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RESEARCH ARTICLE

Fine $g\delta s$ −Separation Axioms in Fine – Topological Sapce

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Abstract

In this paper, we have introduce, study and investigated the following separation axioms: $f-g\delta s-T_i$ space (i=0, 1, 2) and weaker forms of regular and normal space by using the notion of $f-g\delta s$ −open sets.

1. Introduction

Powar P. L. and Rajak K. [16], have investigated a special case of generalized topological space called fine topological space. In this space, they have defined a new class of open sets namely fine-open sets which contains all α −open sets, β −open sets, semi-open sets, pre-open sets, regular open sets etc.. By using these fine-open sets they have defined fine-irresolute mappings which include pre-continuous functions, semi-continuous function, α −continuouos function, β −continuous functions, α −irresolute functions, β −irresolute functions, etc (cf. [12]-[16]).

In this paper, we have defined strongly $g\delta s$ −continuous functions, pre-$g\delta s$ −continuous functions, semi-$g\delta s$ −continuous functions etc. in fine-topological space and investigated their properties. Also defined $g\delta s$ −seperation axioms in fine-topological space.

2. Preliminaries

Throughout this paper, (X,τ) and (Y,σ) means topological spaces on which no separation axioms are assumed. For a subset A of a space X the closure and interior of A with respect to τ are denoted by $cl(A)$ and $int\,(A)$. We use the following definitions:

Definition 2.1 A subset A of a topological space X is said to be regular open if $A=int(cl(A))$ and the compliment of regular open set is called regular closed (cf. [17]).

Definition 2.2 The largest regular open set contained in A is called the δ −interior of A and is denoted by $\delta-int(A)$ (cf. [17]).

Definition 2.3 The set A of X is called δ −open if $A=\delta-int(A)$. The complement of δ −open is called δ −closed (cf. [17]).

Definition 2.4 A subset A of a space (X,τ) is called

1) Semi-open if $A\subset cl\big(int\,(A)\big)$ (cf. [11]).
2) α −open if $A\subset int(cl(int(A)))$ (cf. [8]).

Definition 2.5 The largest semi-open set contained in A is called the semi-interior of A and is denoted by $sint(A)$(cf. [6]).

Definition 2.6 The smallest semi-closed set containing the set A is called semi-closure of A and is denoted by $scl(A)$(cf. [6]).

Definition 2.7 A subset A of X is called $g\delta s$ −closed [1] if $scl(A) \subset U$ whenever $A \subset U$ and U is δ −open in X. The family of all $g\delta s$ −closed subsets of the space X is denoted by $G\delta sc(X)$.
Definition 2.8 The smallest $g\delta s$ −closed set containing the set A is called $g\delta s$ −closure of A and is denoted by $g\delta s - cl(A)$. A set A is $g\delta s$ −closed if $g\delta s - cl(A) = A$ (cf. [1]).
Definition 2.9 The largest $g\delta s$ −open set contained in A is called $g\delta s$ −interior of A and is denoted by $g\delta s - int(A)$. A set A is $g\delta s$ −open if $g\delta s - int\ (A) = A$ (cf. [1]).
Definition 2.10 A function $f: X \to Y$ is called

1) Semi-continuous [11] if $f^{-1}(V)$ is semi-closed in X for every closed set V in Y.
2) $g\delta s$ −continuous [2] if $f^{-1}(V)$ is $g\delta s$ −-closed in X for every closed set V in Y.
3) Semi $g\delta s$ −continuous [2] if $f^{-1}(V)$ is $g\delta s$-closed in X for every semi-closed set V in Y.
4) $g\delta s$ −irresolute [7] if $f^{-1}(V)$ is $g\delta s$-closed in X for every $g\delta s$ −closed set V in Y.
5) $g\delta s$ −open [3] if $f(V)$ is $g\delta s$ −open in Y for every closed set V in X.
6) $pg\delta s$ −open [3] if $f(V)$ is $g\delta s$ −open in Y for every semi-open set V in X.
7) $quasi - g\delta s$ −open [4] if $f(V)$ is open in Y for every $g\delta s$ −open set V in X.
8) Strongly $g\delta s$ −open [4] if $f(V)$ is $g\delta s$ −open in Y for every $g\delta s$ −open set V in X.
9) Semi-closed [9] if $f(V)$ is semi-closed in Y for every closed set V in X.
10) Pre-closed [10] if $f(V)$ is closed in Y for every semi-closed set V in X.

Definition 2.11 A topological space X is said to be $g\delta s - T_0$ space if for each pair of distinct points x and y of X, there exists a $g\delta s$ −open set containing one point but not the other (cf. [5]).
Definition 2.12 A topological space X is said to be $g\delta s - T_1$ space if for any pair of distinct point x and y, there exists a $g\delta s$ −open sets G and H such that $x \in G, y \notin G$ and $x \notin H, y \in H$ (cf. [5]).
Definition 2.13 A topological space X is said to be $g\delta s - T_2$ space if for any pair of distinct points x and y, there exists disjoint $g\delta s$ −open sets G and H such that $x \in G$ and $y \in H$ (cf. [5]).
Definition 2.14 A topological space X is said to be $g\delta s$ −regular if for each closed set F and each point $x \notin F$, there exist disjoint $g\delta s$ −open sets U and V such that $x \in U$ and F⊂ V (cf. [5]).
Definition 2.15 Let (X, τ) be a topological space we define
$\tau(A_\alpha) = \tau_\alpha$ (*say*) = $\{G_\alpha (\neq X) : G_\alpha \cap A_\alpha = \phi$, for $A_\alpha \in \tau$ and $A_\alpha \neq \phi, X$, for some $\alpha \in J$, where *J* is the index set.$\}$
Now, we define

$$\tau_f = \{\phi, X, \cup_{\{\alpha \in J\}} \{\tau_\alpha\}\}$$

The above collection τ_f of subsets of *X* is called the fine collection of subsets of *X* and (X, τ, τ_f) is said to be the fine space *X* generated by the topology τ on *X* (cf. [16]).
Definition 2.16 A subset *U* of a fine space *X* is said to be a fine-open set of *X*, if *U* belongs to the collection τ_f and the complement of every fine-open sets of *X* is called the fine-closed sets of *X* and we denote the collection by F_f (cf. [16]).
Definition 2.17 Let A be a subset of a fine space *X*, we say that a point $x \in X$ is a fine limit point of *A* if every fine-open set of *X* containing *x* must contains at least one point of A other than *x* (cf. [16]).
Definition 2.18 Let *A* be the subset of a fine space *X*, the fine interior of *A* is defined as the union of all fine-open sets contained in the set *A* i.e. the largest fine-open set contained in the set *A* and is denoted by f_{Int} (cf. [16]).
Definition 2.19 Let *A* be the subset of a fine space *X*, the fine closure of *A* is defined as the intersection of all fine-closed sets containing the set *A* i.e. the smallest fine-closed set containing the set *A* and is denoted by f_{cl} (cf. [16]).
Definition 2.20 A function $f : (X, \tau, \tau_f) \to (Y, \tau', \tau_f')$ is called fine-irresolute (or f-irresolute) if $f^{-1}(V)$ is fine-open in *X* for every fine-open set *V* of *Y* (cf. [16]).

3. Fine $g\delta s$ −open sets and Fine $g\delta s$ −continuous functions

In this section, we have defined fine $g\delta s$ −open sets and fine $g\delta s$ −continuous functions.
Definition 3.1 The largest f-regular open set contained in A is called fine-δ −interior of A and is denoted by $f - \delta - Int(A)$.
Definition 3.2 The smallest f-regular closed set containing A is called fine−δ −closure of A and is denoted by $f - \delta - cl(A)$.
Definition 3.3 A subset A of X is said to be fine−$g\delta s$ −closed if $f - s - cl(A) \subset U$ whenever $A \subset U$ and U is fine – δ −open in X. The family of all $f - g\delta s$ −closed sets is denoted by $FG\delta SC(X)$.
Definition 3.4 The intersection of all fine−$g\delta s$ −closed sets containing a set A is called $f - g\delta s$ −closure of A and is denoted by $f - g\delta s - cl(A)$. A set A is $f - g\delta s$ −closed iff $f - g\delta s - cl(A) = A$.
Definition 3.5 The union of all $fg\delta s$ −open sets contained in A is called $f - g\delta s$ −interior of A and is denoted by $f - g\delta s - Int(A)$. A set A is $f - g\delta s$ −open if and only if $f - g\delta s - Int(A) = A$.

Definition 3.6 A function $f: X \to Y$ is called fine$-g\delta s-$continuous if $f^{-1}(V)$ is fine$-g\delta s-$closed in X for every fine-closed set V in Y.
Definition 3.7 A function $f: X \to Y$ is called fine-semi$-g\delta s-$continuous if $f^{-1}(V)$ is fine$-g\delta s-$closed in X for every fine-semi-closed set V in Y.
Definition 3.8 A function $f: X \to Y$ is called fine$-g\delta s-$irresolute if $f^{-1}(V)$ is fine$-g\delta s-$closed in X for every fine$-g\delta s-$closed set V in Y.
Definition 3.9 A function $f: X \to Y$ is called fine$-g\delta s-$open if $f(V)$ is fine$-g\delta s-$open in Y for every fine-open set V in X.
Definition 3.10 A function $f: X \to Y$ is called fine$-pg\delta s-$open if $f(V)$ is fine$-g\delta s-$closed in Y for every fine-semi-open set V in X.
Definition 3.11 A function $f: X \to Y$ is called fine-quasi- $g\delta s-$open if $f(V)$ is fine-open in Y for every $fg\delta s-$open set V in X.
Definition 3.12 A function $f: X \to Y$ is called fine-strongly$-g\delta s-$open if $f(V)$ is fine$-g\delta s-$closed in Y for every fine-$g\delta s$-open set V in X.
Definition 3.13 A function $f: X \to Y$ is called fine-semi-closed if $f(V)$ is fine-semi-closed in Y for every fine-closed set V in X.
Definition 3.14 A function $f: X \to Y$ is called fine-pre closed if $f(V)$ is fine-closed in Y for every fine-semi-open set V in X.

Remark 3.15

1) Every $g\delta s-$open set is fine open and every $g\delta s-$closed set if fine-closed.
2) Every semi-closed and pre-closed set is fine closed.

Remark 3.17 By the Definition 2.20, Definition 2.10 and Remark 3.15, we conclude the following:

$\Rightarrow$ Semi-continuous
$\Rightarrow$ $g\delta s-$continuous
$\Rightarrow$ Semi-$g\delta s$ -continuous
$\Rightarrow$ $g\delta s$ -irresolute
Fine-irresolute mapping $\Rightarrow$ $g\delta s$ $-$open
$\Rightarrow$ $p\, g\delta s$ $-$open
$\Rightarrow$ $quasi - g\delta s$ $-$open
$\Rightarrow$ Strongly $g\delta s$ $-$open
$\Rightarrow$ Semi-open
$\Rightarrow$ Pre-closed

4. Fine $g\delta s$ $-$separaton axioms

In this section, we introduce and study weak separation axioms such as $f - g\delta s - T_0$, $f - g\delta s - T_1$ and $f - g\delta s - T_2$ spaces and obtain some of their properties.

Definition 4.1 A topological space X is said to be $f - g\delta s - T_0$ space if for each pair of distinct points x and y of X, there exists a $f - g\delta s$ $-$open set containing one point but not the other.

Theorem 4.1 A topological space X is a $f - g\delta s - T_0$ space if and only if $f - g\delta s$ $-$closures of distinct points are distinct.

Proof. Let x and y be distinct points of X. Since, X is $f - g\delta s - T_0$ space, there exists a $f - g\delta s$ $-$open set G such that $x \in G$ and $y \notin G$. Consequently, $X - G$ is a $f - g\delta s$ $-$closed set containing y but not x. But, $f - g\delta s - cl(y)$is the intersection of all $f - g\delta s$ $-$closed set containing y. Hence, $y \in f - g\delta s - cl(y)$ but $x \notin f - g\delta s - cl(y)$ as $x \notin X - G$. Therefore, $f - g\delta s - cl(x) \neq f - g\delta s - cl(y)$.
Conversely, let $f - g\delta s - cl(x) \neq f - g\delta s - cl(y)$ for $x \neq y$. Then, there exists at least one point $z \in X$ such that $z \in f - g\delta s - cl(x)$ but $z \notin f - g\delta s - cl(y)$. We claim $x \notin f - g\delta s - cl(y)$, because if $x \in f - g\delta s - cl(y)$ then $\{x\} \subset f - g\delta s - cl(y) \Rightarrow f - g\delta s - cl(x) \subset f - g\delta s - cl(y)$. So $z \in f - g\delta s - cl(y)$, which is a contradiction, hence $x \notin f - g\delta s - cl(y) \Rightarrow x \in X - fg\delta scl(y)$, which is a $f - g\delta s$ $-$open set containing x but not y. Hence, X is a $f - g\delta s - T_0$ space.

Theorem 4.2 If $f: X \to Y$ is a bijection strongly $f - g\delta s$ $-$open and X is $f - g\delta s - T_0$ space, then Y is also $f - g\delta s - T_0$ space.

Proof. Let y_1 and y_2 be two distinct points of Y. Since f is bijection there exist distinct points x_1 and x_2 of X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since, X is $f - g\delta s - T_0$ space there exists a $f - g\delta s$ $-$open set G such that $x_1 \in G$ and $x_2 \in G$. Therefore, $y_1 = f(x_1) \in f(G)$ and $y_2 = f(x_2) \notin f(G)$. Since, f being strongly fine-$g\delta s$ $-$open function, f(G) is fine-$g\delta s$ $-$open in Y. Thus, there exists a $f - g\delta s$ $-$open set f(G) in Y such that $y_1 \in f(G)$ and $y_2 \notin f(G)$. Therefore, Y is $f - g\delta s - T_0$ space.

Definition 4.2 A topological space X is said to be $f - g\delta s - T_1$ space if for any pair of distinct points x and y there exist a $f - g\delta s$ −open sets G and H such that $x \in G, y \notin G$ and $x \notin H, y \in H$.

Theorem 4.3 A topological space X is $f - g\delta s - T_1$space if and only if singletons are $f - g\delta s$ −closed sets.

Proof. Let X be a $f - g\delta s - T_1$space and $x \in X$. Let $y \in X - \{x\}$. Then, for $x \neq y$, there exists $f - g\delta s$ −open set U_y such that $y \in U_y$ and $x \notin U_y$. Consequently, $y \in U_y \subset X - \{x\}$. That is $X - \{x\} = \cup \{U_y : y \in X - \{x\}\}$, which is $f - g\delta s$ −open. Hence, $\{x\}$ is $f - g\delta s$ −closed set.

Conversely, suppose $\{x\}$ is $f - g\delta s$ −closed set for every $x \in X$. Let x and $y \in X$ with $x \neq y$. Now, $x \neq y \Rightarrow y \in X - \{x\}$. Hence, $X - \{x\}$ is $f - g\delta s$ −open set containing y but not x. Similarly, $X - \{y\}$ is $f - g\delta s$ −open set containing x but not y. Therefore, X is $f - g\delta s - T_1$ space.

Theorem 4.4 The property being $f - g\delta s - T_1$ space is preserved under bijection and strongly fine-$g\delta s$ −open function.

Proof. Let $f: X \to Y$ be bijection and strongly fine−$g\delta s$ −open function. Let X be a $f - g\delta s - T_1$ space and y_1 and y_2 be two distinct points of Y. Since f is bijective there exists distinct points x_1, x_2 of X such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Now, X being a $f - g\delta s - T_1$ space, there exist $f - g\delta s$ −open sets G and H such that $x_1 \in G, x_2 \notin G$ and $x_1 \notin H, x_2 \in H$. Therefore, $y_1 = f(x_1) \in f(G)$ but $y_2 = f(x_2) \notin f(G)$ and $y_2 = f(x_2) \in f(H)$ and $y_1 = f(x_1) \notin f(H)$. Now, f being strongly $f - g\delta s$ −open, f(G) and f(H) are $f - g\delta s$ −open subsets of Y such that $y_1 \in f(G)$ but $y_2 \notin f(G)$ and $y_2 \in f(H)$ and $y_1 \notin f(H)$. Hence, Y is $f - g\delta s - T_1$ space.

Theorem 4.5 If $f: X \to Y$ be $f - g\delta s$ −continuous injection and Y be T_1, then X is $f - g\delta s - T_1$.

Proof. If $f: X \to Y$ be $f - g\delta s$ −continuous injection and Y be T_1. For any two distinct points x_1, x_2 of X there exist distinct points y_1, y_2 of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since, Y is T_1 −space there exist fine-open sets U and V in Y such that $y_1 \in U, y_2 \notin U$and $y_1 \notin V, y_2 \in V$. That is $x_1 \in f^{-1}(U), x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V), x_2 \notin f^{-1}(U)$. Since, f is $f - g\delta s$ −continuous, $f^{-1}(U), f^{-1}(V)$ are $f - g\delta s$ −open sets in X. Thus, for two distinct points x_1, x_2 of X there exist $f - g\delta s$ −open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U), x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V), x_2 \notin f^{-1}(U)$. Therefore, X is $f - g\delta s - T_1$ space.

Theorem 4.6 If $f: X \to Y$ be $f - g\delta s$ −irresolute injective and Y $f - g\delta s - T_1$ space, then X is $f - g\delta s - T_1$ space.

Proof. Let x_1, x_2 be pair of distinct points in X. Since f is injective there exist distinct points y_1, y_2 of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since, Y is $f - g\delta s - T_1$ space there exist $f - g\delta s$ −open sets U and V in Y such that $y_1 \in U, y_2 \notin U$ and $y_1 \notin V, y_2 \in V$. That is $x_1 \in f^{-1}(U), x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V), x_2 \notin f^{-1}(U)$. Since f is $f - g\delta s$ −irresolute $f^{-1}(U), f^{-1}(V)$ are $f - g\delta s$ −open sets in X. Thus for two distinct points x_1, x_2 of X there exist $f - g\delta s$ −open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U), x_1 \notin f^{-1}(V)$ and $x_2 \in f^{-1}(V)$, $x_2 \notin f^{-1}(U)$. Therefore, X is $f - g\delta s - T_1$ space.

Definition 4.3 A topological space X is said to be $f - g\delta s - T_2$ space if for any pair of distinct points x and y, there exist disjoint $f - g\delta s$ −open sets G and H such that $x \in G$ and $y \in H$.

Theorem 4.7 If $f: X \to Y$ is $f - g\delta s$ −continuous injection and Y is T_2then X is $f - g\delta s - T_2$ space.

Proof. Let $f: X \to Y$ be $f - g\delta s$ −continuous injection and Y is T_2. For any two distinct points $x_1, x_2$of X there exist distinct points y_1, y_2 of Y such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$. Since, Y is T_2 space there exist disjoint fine-open sets U and V in Y such that $y_1 \in U$ and $y_2 \in V$. That is $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Since, f is fine-$g\delta s$ −continuous $f^{-1}(U)$ and $f^{-1}(V)$ are $f - g\delta s$ −open sets in X. Further f is injective, $f^{-1}(U) \cap f^{-1}(V) = f^{-1}(U \cap V) = f^{-1}(\phi) = \phi$. Thus, for two disjoint points x_1, x_2 of X there exist disjoint $f - g\delta s$ −open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Therefore, X is $f - g\delta s - T_2$space.

Theorem 4.8 If $f: X \to Y$ is $f - g\delta s$ −irresolute injective function and Y is $f - g\delta s - T_2$ space then X is $f - g\delta s - T_2$ space.

Proof. Let x_1, x_2 be pair of distinct points in X. Since f is injective there exist distinct points y_1, y_2 of Y such that $y_1 = f(x_1), y_2 = f(x_2)$. Since Y is $f - g\delta s - T_2$ space there exist disjoint $f - g\delta s$ −open sets U and V in Y such that $y_1 \in U$ and $y_2 \in V$ in Y. That is $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Since, f is $f - g\delta s$ −irresolute injective $f^{-1}(U), f^{-1}(V)$ are distinct $f - g\delta s$ −open sets in X. Thus, for two distinct points x_1, x_2 of X there exist disjoint $f - g\delta s$ −open sets $f^{-1}(U)$ and $f^{-1}(V)$ such that $x_1 \in f^{-1}(U)$ and $x_2 \in f^{-1}(V)$. Therefore, X is $f - g\delta s - T_2$ space.

Theorem 4.9 In any fine-topological space the following are equivalent:

1) X is $f - g\delta s - T_2$ space.
2) For each $x \neq y$, there exists a $f - g\delta s$ −open set U such that $x \in U$ and $y \notin f - g\delta s - cl(U)$.

3) For each $x \in X, \{x\} = \cap\{f - g\delta s - cl(U): U$ is $f - g\delta s$ −open set in X and $x \in U$.

Proof.

1)⟹2) Assume (1) holds. Let $x \in X$ and $x \neq y$, then there exist disjoint $f - g\delta s$ −open sets U and V such that $x \in U$ and $y \in V$. Clearly, $X - V$ is $f - g\delta s$ −closed set. Since, $U \cap V = \phi, U \subset X - V$. Therefore, $f - g\delta s - cl(U) \subset f - g\delta s - cl(X - V) = X - V$. Now, $y \notin X - V \Rightarrow y \notin f - g\delta s - cl(U)$.

2)⟹3) For each $x \neq y$, there exust a $f - g\delta s$ −open set U such that $x \in U$ and $y \notin f - g\delta s - cl(U)$. So, $y \notin \cap\{f - g\delta s - cl(U): U$is $fg\delta s$ −open in X and $x \in U\} = \{x\}$.

3)⟹1) Let $x, y \in X$ and $x \neq y$. By hypothesis there exist a $fg\delta s$ −open set U such that $x \in U$ and $y \notin fg\delta s - cl(U)$. This implies there exists a $fg\delta s$ −closed set V such that $y \notin V$. Therefore, $y \in X - V$ and $X - V$ is $fg\delta s$ −open set. Thus, there exist two disjoint $fg\delta s$ −open sets U and X-V such that $x \in U$and $y \in X - V$. Therefore X is $fg\delta s - T_2$ space.

5. Fine−$g\delta s$ −regular space

In this section, we introduce and study $fg\delta s$ −regular space and some of their properties.

Definition 5.1 A topological space X is said to be $fg\delta s$ −regular space if for each fine-cloed set F and each point $x \notin F$, there exist disjoint $fg\delta s$ −open sets U and V such that $x \in U$ and $F \subset V$.

Theorem 5.1 Every $fg\delta s$ −regular T_0 space is$fg\delta s - T_2$.

Proof. Let $x, y \in X$ such that $x \neq y$. Let X be a T_0 space and V be a fine –open set which contains x but not y. Then, X-V is a closed set containing y but not x. Now, by $fg\delta s$ −regularity of X there exist disjoint $fg\delta s$ −open sets U and W such that $x \in U$ and $X - V \subset W$. Since, $y \in X - V, y \in W$. Thus, for $x, y \in X$ with $x \neq y$, there exist disjoint open sets U and W such that $x \in U$ and$y \in W$. Hence, X is $fg\delta s - T_2$.

Theorem 5.2 If $f: X \to Y$ is fine-continuous bijective $fg\delta s$ −open function and X is a fine-regular space, then Y is $fg\delta s$ −regular.

Proof. Let F be a fine-closed set in Y and $y \notin F$. take $y = f(x)$ for some $x \in X$. Since f is continuous surjective $f^{-1}(F)$ is fine-closed set in X and $x \notin f^{-1}(F)$. Now, since X is fine−regular, there exist disjoint fine-open sets U and V such that $x \in U$ and $f^{-1}(F) \subset V$. That is $y = f(x) \in f(U)$ and $F \subset f(V)$. Since, f is $fg\delta s$ −open function f(U) and f(V) are $fg\delta s$ −open sets in Y and f is bijective. $f(U) \cap f(V) = f(U \cap V) = f(\phi) = \phi$. Therefore, Y is $fg\delta s$ −regular.

Theorem 5.3 If $f: X \to Y$ is $fg\delta s$ −continuous, fine-closed injection and Y is fine-regular, then X is $fg\delta s$ −regular.

Proof. Let F be a fine-closed set in X and $x \notin F$. Since, f is closed injection f(F) is fine-closed set in Y such that $f(x) \notin f(F)$. Now, Y is fine-regular there exist disjoint fine-open sets G and H such $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since, f is $fg\delta s$ −continuous, $f^{-1}(G)$ and $f^{-1}(H)$ are $fg\delta s$ −open sets in X. Further $f^{-1}(G) \cap f^{-1}(H) = \phi$. Hence, X is $fg\delta s$ −regular.

Theorem 5.4 If $f: X \to Y$ is fine-semi-$g\delta s$ −continuous, fine-closed injection and Y is fine-semi regular, then X is $fg\delta s$ −regular.

Proof. Let F be a fine-closed set in X and $x \notin F$. Since, f is fine-closed injection f(F) is fine-closed set in Y such that $f(x) \notin f(F)$. Now, Y is fine-semi-regular, there exist disjoint fine-semi-open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since, f is fine-semi-$f\delta s$ −continuous $f^{-1}(G)$ and $f^{-1}(H)$ are $fg\delta s$ −open sets in X. Further, $f^{-1}(G) \cap f^{-1}(H) = \phi$. Hence, X is $fg\delta s$ −regular.

Theorem 5.5 If $f: X \to Y$ is $fg\delta s$ −irresolute, fine-closed injection and Y is $fg\delta s$ −regular then X is $fg\delta s$ −regular.

Proof. Let F be a fine-closed set in X and $x \notin F$. Since, f is fine-closed injection f(F) is fine-closed set in Y such that $f(x) \notin f(F)$. Now, Y is $fg\delta s$ −regular, there exist diajoint $fg\delta s$ −open sets G and H such that $f(x) \in G$ and $f(F) \subset H$. This implies $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since, f is $fg\delta s$ −irresolute $f^{-1}(G)$ and $f^{-1}(H)$ are $fg\delta s$ −open sets in X. Further $f^{-1}(G) \cap f^{-1}(H) = \phi$. Hence, X is $fg\delta s$ −regular.

6. Fine-$g\delta s$ −normal space

In this section, we introduce and study $fg\delta s$ −normal spaces and some of their properties.

Definition 6.1 A topological space X is said to be $fg\delta s$ −normal if every pair of disjoint fine-closed sets E and F of X there exist disjoint $fg\delta s$ −open sets U and V such that $E \subset U$ and $F \subset V$.

Theorem 6.1 The following statements are equivalent for a fine-topological space X:

1) X is $fg\delta s$ −normal.
2) For each fine-closed set A and for each fine-open set U containing A, there exist a $fg\delta s$ −open set V containing A such that $fg\delta s - cl(V) \subset U$.

3) For each pair of disjoint fine-closed sets A and B there exists a $fg\delta s$ $-$open set U containing A such that $fg\delta s-cl(U)\cap B=\phi$.

Proof.

1)⟹2) Let A be a fine-closed set and U be a fine-open set containing A. Then, $A\cap(X-U)=\phi$ and therefore they are disjoint fine-closed sets in X. Since, X is $fg\delta s$ $-$normal, there exist disjoint $fg\delta s$ $-$open sets V and W such that $A\subset U, X-U\subset W$ that is $X-W\subset U$. Now $V\cap W=\phi$, implies $V\subset X-W$. Therefore, $fg\delta s-cl(V)\subset fg\delta s-cl(X-W)=X-W$, because X-W is $fg\delta s$ $-$closed set. Thus, $A\subset V\subset fg\delta scl(V)\subset X-W\subset U$. That is $A\subset V\subset fg\delta scl(V)\subset U$.

2)⟹3) Let A and B be disjoint fine-closed sets in X, then $A\subset X-B$ and X-B is a fine-open set containing A. By (2), there exists a $fg\delta s$ $-$open set U such that $A\subset U$ and $fg\delta s-cl(U)\subset X-B$, which implies $fg\delta s-cl(U)\cap B=\phi$.

3)⟹ 1) Let A and B be disjoint fine-closed sets in X. By (3) there exist a $fg\delta s$ $-$open set U such that $A\subset U$ and $fg\delta s-cl(U)\cap B=\phi$ or $B\subset X-fg\delta s-cl(U.)$ Now, U and $X-fg\delta s-cl(U)$ are disjoint $fg\delta s$ $-$open sets of X such that $A\subset U$ and $B\subset X-fg\delta s-cl(U)$. Hence, X is $fg\delta s$ $-$normal.

Theorem 6.2 If X is fine semi-normal, then the following statements are true:

1) For each $f-\delta$ $-$closed set A and every $f-g\delta s$ $-$open set B such that $A\subset B$ there exists a fine-semi-open set U such that $A\subset U\subset f-c-cl(U)\subset B$.
2) For every $fg\delta s$ $-$closed set A and every $f-\delta$ $-$open set containing A, there exists a fine-semi-open set containing U such that $A\subset U\subset f_s-cl(U)\subset B$.

Proof.

1) Let A be a $f-\delta$ $-$closed set and B be a $fg\delta s$ $-$open set such that $A\subset B$. Then, $A\cap(X-B)=\phi$. Since, A is a $f-g$ $-$closed set and $X-B$ be a $fg\delta s$ $-$closed, by Theorem 6.1, there exists fine-semi-open sets U and V such that $A\subset U, X-B\subset V$ and $U\cap V=\phi$. Thus, $A\subset U\subset X-V\subset B$. Since, $X-V$ is fine-semi-closed $fscl(U)\subset X-V$. Therefore, $A\subset U\subset f_s-cl(U)\subset B$.
2) Let A be a $fg\delta s$ $-$closed set and B be a $f-\delta$ $-$open set such that $A\subset B$. Then, $X-B\subset X-A$. Since, X is fine-semi-normal and X-A is a $fg\delta s$ $-$open set containing $f-\delta$ $-$closed set $X-B$, by(1) there exists a fine-semi-open set G such that $X-B\subset G\subset f_s cl(G)\subset X-A$. That is, $A\subset X-f_s-cl(G)\subset X-G\subset B$. Let $U=X-f_s-cl(G)$, then U is fine-semi-open set and $A\subset U\subset f_s-cl(U)\subset B$.

7. Conclusion

The author has defined strongly $g\delta s$ $-$continuous functions, pre-$g\delta s$ $-$continuous functions, semi-$g\delta s$ $-$continuous functions etc. in fine-topological space and investigated their properties also defined $g\delta s$ $-$seperation axioms in fine-topological space. This concept may be useful in Quantum physics, Quantum Mechanics, Quantum gravity etc.

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