero-mass scalar field. In particular we consider the cases : 4(a) the radius of the universe is directly proportional to the cosmic time t and 4(b) the radius of the universe is an exponentially increasing function of time t. In case 4(a), the radius of the universe R(t), the charge Q increase with time and vanish at t = 0, while the bulk viscosity ζ, the pressure p and the density ρ decrease with time. At t = 0, ρ is infinite and so there is an occurrence of big-bang at the initial state. Since $\rho \geq 0$ for a physically realistic solution, the solution will remain physically realistic as t $\rightarrow\infty$ provided the cosmological constant $\wedge = 0$. In case 4(b), it is observed that the bulk viscosity ζ tends to a constant as t $\rightarrow\infty$. The mass density ρ also tends to a constant as t $\rightarrow\infty$ and the solution for ρ at any time t is physically realistic provided $A^2\alpha^2 e^{2\alpha t} + 3K > 2 C^2_0 r^{-4}$. This solution is found to be physically realistic only for closed model of the universe in the case where $\wedge = 0$.

In case 5(a), the bulk viscosity ζ, the pressure p and the mass density ρ tend to zero as t $\rightarrow\infty$ when $\wedge = 0$. At t = 0, ρ is infinite and so there is an occurrence of big-bang at the initial stage as in case 4(a).

In case 5(b), as time t increases, the radius of the universe R(t) and the charge Q increase. At t = 0, the radius R(t), the charge Q, the pressure p and the density ρ are finite. So, this expanding model avoids the big-bang at the initial epoch. It is also observed that the mass density ρ is found to be negative corresponding to the values of the curvature index K = 0, -1. Hence, for a physically realistic solution for all values of time in this case we shall take the value of the curvature index K = +1 thereby showing that the model under consideration must be a closed one.

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