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RESEARCH ARTICLE

α-δ_p-ALMOST COMPACTNESS FOR CRISP SUBSETS IN A FUZZY TOPOLOGICAL SPACE

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Abstract

In this paper,using the notion of α-shading of Gantner et al. [5], the idea of α-δ_p-almost compactness for crisp subsets of a space X is introduced, where the underlying structure on X is a fuzzy topology. Mainly several characterizations of such subsets are obtained, where among other things, ordinary nets and power-set filterbases are taken as supporting appliances.

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Introduction

It is seen from the literature that many mathematicians have keen interest to introduce various types of compactness in a fuzzy topological space. In 1978, Gantner, Steinlage and Warren [5] paved a new idea of compactness, called α-compactness, by means of a sort of α-level covering termed α-shading. We take resort to the same concept here to define the proposed idea of α-δ_p-almost compactness in a fuzzy topological space (henceforth to be abbreviated as fts). This new idea is also characterized by ordinary nets and power-set filters as basic appliances with the notion of adherence suitably defined via fuzzy topology of the space, the characterizations being also true for α-δ_p-almost compactness of X if one puts $A = X$.

In what follows, by (X, τ) or simply by X, we mean an fts in the sense of Chang [3]. By a crisp subset or just a subset A of an fts X, we mean that A is an ordinary subset of the set X, the underlying structure of the set X being a fuzzy topology τ, whereas a fuzzy set A in an fts X denotes, as usual, a function from X to the closed interval $I = [0, 1]$ of the real line, i.e., $A \in I^X$ [8]. The closure and interior of a fuzzy set A in X will be denoted by $cl\ A$ and $int\ A$ respectively. The support of a fuzzy set A in X will be denoted by $suppA = \{x \in X : A(x) \neq 0\}$. A fuzzy point [7] with the singleton support $x \in X$ and the value α ($0 < \alpha \leq 1$) at x will be denoted by x_α. 0_X and 1_X are the constant fuzzy sets taking respectively the constant values 0 and 1 on X. The complement of a fuzzy set A in X will be denoted by $1_X \setminus A$ [8], defined by $(1_X \setminus A)(x) = 1 - A(x)$, for each $x \in X$. For two fuzzy sets A and B in X, we write $A \leq B$ iff $A(x) \leq B(x)$, for each $x \in X$, while we write A q B to mean A is quasi-coincident (q-coincident, for short) with B [7] if there is some $x \in X$ such that $A(x) + B(x) > 1$; the negation of A q B is written as A $\bar{q}$ B. A fuzzy set A in X is called fuzzy regular open [1] if $A = int\ cl\ A$. A fuzzy set B is called a quasi-neighbourhood (q-nbd, for short) of a fuzzy set A if there is a fuzzy open set U in X such that $AqU \leq B$ [7]. If, in addition, B is fuzzy open (resp. fuzzy regular open), then B is called a fuzzy open (resp. fuzzy regular open) q-nbd of A. A fuzzy nbd [7] A of a fuzzy point x_α in an fts X is define in the usual way, i.e., whenever for some fuzzy open set U in X, $x_\alpha \leq U \leq A$; A is a fuzzy open nbd of x_α if A is fuzzy open, in addition. A fuzzy point x_α is said to be a fuzzy δ-cluster point of a fuzzy set A in an fts X if every fuzzy regular open q-nbd$U$ of x_α is q-coincident with A [4]. The union of all fuzzy δ-cluster points of A is called the fuzzy δ-closure of A and is denoted by δclA [4].

§ 1. FUZZY δ-PREOPEN AND δ-PRECLOSED SETS : SOME RESULTS

In this section, we recall some definitions and theorems from [2] for ready references.

DEFINITION 1.1. A fuzzy set A in an fts X is said to be fuzzy δ-preopen if $A \leq int\ (\delta clA)$. The complement of a fuzzy δ-preopen set is called fuzzy δ-preclosed.

DEFINITION 1.2. A fuzzy set A in an fts X is called a fuzzy δ-pre-q-nbd of a fuzzy point x_α in X if there exists a fuzzy δ-preopen set V in X such that $x_\alpha qV \leq A$.

DEFINITION 1.3. A fuzzy point x_α in an fts X is called a fuzzy δ-precluster point of a fuzzy set A in X if every fuzzy δ-pre-q-nbd of x_α is q-coincident with A.

The union of all fuzzy δ-precluster points of A is called the fuzzy δ-preclosure of A and will be denoted by $\delta - pclA$.

DEFINITION 1.4. The union of all fuzzy δ-preopen sets in an fts X, each contained in a fuzzy set A in X, is called the fuzzy δ-preinterior of A and is denoted by $\delta - pintA$.

THEOREM 1.5. The union (intersection) of any collection of fuzzy δ-preopen (δ-preclosed) sets in an fts X is also fuzzy δ-preopen (δ-preclosed).

THEOREM 1.6. In an fts X, the following statements hold :

(a) A fuzzy set A in X is δ-preopen (δ-preclosed) iff $A = \delta - pintA$ (resp. $A = \delta - pclA$).

(b) $\delta - pcl(1_X \backslash A) = 1_X \backslash \delta - pintA$, for any fuzzy set A in X.

(c) $\bigcup_{i=1}^{n} \delta - pclA_i = \delta - pcl\ (\bigcup_{i=1}^{n} A_i)$, for any finite collection $\{A_1, A_2, \dots, A_n\}$ of fuzzy sets A_1, $A_2, \dots, A_n$ in X.

(d) $\delta - pcl(\delta - pclA) = \delta - pclA$, for any fuzzy set A in X.

(e) $\delta - pintA$ (resp., $\delta - pclA$) is a fuzzy δ-preopen (resp., δ-preclosed) set in X, for any fuzzy set A in X.

From the above definitions, we get the following two results.

RESULT 1.7. For any two fuzzy δ-preopen sets A, B, $A\bar{q}B \Longrightarrow \delta - pclA\bar{q}B$ and $A\bar{q}\delta - pclB$.

PROOF. If possible, let $\delta - pclAqB$. Then there exists $x \in X$ such that $(\delta - pclA)(x) + B(x) > 1$. Let $(\delta - pclA)(x) = \alpha$. Then $x_\alpha \in \delta - pclA$and $x_\alpha qB$. As $x_\alpha \in \delta - pclA$, by Definition 1.2 and Definition 1.3, BqA, a contradiction.

Similarly, it can be proved that $A\bar{q}\delta - pclB$.

RESULT 1.8. For a fuzzy δ-preopen set U, $\delta - pcl\big(\delta - pint(\delta - pclU)\big) = \delta - pclU$.

PROOF.$U \leq \delta - pclU \Longrightarrow \delta - pintU = U \leq \delta - pint(\delta - pclU) \Longrightarrow \delta - pclU \leq \delta - pcl\ (\delta - pint(\delta - pclU))$. Again, $\delta - pclU = \delta - pcl\ (\delta - pclU)$ (by Theorem 1.6 (d)) $\geq \delta - pcl\ (\delta - pint(\delta - pclU))$. Hence $\delta - pclU = \delta - pcl\ (\delta - pint(\delta - pclU))$.

§ 2. α-δ_p-ALMOST COMPACTNESS : CHARACTERIZATIONS

As already mentioned, the notion of α-shading for an fts X was first given by Gantner et al. [5]. The concept when applied to arbitrary crisp subsets of X gets the following description.

DEFINITION 2.1. Let A be a crisp subset of an fts X . A collection $\mathcal{U}$ of fuzzy sets in X is called an α-shading (where 0 <α< 1) of A if for each $x \in A$, there is some $U_x \in \mathcal{U}$ such that $U_x(x)$ >α. If, in addition, the members of $\mathcal{U}$ are fuzzy open (δ-preopen) then $\mathcal{U}$ is called a fuzzy open (resp. δ-preopen) α-shading of A.

DEFINITION 2.2. Let X be an fts and A be a crisp subset of X . A is said to be α-compact [5] (resp., α-almost compact [6]) if each α-shading (0 <α< 1) of A by fuzzy open sets of X has a finite (resp., finite proximate) α-subshading, i.e., there is a finite subcollection$\mathcal{U}_0$ of $\mathcal{U}$ such that $\{U\colon U \in \mathcal{U}_0\}$ (resp., $\{cl\ U : U \in \mathcal{U}_0\}$is again an α-shading of A. If $A = X$ in addition, then X is called an α-compact(resp., α-almost compact) space.

We now set the following definition.

DEFINITION 2.3. Let X be an ftsand A, a crisp subset of X . A is said to be α-δ_p-almost compact if each α-shading of A by fuzzy δ-preopen sets of X has a finite δ_p-proximate α-subshading, i.e., there exists a finite subcollection$\mathcal{U}_0$ of $\mathcal{U}$ such that $\{\delta - pclU : U \in \mathcal{U}_0\}$ is again an α-shading of A. If, in addition, $A = X$, then X is called an α-δ_p-almost compact space.

It is immediate from Definition 2.3 and Theorem 1.6 that

THEOREM 2.4.(a) Every finite subset of an fts X is α-δ_p-almost compact.

(b) If A_1 and A_2 are α-δ_p-almost compact subsets of an fts X , then so is $A_1 \cup A_2$.

(c) X is α-δ_p-almost compact if X can be written as the union of finite number of α-δ_p-almost compact sets in X .

As $\delta - pclA \leq clA$, for any fuzzy set A in an fts X , it is clear from definition that α-δ_p-almost compactness imply α-almost compactness. In order to arrive at a condition, under which α-δ_p-almost compactness may imply α-compactness and hence α-almost compactness, we need to define a sort of regularity condition in our setting. The following definition serves our purpose.

DEFINITION 2.5. An fts X is said to be α-δ_p-regular, if for each point $x \in X$ and each fuzzy open set U_x in X with $U_x(x) > \alpha$, there exists a fuzzy δ-preopen set V_x in X with $V_x(x) > \alpha$ such that $\delta - pclV_x \leq U_x$.

Two other equivalent ways of defining α-δ_p-regularity are given by the following result.

THEOREM 2.6. For an fts X , the following are equivalent :

(a) X is α-δ_p-regular.

(b) For each point $x \in X$ and each fuzzy closed set $F$with $F(x) < 1 - \alpha$, there is a fuzzy δ-preopen set U such that $(\delta - pclU)(x) < 1 - \alpha$ and $F \leq U$.

(c) For each $x \in X$ and each fuzzy closed set $F$with $F(x) < 1 - \alpha$, there exist fuzzy δ-preopen sets U and V such that $V(x) > \alpha, F \leq U$ and $U\bar{q}V$.

PROOF. (a) ⇒ (b) : Let $x \in X$ and F be a fuzzy closed set with $F(x) < 1 - \alpha$. Put $V = 1_X \backslash F$. Then V is a fuzzy open set and $V(x) > \alpha$. By (a), there is a fuzzy δ-preopen set W in X with $W(x) > \alpha$and $\delta - pclW \leq V = 1_X \backslash F$. Then $F \leq 1_X \backslash \delta - pclW = \delta - pint(1_X \backslash W) = U$ (say). Then U is fuzzy δ-preopen in X . Also, $\delta - pclU = \delta - pcl(\delta - pint(1_X \backslash W)) = \delta - pcl(1_X \backslash \delta - pclW) = 1_X \backslash \delta - pint(\delta - pclW) \leq 1_X \backslash W$. Thus $(\delta - pclU)(x) \leq (1_X \backslash W)(x) < 1 - \alpha$.

(b) ⇒ (a) : Let $x \in X$ and U be any fuzzy open set in X with $U(x) > \alpha$. Let $F = 1_X \backslash U$. Then F is a fuzzy closed set in X with $F(x) < 1 - \alpha$. By (b), there is a fuzzy δ-preopen set V such that $(\delta - pclV)(x) < 1 - \alpha$and $F \leq V$. So $(1_X \backslash \delta - pclV)(x) > \alpha$, i.e., $W(x) > \alpha$ where $W = 1_X \backslash \delta - pclV = \delta - pint(1_X \backslash V)$ is a fuzzy δ-preopen set in X . Now $\delta - pclW = \delta - pcl(1_X \backslash \delta - pclV) = 1_X \backslash \delta - pint(\delta - pclV) \leq 1_X \backslash V \leq 1_X \backslash F = U$. Hence (a) follows.

(b) ⇒ (c) : For a given $x \in X$ and a fuzzy closed set F with $F(x) < 1 - \alpha$, there exists (by (b)) a fuzzy δ-preopen set U such that $(\delta - pclU)(x) < 1 - \alpha$ and $F \leq U$. Then the fuzzy point $x_{1-\alpha} \notin \delta - pclU$. Hence by Definition 1.2 and Definition 1.3, there is a fuzzy δ-preopen set V in X such that $x_{1-\alpha} qV$and $V\bar{q}U$, i.e., $V(x) + 1 - \alpha > 1 \Rightarrow V(x) > \alpha$.

(c) $\Rightarrow$ (b) : Let $x \in X$, and F, a fuzzy closed set in X with $F(x) < 1 - \alpha$. By (c), there exist fuzzy δ-preopen sets U and V such that $V(x) > \alpha, F \leq U$and $U\bar{q}V$. Now $V(x) > \alpha \Longrightarrow x_{1-\alpha}qV$. Then as $U\bar{q}V$, by Result 1.7,$\delta - pclU\bar{q}V \Longrightarrow (\delta - pclU)(x) \leq 1 - V(x) < 1 - \alpha$.

THEOREM 2.7. In an α-δ_p-regular fts X, the α-δ_p-almost compactness of a crisp subset A of X implies its α-compactness (and hence α-almost compactness).

PROOF. Let $\mathcal{U}$ be a fuzzy open α-shading of an α-δ_p-almost compact set A in an α-δ_p-regular fts X. Then for each $a \in A$, there exists $U_a \in \mathcal{U}$ such that $U_a(a) > \alpha$. By α-δ_p-regularity of X, there is a fuzzy δ-preopen set V_a in X with $V_a(a) > \alpha$ such that $\delta - pclV_a \leq U_a$ … (1).
Let $\mathcal{V}$ $=\{V_a : a \in A\}$. Then $\mathcal{V}$ is a fuzzy δ-preopen α-shading of A. By α-δ_p-almost compactness of A, there is a finite subset A_0 of A such that $\mathcal{V}_0$ $=\{\delta - pclV_a : a \in A_0\}$ is an α-shading of A. By (1), $\mathcal{U}_0 = \{U_a : a \in A_0\}$ is then a finite α-subshading of $\mathcal{U}$. Hence A is α-compact (and hence α-almost compact).

In what follows in the rest of this paper we would like to give different characterizations of α-δ_p-almost compact sets (space) via different approaches.

THEOREM 2.8. A crisp subset A of an fts X is α-δ_p-almost compact iff every family of fuzzy δ-preopen sets, the δ-preinteriors of whose δ-preclosures form an α-shading of A, contains a finite subfamily, the δ-preclosures of whose members form an α-shading of A.

PROOF. It is sufficient to observe that for a fuzzy δ-preopen set U, $U \leq \delta - pint\ (\delta - pcl\, U) \leq \delta - pcl\ (\delta - pint\ (\delta - pcl\, U)) = \delta - pcl\, U$ (by Result 1.8).

THEOREM 2.9. A crisp subset A of an fts X is α-δ_p-almost commpact iff for every collection $\{F_i : i \in \Lambda\}$ of fuzzy δ-preopen sets with the property that for each finite subset Λ_0 of Λ, there is $x \in A$ such that $\inf_{i \in \Lambda_0} F_i(x) \geq 1 - \alpha$, one has $\inf_{i \in \Lambda} (\delta - pcl\ F_i)(y) \geq 1 - \alpha$, for some $y \in A$.

PROOF. Let A be α-δ_p-almost compact, and if possible, let for a collection $\{F_i : i \in \Lambda\}$ of fuzzy δ-preopen sets in X with the stated property, $(\bigcap_{i \in \Lambda} \delta - pcl\ F_i)(x) < 1 - \alpha$, for each $x \in A$. Then $\alpha < (1_X \setminus \bigcap_{i \in \Lambda} \delta - pcl\ F_i)(x) =$

$[\bigcup_{i\in\Lambda}(1_X\setminus\delta-pcl\,F_i)](x)$, for each $x\in A$ which shows that $\{1_X\setminus\delta-pcl\,F_i : i\in\Lambda\}$ is a fuzzy δ-preopen α-shading of A. By α-δ_p-almost compactness of A, there is a finite subset Λ_0 of Λ such that $\{\delta-pcl\,(1_X\setminus\delta-pcl\,F_i) : i\in\Lambda_0\}$ is an α-shading of A. Hence α< $[\bigcup_{i\in\Lambda_0}(1_X\setminus\delta-pint\,(\delta-pcl\,F_i))](x)$ = $[1_X\setminus\bigcap_{i\in\Lambda_0}\delta-pint\,(\delta-pcl\,F_i)](x)$, for each $x\in A$. Then $(\bigcap_{i\in\Lambda_0}F_i)(x)\le[\bigcap_{i\in\Lambda_0}\delta-pint\,(\delta-pcl\,F_i)](x)<1-\alpha$, for each $x\in A$, a contradiction.

Conversely, let under the given hypothesis, A be not α-δ_p-almost compact. Then there is a fuzzy δ-preopen α-shading $\mathcal{U}=\{U_i : i\in\Lambda\}$ of A such that for every finite subset Λ_0 of Λ, $\{\delta-pcl\,U_i : i\in\Lambda_0\}$ is not an α-shading of A, i.e., there exists $x\in A$ such that $\sup_{i\in\Lambda_0}(\delta-pcl\,U_i)(x)\le\alpha$, i.e., $1-\sup_{i\in\Lambda_0}(\delta-pcl\,U_i)(x)=\inf_{i\in\Lambda_0}[1_X\setminus(\delta-pcl U_i)(x)]\ge 1-\alpha$. Hence $\{1_X\setminus\delta-pcl\,U_i : i\in\Lambda\}$ is a family of fuzzy δ-preopen sets with the stated property. Consequently, there is some $y\in A$ such that $\inf_{i\in\Lambda}[\delta-pcl\,(1_X\setminus\delta-pcl\,U_i)](y)\ge 1-\alpha$.Then $\sup_{i\in\Lambda}U_i(y)\le\sup_{i\in\Lambda}[\delta-pint(\delta-pcl\,U_i)](y)=1-\inf_{i\in\Lambda}[1_X\setminus\delta-pint\,(\delta-\mathrm{pcl}\,U_i)](y)=1-\inf_{i\in\Lambda}[\delta-pcl\,(1_X\setminus\delta-pcl U_i)](y)\le\alpha$. This shows that $\{U_i : i\in\Lambda\}$ fails to be an α-shading of A, a contradiction.

Let us now introduce the following definition :

DEFINITION 2.10. A family $\{F_i : i\in\Lambda\}$ of fuzzy sets in an fts X is said to have α-δ_p-interiorly finite intersection property (α-δ_p-IFIP, for short) in a subset A of X, if for each finite subset Λ_0 of Λ, there exists $x\in A$ such that $[\bigcap_{i\in\Lambda_0}\delta-pint\,F_i](x)\ge 1-\alpha$.

THEOREM 2.11. A crisp subset A of an fts X is α-δ_p-almost compact iff for every family $\mathcal{F}=\{F_i : i\in\Lambda\}$ of fuzzy δ-preclosed sets in X with α-δ_p-IFIP in A, there exists $x\in A$ such that $\inf_{i\in\Lambda}F_i(x)\ge 1-\alpha$.

PROOF. Let $\mathcal{F}=\{F_i : i\in\Lambda\}$ be a family of fuzzy δ-preclosed sets in X with α-δ_p-IFIP in A where A is an α-δ_p-almost compact subset of X. If possible, let for each $x\in A$, $\inf_{i\in\Lambda}F_i(x)<1-\alpha$, i.e., $(\bigcap_{i\in\Lambda}F_i)(x)<1-\alpha$ i.e., $1-(\bigcap_{i\in\Lambda}F_i)(x)>\alpha\Rightarrow[\bigcup_{i\in\Lambda}(1_X\setminus F_i)](x)>\alpha$. Therefore, $\mathcal{U}=\{1_X\setminus F_i : i\in\Lambda\}$ is a fuzzy δ-preopen α-shading of A.

By α-δ_p-almost compactness of A, there exists a finite subfamily Λ_0 of Λ such that $[\bigcup_{i\in\Lambda_0} \delta - pcl\,(1_X \setminus F_i)](x)$ = 1 – $(\bigcap_{i\in\Lambda_0} \delta - p\mathrm{int}\, F_i)(x)$ >α, i.e., $(\bigcap_{i\in\Lambda_0} \delta - pint\, F_i)(x)$ < 1–α,for each $x \in A$, which shows that $\mathscr{F}$ does not have α-δ_p-IFIP in A, a contradiction.

Conversely, let $\mathscr{U}$ = $\{U_i : i \in \Lambda\}$ be a fuzzy δ-preopen α-shading of A. Then $\mathscr{F}$ = $\{1_X \setminus U_i : i \in \Lambda\}$ is a family of fuzzy δ-preclosed sets in $X$with $\inf_{i\in\Lambda} (1_X \setminus U_i)(x)$ < 1 – α, for each $x \in A$. Then by hypothesis,$\mathscr{F}$ cannot have α-δ_p-IFIP in A. Hence for some finite subset Λ_0 of Λ, we have for each $x \in A$, $[\bigcap_{i\in\Lambda_0} \delta - pint\,(1_X \setminus U_i)](x)$ < 1 – α⇒ 1 – $(\bigcup_{i\in\Lambda_0} \delta - pcl\, U_i)(x)$ < 1 – α, for each $x \in A \Rightarrow$ $(\bigcup_{i\in\Lambda_0} \delta - pcl\, U_i)(x)$ >α, for each $x \in A \Rightarrow A$ is α-δ_p-almost compact.

§ 3. CHARACTERIZATIONS OF α-δ_p-ALMOST COMPACTNESS VIA ORDINARY NETS AND POWER-SET FILTERBASES

In this section, we characterize α-δ_p-almost compactness of a crisp subset A of an fts X via δ_p^α-adherent point of ordinary nets and power-set filterbases.

DEFINITION 3.1. Let $\{S_n : n \in (D, \geq)\}$ (where $(D, \geq)$ is a directed set) be an ordinary net in A and $\mathscr{F}$ be a power-set filterbase on A, and $x \in X$ be any crisp point. Then x is called an δ_p^α-adherent point of

(a) the net $\{S_n\}$ if for each fuzzy δ-preopen set U in X with $U(x)$ >α and for each $m \in D$, there exists $k \in D$ such that $k \geq m$ in D and $(\delta - pcl\, U)(S_k)$ >α,

(b) the filterbase$\mathscr{F}$ if for each fuzzy δ-preopen set U with $U(x)$ >α and foreach $F \in \mathscr{F}$, there exists a crisp point x_F in F such that $(\delta - pcl\, U)(x_F)$ >α.

THEOREM 3.2. A crisp subset A of an fts X is α-δ_p-almost compact iff every net in A has aδ_p^α-adherent point in A.

PROOF. Suppose A isα-δ_p-almost compact, but there is a net $\{S_n : n \in (D, \geq)\}$ in A ($(D, \geq)$ being a directed set, as usual) having no δ_p^α-adherent point in A. Then for each $x \in A$, there is a fuzzy δ-preopen set U_x in $X$with $U_x(x)$ >α, and an $m_x \in D$ such that $(\delta - pcl\, U_x)(S_n)$ ≤α, for all $n \geq m_x$ ($n \in D$). Now, $\mathscr{U}$ = $\{1_X \setminus \delta - pcl\, U_x : x \in A\}$ is a collection of fuzzy δ-preopen sets such that for any of its finite subcollection $\{1_X \setminus$

$\delta - pcl\, U_{x_1}, \ldots,\ 1_X \backslash \delta - pcl\, U_{x_k}$ } (say) , there exists $m \in D$ with $m \geq m_{x_1}, \ldots, m_{x_k}$ in D such that $(\bigcup_{i=1}^{k} \delta\text{-pcl}\, U_{x_i})(S_n) \leq$α,for all $n \geq m$ $(n \in D)$,i.e., $\inf_{1 \leq i \leq k}$ $(1_X \backslash \delta - pcl\, U_{x_i})(S_n) \geq 1 - α$, for all $n \geq m$. Hence by Theorem 2.9, there exists some $y \in A$ such that $\inf_{x \in A} [\delta - pcl\,(1_X \setminus \delta - pcl\, U_x)](y) \geq 1–α$, i.e., $(\bigcup_{x \in A} U_x)(y) \leq$ $[\bigcup_{x \in A} \delta\text{-pint}\,(\delta\text{-pcl}\, U_x)](y) =$ 1– [1– $(\bigcup_{x \in A}(\delta - pint\,(\delta\text{-pcl}\, U_x))(y)]$ = 1– $\inf_{x \in A} [\delta - pcl\,(1_X \backslash \delta - pcl\, U_x)](y) \leq$ 1 – 1 + α= α. We have, in particular, $U_y(y) \leq$α, going against the definition of U_y.

Conversely, let every net in A have a δ_p^α-adherent point in A and suppose $\{F_i : i \in \Lambda\}$ be an arbitrary collection of fuzzy δ-preopen sets in X. Let Λ_f denote the collection of all finite subsets of Λ, then (Λ_f, ≥) is a directed set, where for μ, λ∈ Λ_f, μ≥λiffμ⊇λ. For each μ∈ Λ_f, put $F_\mu = \cap\{F_i : i \in \mu\}$. Let for each μ∈ Λ_f, there be a point $x_\mu \in A$ such that $\inf_{i \in \mu} F_i(x_\mu) \geq 1 - α$ …(1). It is then enough to prove, in view of Theorem 2.9, that $\inf_{i \in \Lambda} (\delta - pcl\, F_i)(z) \geq 1 - α$ for some $z \in A$. If possible, let $\inf_{i \in \Lambda} (\delta - pcl\, F_i)(z) < 1 - α$, for each $z \in A$ …(2). Now, $S = \{ x_\mu : \mu \in (\Lambda_f, \geq) \}$ is clearly a net of points in A. By hypothesis, there is a δ_p^α-adherent point z in A of this net. By (2), $\inf_{i \in \Lambda} (\delta - pcl\, F_i)(z) < 1 - α$and hence there is some $i_0 \in \Lambda$ such that $(\delta - pcl\, F_{i_0})(z) < 1 - α$, i.e., $(1_X \backslash \delta - pcl\, F_{i_0})(z) >$α. Since z is a δ_p^α-adherent point of S, for the index $\{i_0\} \in \Lambda_f$, there is $\mu_0 \in \Lambda_f$ with $\mu_0 \geq \{i_0\}$ (i.e., $i_0 \in \mu_0$) such that $\delta - pcl\,(1_X \setminus \delta - pcl\, F_{i_0})(x_{\mu_0}) >$α, i.e., $\delta - pint\, \delta\text{-pcl}\, F_{i_0}(x_{\mu_0}) < 1 - α$. Since $i_0 \in \mu_0$, $\inf_{i \in \mu_0} F_i(x_{\mu_0}) \leq F_{i_0}(x_{\mu_0}) \leq \delta - pint\, \delta\text{-pcl}\, F_{i_0}(x_{\mu_0}) < 1 -$ α, which contradicts (1). This completes the proof.

THEOREM 3.3. A crisp subset A of an fts X is α-δ_p-almost compact iff every filterbase $\mathcal{F}$ on A has a δ_p^α-adherent point in A.

PROOF. Let A be α-δ_p-almost compact and let there exist, if possible, a filterbase $\mathcal{F}$ on A having no δ_p^α-adherent point in A. Then for each $x \in A$, there exists a fuzzy δ-preopen set U_x with $U_x(x) >$α, and an $F_x \in \mathcal{F}$ such that $(\delta - pcl\, U_x)(y) \leq$α, for each $y \in F_x$. Then $\mathcal{U} = \{U_x : x \in A\}$ is a fuzzy δ-preopen α-shading of A. By α-δ_p-almost compactness of *A*, there are finitely many points $x_1, x_2, \ldots, x_n$ in A such that $\mathcal{U}_0 = \{\delta - pcl\, U_{x_i} : i = 1, 2, \ldots, n\}$ is again an α-shading of A. Now let $F \in \mathcal{F}$ be such that $F \leq F_{x_1} \cap F_{x_2} \cap \ldots \cap F_{x_n}$. Then

$(\delta - pcl\, U_{x_i})(y) \leq \alpha$, for all $y \in F$ and for i = 1, 2, . . . , n. Thus $\mathcal{U}_0$ fails to be an α-shading of A, a contradiction.

Conversely, let the condition hold and suppose, if possible, { y_n : $n \in (D, \geq)$} be a net in A having no δ_p^α-adherent point in A ($(D, \geq)$ being a directed set, as usual). Then for each $x \in A$, there are a fuzzy δ-preopen set U_x with $U_x(x) > \alpha$ and an $m_x \in D$ such that $(\delta - pcl\, U_x)(y_n) \leq \alpha$, for all $n \geq m_x$ ($n \in D$). Thus $\mathcal{B}$ ={ $F_x : x \in A$ }, where F_x = { y_n : $n \geq m_x$ }, is a subbase for a filterbase $\mathcal{F}$ on A, where $\mathcal{F}$ consists of all finite intersections of members of $\mathcal{B}$. By hypothesis, $\mathcal{F}$ has a δ_p^α-adherent point z (say) in A. But there are a fuzzy δ-preopen set U_z with $U_z(z) > \alpha$ and an $m_z \in D$ such that $(\delta - pcl\, U_z)(y_n) \leq \alpha$, for all $n \geq m_z$, i.e., for all $p \in F_z \in \mathcal{B}$ ($\subseteq \mathcal{F}$), $(\delta - pcl\, U_z)(p) \leq \alpha$. Hence z cannot be a δ_p^α-adherent point of the filterbase $\mathcal{F}$, a contradiction. Hence by Theorem 3.2, A is α-δ_p-almost compact.

Putting A = X in the characterization theorems so far for α-δ_p-almost compact crisp subset A in an fts X, we arrive at the following formulations for α-δ_p-almost compactness of X.

THEOREM 3.4. For an fts (X , τ), the following are equivalent :

(a) X is α-δ_p-almost compact.

(b) For every family $\mathcal{U}$ = $\{U_i : i \in \Lambda\}$ of fuzzy δ-preopen sets in X such that $\{\delta - pint(\delta - pclU_i) : i \in \Lambda\}$ is an α-shading of X ,there exists a finite subset Λ_0 of Λ such that $\{\delta - pclU_i : i \in \Lambda_0\}$ is an α-shading of X.

(c) For every collection $\{F_i : i \in \Lambda\}$ of fuzzy δ-preopen sets in X with the property that for each finite subset Λ_0 of Λ, there is $x \in X$ such that $\inf_{i \in \Lambda_0} F_i(x) \geq 1 - \alpha$, one has $\inf_{i \in \Lambda} (\delta - pcl\, F_i)(y) \geq 1 - \alpha$, for some $y \in X$.

(d) For every family $\mathcal{F}$ = $\{F_i : i \in \Lambda\}$ of fuzzy δ-preclosed sets in X with α-δ_p-IFIP in X , there exists $x \in X$ such that $\inf_{i \in \Lambda} F_i(x) \geq 1 - \alpha$.

(e) Every net in X has a δ_p^α-adherent point in X .

(f) Every filterbase on X has a δ_p^α-adherent point in X .

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