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**RESEARCH ARTICLE**

# A Comparison of the Classical Estimators with the Bayes Estimators of One Parameter Inverse Rayleigh Distribution

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***Abstract***

In this paper, we obtained the best estimate for the scale parameter (θ) of one parameter Inverse Rayleigh distribution, through the comparison of some classical estimators [Maximum Likelihood Estimator (MLE), Uniformly Minimum Variance Unbiased Estimator (UMVUE), and Minimum Mean Squared Error Estimator (MinMSE)]and Bayes estimators under Generalized squared error loss function. In order to get a better understanding of our Bayesian analysis we consider the non-informative prior for θ using Jefferys prior information, as well as informative prior density represented by Exponential distribution.

The comparison was based on a Monte Carlo simulation study, on the performance of these estimators with respect to the mean square error (MSE). The results showed that the best estimator for the one parameter Inverse Rayleigh distribution is Bayes estimator under Generalized squared error loss function (when $a_0$ is much greater than $a_1$, and $a_1$ is *greater* than $a_2$) with Exponential prior when the scale parameter of Exponential prior is less than 1.



## INTRODUCTION

The Inverse Rayleigh Distribution has many applications in the area of reliability studies. It was introduced in literature by Trayer (1964), in reliability and survival studies, many life distributions are characterized by a monotonic failure rate. Voda (1972) has mentioned that the distribution of lifetimes of several types of experimental units can be approximated by the Inverse Rayleigh distribution [11]. Gharraph (1993) derived five measures of location for the inverse Rayleigh distribution. These measures are the mean, harmonic mean, geometric mean, mode, and the median. He, also, estimated the unknown parameter using different methods of estimation. A comparison of these estimates was discussed numerically in term of their bias and root mean square error [5]. Abdel-Monem (2003) developed some estimation and prediction results for the inverse Rayleigh distribution [1]. Also, Shawky, and Bakoban (2010) have derived moments and moment generating functions from EG distribution and have made some statistical inferences based on record values [9].

The object of the present paper is to obtain the best estimate for the scale parameter (θ) of one parameter Inverse Rayleigh distribution, through the comparison of some Classical estimators with the Bayesian estimators. Also, we have mainly considered different estimators and compare their performance through Monte-Carlo simulation. The performance of these fifth estimators compared according to the mean squared error (MSE).

## 1. ONE PARAMETER INVERSE RAYLEIGH DISTRIBUTION

The one parameter Inverse Rayleigh Distribution with probability density function (p.d.f) with scale parameter (θ) is given by [2]:

$$f\left( x;\theta \right)=\frac{2\theta}{x^3}\,.\,e^{-\frac{\theta}{x^2}} \quad ; \quad x>0\ ,\ \theta>0 \qquad (1)$$

The corresponding cumulative distribution function (C.D.F) is:

$$F\left( x;\theta \right)= e^{-\frac{\theta}{x^2}} \quad ; \quad x>0\ ,\ \theta>0 \qquad (2)$$

The Reliability, failure rate and the Cumulative failure rate (Hazard Rate) functions of Inverse Rayleigh distribution are given, respectively, by:

$$R\left( t,\theta \right)= 1-F\left( t;\theta \right)= 1-e^{-\frac{\theta}{t^2}} \qquad (3)$$

$$h\left( t,\theta \right) = \frac{f\left( t;\theta \right)}{R\left( t;\theta \right)} \qquad (4)$$

$$H\left( t,\theta \right)= -Ln\,R\left( t \right)= -Ln\,(1-e^{-\frac{\theta}{t^2}}) \qquad (5)$$

### 2. SOME CLASSICAL ESTIMATORS

In this section, some classical estimators of the scale parameter (θ) of the one parameter Inverse Rayleigh distribution, such as the Maximum Likelihood Estimator, the Uniformly Minimum Variance Unbiased Estimator, and the Minimum Mean Squared Error Estimator are obtained with the assumption that , the scale parameter (θ) is fixed and constant , but it is unknown . Also, these methods are called Non-Bayesian methods. This section includes derivative and discussion of (MSE's) for these three classical estimators for the scale parameter (θ).

### 3.1 MAXIMUM LIKELIHOOD ESTIMATOR (MLE)

The Maximum likelihood method was proposed by R. A. Fisher (1920)[7], and has been widely used since . This method is the most popular procedure in estimating the unknown parameter θ which specifies a probability function $f(x;\theta)$ , based on the observations $(x_1, x_2, x_3, ...., x_n)$ which were independently sample from the inverse Rayleigh distribution .

The maximum likelihood estimator ($\hat{\theta}$) of the parameter (θ) that maximize the likelihood function from (1) is given by:

$$L\left( x_1, x_2, x_3, ...., x_n;\theta \right)= 2^n\,\theta^n \prod_{i=1}^{n}\frac{1}{x_i^3}\,.\exp\left[ -\theta\sum_{i=1}^{n}\frac{1}{x_i^2} \right] \qquad (6)$$

Taking the logarithm for the likelihood function and differentiating with respect to (θ), we get:

$$\frac{\partial LnL\left( x_i,\theta \right)}{\partial\theta} = \frac{n}{\theta}-\sum_{i=1}^{n}\frac{1}{x_i^2}$$

$$Let \quad \frac{\partial LnL\left( x_i,\theta \right)}{\partial\theta} = 0 \quad ; \quad Hence:$$

$$\hat{\theta}_{ML} = \frac{n}{\sum_{i=1}^{n}\frac{1}{x_i^2}} = \frac{n}{T} \quad , \text{ where, } \quad T=\sum_{i=1}^{n}\frac{1}{x_i^2} \qquad (7)$$

### 3.2 UNIFORMLY MINIMUM VARIANCE UNBIASED ESTIMATORS (UMVUE)

The first Uniformly Minimum variance unbiased estimator was obtained by Aitken and Silverstone (1942) in the situation in which the information inequality yields the same result.

Here, we obtain the Uniformly Minimum Variance Unbiased Estimator (UMVUE) of the scale parameter (θ) for the one parameter Inverse Rayleigh distribution which belongs to the exponential family, where:

$$a(\theta) = 2\theta \quad ; \quad b(x) = x^{-3} \quad ; \quad c(\theta) = -\theta \quad ; \quad d(x) = x^{-2}$$

Therefore, the statistic $T = \sum_{i=1}^{n}\left(\frac{1}{x_i^2}\right)$ is a complete sufficient statistic for (θ).

Let $Y = \frac{1}{X^2}$, which implies that, $X = Y^{-\frac{1}{2}}$, then:

$$g(y) = f\left(x = y^{-\frac{1}{2}}\right) . \left|\frac{dx}{dy}\right|$$

$$g(y) = \theta\, e^{-\theta y} \qquad , \qquad 0 < \mathrm{y}$$

Therefore, $Y \sim Exp(\theta)\ and\ hence\ , T = \sum_{i=1}^{n} Y_i \sim \Gamma(n, \theta)$ with the density function $\prod(t)$, where

$$\prod(t) = \frac{\theta^n}{\Gamma(n)} t^{n-1} . e^{-\theta t} \quad ; \quad t > 0 \ , \ \theta > 0 \ , \ n > 0$$

Now,

$$E\left(\frac{1}{T}\right) = \int_0^\infty \frac{1}{t} . \frac{\theta^n}{\Gamma(n)} t^{n-1} . e^{-\theta t}\, dt \ = \ \frac{\theta}{n-1} \int_0^\infty \frac{\theta^{n-1}}{\Gamma(n-1)} t^{n-2} . e^{-\theta t}\, dt$$

$$E\left(\frac{1}{T}\right) = \frac{\theta}{n-1}$$

Then, $\left[\frac{n-1}{T}\right]$ is an unbiased estimator for (θ), and we proved that (T) is a complete sufficient statistic for (θ), thus, by theorem of Lehmann – Scheffe, the (UMVUE) of (θ) denoted by ( $\hat{\theta}_{UMVU}$ ) is given by:

$$\hat{\theta}_{UMVU} = \frac{n-1}{\sum_{i=1}^{n}\left(\frac{1}{x_i^2}\right)} = \frac{n-1}{T} \tag{8}$$

The Mean Squared Error (MSE) of an estimator is the difference between values implied by an estimator and the true values of the quantity being estimated, or, the average of the squares of the "errors". The error is the amount by which the value implied by the estimator differs from the quantity to be estimated. And the difference occurs because of randomness or because the estimator doesn't account for information that could produce a more accurate estimate [11].

The Basic Properties of the (MSE) are:

- The (MSE) is the second moment (about the origin) of the error.
- The (MSE) has the same units of measurement as the square of the quantity being estimated.
- The square root of (MSE) yield the root mean square error or root mean square deviation (RMSE or RMSD) , which has the same units as the quantity being estimated .
- The (MSE) is a risk function, corresponding to the expected value of the squared error loss or quadratic loss.
- For an unbiased estimator, the (MSE) is the variance. And the (RMSE) is the square root of the variance, which known as the standard deviation (SD).

The (MSE) of an estimator ($\hat{\theta}$) with respect to the estimated parameter $(\theta)$ is defined as follows:

$$MSE\ (\hat{\theta})\ =\ E\left[\hat{\theta}-\theta\right]^2\ =\ Var\ (\hat{\theta})+Bias^{\,2}\ (\hat{\theta})$$

Thus, the (MSE) assesses the quality of an estimator in terms of its variation and unbiasedness.

Minimum Mean Squared Error Estimator for (θ), or ($\hat{\theta}_{Min\,MSE}$) can be found by assuming that:

$$\hat{\theta}_{Min\,MSE}\ =\ \frac{c}{T} \tag{9}$$

Therefore, the MSE considered as the following:

$$MSE\ (\hat{\theta}_{Min\,MSE})=c^2\ E\left(\frac{1}{T}\right)^2-\ 2\,c\,\theta\,E\left(\frac{1}{T}\right)+\theta^2$$

To minimize (MSE) for ($\hat{\theta}_{Min\,MSE}$), we take the partial derivative with respect to (c) and then equating it to zero , as follows:

$$\frac{\partial}{\partial c}\,MSE\ (\hat{\theta}_{Min\,MSE})\ =2\,c\,E\left(\frac{1}{T}\right)^2-2\,\theta\,E\left(\frac{1}{T}\right)$$

Hence, $$c\ =\ \frac{\theta\,E\left(\frac{1}{T}\right)}{E\left(\frac{1}{T}\right)^2} \tag{10}$$

$$E\left(\frac{1}{T}\right)^2\ =\ \int_0^\infty \frac{1}{t^2}\cdot\frac{\theta^n}{\Gamma(n)}\,t^{n-1}\,.\,e^{-\theta t}\ dt\ =\ \frac{\theta^2}{(n-1)(n-2)}\int_0^\infty \frac{\theta^{n-2}}{\Gamma(n-2)}\,t^{n-3}\,.\,e^{-\theta t}\ dt$$

$$E\left(\frac{1}{T}\right)^2\ =\ \frac{\theta^2}{(n-1)(n-2)}$$

Rec-all that $E\left(\frac{1}{T}\right)=\ \frac{\theta}{n-1}$

Therefore, after substituting into (10), we get $c=\ (n-2)$, and hence, the MinMSE estimator for (θ) is:

$$\hat{\theta}_{Min\,MSE}=\frac{(\,n-2\,)}{T} \tag{11}$$

To compare the three classical estimators theoretically, according to their (MSE), we'll derive their (MSE) as follows:

$$MSE\ (\hat{\theta}_{ML})=E\left[\frac{n}{T}-\theta\right]^2=n^2\ E\left(\frac{1}{T}\right)^2-2\,n\,\theta\,E\left(\frac{1}{T}\right)+\theta^2 \tag{12}$$

After substituting into (12), we get:

$$MSE\left(\hat{\theta}_{ML}\right)=\ \frac{n^2\,\theta^2}{(n-1)(n-2)}\ -\ \frac{2\,n\,\theta^2}{(n-1)}\ +\ \theta^2\ =\ \frac{\theta^2\,(n+2)}{(n-1)(n-2)}$$

$$\therefore\quad MSE\left(\hat{\theta}_{ML}\right)\ =\ \frac{\theta^2\,(n+2)}{(n-1)(n-2)} \tag{13}$$

Hence, (MSE) for (UMVUE) and (MinMSE) are obtained by the same way, as follows:

$$MSE\,(\hat{\theta}_{UMVU}) = E\left[\left(\frac{n-1}{T} - \theta\right)^2\right]$$

$$MSE\,(\hat{\theta}_{UMVU}) = (n-1)^2 \operatorname{var}\left(\frac{1}{T}\right) + \left[E\left(\frac{n-1}{T}\right) - \theta\right]^2$$

As we know that, (UMVUE) is Unbiased Estimator for (θ), therefore:

$$MSE\,(\hat{\theta}_{UMVU}) = (n-1)^2 \operatorname{var}\left(\frac{1}{T}\right) = (n-1)^2\left[\frac{\theta^2}{(n-1)(n-2)} - \frac{\theta^2}{(n-1)^2}\right]$$

$$MSE\,(\hat{\theta}_{UMVU}) = \frac{\theta^2}{(n-2)} \tag{14}$$

$$Now,\ MSE\,(\hat{\theta}_{Min\,MSE}) = E\left[\frac{n-2}{T} - \theta\right]^2 = (n-2)^2 \operatorname{var}\left(\frac{1}{T}\right) + \left[E\left(\frac{n-2}{T}\right) - \theta\right]^2$$

$$MSE\,(\hat{\theta}_{Min\,MSE}) = \frac{(n-2)\theta^2}{(n-1)^2} + \frac{\theta^2}{(n-1)^2} = \frac{\theta^2}{(n-1)}$$

$$MSE\,(\hat{\theta}_{Min\,MSE}) = \frac{\theta^2}{(n-1)} \tag{15}$$

From the equations (13), (14), and (15), we can notice that:

$$MSE\,(\hat{\theta}_{Min\,MSE}) \le MSE\,(\hat{\theta}_{UMVU}) \le MSE\,(\hat{\theta}_{ML})$$

Now, we can say that, Minimum mean squared error (MinMSE) is the best estimator among the Maximum likelihood estimator (MLE), and the Uniformly minimum variance unbiased estimator (UMVUE), while the Maximum Likelihood Estimator is the worse among these three estimators.

## 3. STANDARD BAYES ESTIMATOR

In this section, we describe another approach to estimate the parameter (θ) which is called Bayesian methods where the unknown parameter $(\theta)$ is regarded as being the value of a random variable from a given probability distribution, with the knowledge of some information about the value of $(\theta)$ prior to observing the data $(x_1, x_2, x_3, ...., x_n)$.[8]

## 4.1 BAYES ESTIMATOR UNDER GENERALIZED SQUARE ERROR LOSS FUNCTION (GS)

Al-Nasser and Saleh (2006) suggested a new loss function called it Generalized square error loss function in estimating the scale parameter and the reliability function for Weibull distribution , which introduced as follows[6],[9]:

$$L\,(\hat{\theta}, \theta) = \left(\sum_{j=0}^{k} a_j\, \theta^j\right)(\hat{\theta} - \theta)^2 \quad ; k = 0, 1, 2, 3, ...$$

$$Where,\ a_j\ ,\ \left(j = 0, 1, 2, 3, ..............., k\right)\ is\ a\ consant\ .$$

$$L\,(\hat{\theta}, \theta) = \left(a_0 + a_1\,\theta + ..... + a_k\,\theta^k\right)(\hat{\theta} - \theta)^2$$

Then, the risk function under the Generalized square error loss function is denoted by $R_{GS}(\hat{\theta}, \theta)$ is:

$$R_{GS}(\hat{\theta},\theta)=E\left[L(\hat{\theta},\theta)\right]=\int_0^\infty L(\hat{\theta},\theta).h(\theta\,|\,\underline{x})\,d\theta$$

$$R_{GS}(\hat{\theta},\theta)=\int_0^\infty \left(a_0+a_1\,\theta+.....+a_k\,\theta^k\right)(\hat{\theta}^2-2\hat{\theta}\theta+\theta^2)\,h(\theta\,|\,\underline{x})\,d\theta$$

$$R_{GS}(\hat{\theta},\theta)=a_0\,\hat{\theta}^2-2\,a_0\,\hat{\theta}E\left(\theta\,|\,\underline{x}\right)+a_0\,E\left(\theta^2|\,\underline{x}\right)+a_1\,\hat{\theta}^2E\left(\theta\,|\,\underline{x}\right)-2\,a_1\,\hat{\theta}E\left(\theta^2|\,\underline{x}\right)+a_1\,E\left(\theta^3|\,\underline{x}\right)+$$
$$+\cdots\cdots+a_k\,\hat{\theta}^2\,E\left(\theta^k|\,\underline{x}\right)-2\,a_k\,\hat{\theta}E\left(\theta^{k+1}|\,\underline{x}\right)+a_k\,E\left(\theta^{k+2}|\,\underline{x}\right)$$

Taking the partial derivative for $R_{GS}(\hat{\theta},\theta)$ with respect to $\hat{\theta}$ and setting it equal to zero, yields:

$$\frac{\partial\,R_{GS}(\hat{\theta},\theta)}{\partial\,\hat{\theta}}=2a_0\,\hat{\theta}-2\,a_0\,E\left(\theta\,|\,\underline{x}\right)+2a_1\,\hat{\theta}E\left(\theta|\,\underline{x}\right)-2\,a_1E\left(\theta^2|\,\underline{x}\right)+\cdots\cdots\cdots+$$
$$+2a_k\,\hat{\theta}\,E\left(\theta^k|\,\underline{x}\right)-2\,a_k\,E\left(\theta^{k+1}|\,\underline{x}\right)\;=\;0$$

$$\hat{\theta}=\frac{a_0\,E\,(\theta\,|\,\underline{x})+a_1\,E\,(\theta^2\,|\,\underline{x})+........+a_k\,E\,(\theta^{k+1}\,|\,\underline{x})}{a_0+a_1\,E\,(\theta\,|\,\underline{x})+......................+a_k\,E\,(\theta^k\,|\,\underline{x})} \tag{16}$$

In this study we consider informative as well as non-informative prior as the follows:

### 4.2 BAYES ESTIMATOR UNDER JEFFREYS PRIOR INFORMATION

Let us assume that (θ) has non informative prior density defined as using Jefferys prior information g(θ) , which is given by[4]:

$$g_1\left(\theta\right)\propto\sqrt{I(\theta)}$$

Where, I (θ) represented Fisher Information which defined as follows:

$$I\left(\theta\right)=-\,n\,E\left(\frac{\partial^2\,Ln\,f\left(x_i\,;\theta\right)}{\partial\,\theta^2}\right)\quad,\quad Hence:$$

$$g_1\left(\theta\right)=b\,\sqrt{-\,n\,E\left(\frac{\partial^2\,Ln\,f\left(x_i\,;\theta\right)}{\partial\,\theta^2}\right)} \tag{17}$$

$$Ln\;f\left(x_i\,;\theta\right)=Ln\left(\theta\right)+\theta\,Ln\left(\alpha\right)-\left(\theta+1\right)Ln\left(x_i\right)$$

$$\frac{\partial\,Ln\;f\left(x_i\,;\theta\right)}{\partial\,\theta}\;=\;\frac{1}{\theta}+Ln\left(\alpha\right)-Ln\left(x_i\right)\quad=\frac{1}{\theta}-\frac{1}{x_i^2}\quad=-\frac{1}{\theta^2}$$

$$\frac{\partial^2\,Ln\;f\left(x_i\,,\theta\right)}{\partial\,\theta^2}\;=\;\frac{-1}{\theta^2}$$

Hence, we get:

$$E\left(\frac{\partial^2\,Ln\;f\left(x_i\,,\theta\right)}{\partial\,\theta^2}\right)\;=\;\frac{-1}{\theta^2}$$

After substitution into (17), we find that:

$$g_1(\theta)=\frac{1}{\theta}\sqrt{n} \quad , \quad \theta>0$$

The posterior density function is:

$$h_1(\theta \mid x_1, x_2, \ldots\ldots, x_n)=\frac{g_1(\theta).\, L(\theta; x_1, x_2, \ldots\ldots, x_n)}{\int_0^\infty g_1(\theta).\, L(\theta; x_1, x_2, \ldots\ldots, x_n)\, d\theta}$$

$$h_1(\theta \mid x_1, x_2, \ldots\ldots, x_n)=\frac{\theta^{n-1}.\, e^{-\theta T}}{\int_0^\infty \theta^{n-1}.\, e^{-\theta T}\, d\theta}$$

Hence, the posterior density functions of (θ) with Jefferys prior is:

$$h_1(\theta \mid x_1, x_2, \ldots\ldots, x_n) = \frac{T^n.\theta^{n-1}.e^{-\theta T}}{\Gamma n} \tag{18}$$

The posterior density function is recognized as the density of the Gamma distribution:

$$\theta \sim Gamma(n, T) \quad , \quad with:$$

$$E(\theta)=\frac{n}{T} \quad , \quad Var(\theta)=\frac{n}{T^2}$$

$$E(\theta^m)=\int_{\forall\theta} \theta^m\, h_1(\theta \mid \underline{X})\, d\theta = \frac{\Gamma(n+m)}{\Gamma(n)T^m}$$

After substituting into (16), we have:

$$\hat{\theta}_J = \frac{a_0 \cdot \frac{\Gamma(n+1)}{\Gamma(n)T} + a_1 \cdot \frac{\Gamma(n+2)}{\Gamma(n)T^2} + \cdots\cdots + a_k \cdot \frac{\Gamma(n+k+1)}{\Gamma(n)T^{k+1}}}{a_0 + a_1 \cdot \frac{\Gamma(n+1)}{\Gamma(n)T} + a_2 \cdot \frac{\Gamma(n+2)}{\Gamma(n)T^2} + \cdots\cdots + a_k \cdot \frac{\Gamma(n+k)}{\Gamma(n)T^k}}$$

$$\hat{\theta}_J = \frac{a_0 \frac{n}{T} + a_1 \frac{(n+1)n}{T^2} + \cdots + a_k \frac{(n+k)(n+k-1)\cdots\cdots(n+1)n}{T^{k+1}}}{a_0 + a_1 \frac{n}{T} + \cdots\cdots + a_k \frac{(n+k-1)(n+k-2)\cdots\cdots(n+1)n}{T^k}}$$

Therefore, the Bayes estimator for (θ) of one parameter Inverse Rayleigh Distribution under Generalized square error loss function with Jefferys prior denoted by ($\hat{\theta}_J$) can be written as:

$$\hat{\theta}_J = \frac{\sum_{j=0}^{k} a_j \frac{\Gamma(n+1+j)}{T^{j+1}.\Gamma(n)}}{\sum_{j=0}^{k} a_j \frac{\Gamma(n+j)}{T^{j}.\Gamma(n)}} \tag{19}$$

In this paper, we'll use the first and second polynomials as follows:

$$\hat{\theta}_{J1} = \frac{a_0 \frac{n}{T} + a_1 \frac{(n+1)n}{T^2}}{a_0 + a_1 \frac{n}{T}} \tag{20}$$

$$\hat{\theta}_{J2} = \frac{a_0 \frac{n}{T} + a_1 \frac{(n+1)n}{T^2} + a_2 \frac{(n+2)(n+1)n}{T^3}}{a_0 + a_1 \frac{n}{T} + a_2 \frac{n(n+1)}{T^2}} \tag{21}$$

### 4.3 BAYES ESTIMATOR UNDER EXPONENTIAL PRIOR DISTRIBUTION

Assuming that, (θ) has informative prior as Exponential distribution:

$$g_2(\theta) = \frac{1}{\lambda} . e^{-\frac{\theta}{\lambda}} \quad ; \quad \theta > 0 \ , \ \lambda > 0 \tag{22}$$

Since, the posterior distribution of $(\theta)$ will be as follows:

$$h_2(\theta \mid \underline{x}) = \frac{L(x_1, x_2, ......., x_n \mid \theta)\, g_2(\theta)}{\int_0^\infty L(x_1, x_2, ......., x_n \mid \theta)\, g_2(\theta)\, d\theta} = \frac{\theta^n e^{-\theta\left(T + \frac{1}{\lambda}\right)}}{\int_0^\infty \theta^n e^{-\theta\left(T + \frac{1}{\lambda}\right)} d\theta}$$

$$h_2(\theta \mid \underline{x}) = \frac{\left(T + \frac{1}{\lambda}\right)^{n+1}}{\Gamma(n+1)} \theta^n e^{-\theta\left(T + \frac{1}{\lambda}\right)} \tag{23}$$

$$\text{Notice that}: \quad \theta \sim \Gamma\left( (n+1) \ , \ p \right) \quad , \quad \text{Where}, \quad P = \sum_{i=1}^{n} \frac{1}{x_i^2} + \frac{1}{\lambda} = T + \frac{1}{\lambda}$$

$$E(\theta^m) = \int_{\forall \theta} \theta^m\, h_2(\theta \mid \underline{X})\, d\theta = \frac{\Gamma(n+1+m)}{\Gamma(n+1)\, P^m}$$

After substituting, we get, the Bayes estimator for (θ) of one parameter Inverse Rayleigh Distribution under Generalized square error loss function with Exponential prior denoted by ($\hat{\theta}_E$) can be written as:

$$\hat{\theta}_E = \frac{a_0\left(\frac{(n+1)}{p}\right) + a_1\left(\frac{(n+2)(n+1)}{p^2}\right) + \cdots\cdots + a_k\left(\frac{(n+k+1)(n+k).......(n+1)}{p^{k+1}}\right)}{a_0 + a_1\left(\frac{(n+1)}{p}\right) + a_2\left(\frac{(n+2)(n+1)}{p^2}\right) + \cdots\cdots + a_k\left(\frac{(n+k)......\ (n+1)}{p^k}\right)} \tag{24}$$

In this paper, we'll use the first and second polynomials as follows:

$$\hat{\theta}_{E1} = \frac{a_0\left(\frac{(n+1)}{p}\right) + a_1\left(\frac{(n+2)(n+1)}{p^2}\right)}{a_0 + a_1\left(\frac{(n+1)}{p}\right)} \tag{25}$$

$$\hat{\theta}_{E2} = \frac{a_0\left(\frac{(n+1)}{p}\right) + a_1\left(\frac{(n+2)(n+1)}{p^2}\right) + a_2\left(\frac{(n+3)(n+2)(n+1)}{p^3}\right)}{a_0 + a_1\left(\frac{(n+1)}{p}\right) + a_2\left(\frac{(n+2)(n+1)}{p^2}\right)} \tag{26}$$

## 4. SIMULATION RESULTS

In This section, Monte Carlo simulation study is performed to compare five estimators for the scale parameter $\theta$ of one parameter Inverse Rayleigh distribution, using Mean Square Error (MSE) of an estimator which is defined as follows:

$$MSE(\hat{\theta}) = \frac{1}{R} \cdot \sum_{i=1}^{R} (\hat{\theta}_i - \theta)^2 \quad ; \quad i = 1, 2, 3, \cdots, R \tag{27}$$

Where, R represents the number of replications.
We generated R = 5000 samples of sizes (n=5, 10, 30, 50, 100) from the Inverse Rayleigh distribution.
The results are summarized and tabulated in tables (1-8) which contain the expected values and (MSE's) for estimating the scale parameter, and we have observed that:

1. Table (1), shows that, the performance of Bayes estimator under Exponential prior with first polynomials (G1) when (**λ=0.8**) is the best estimator with small sample sizes (n=5,10), while the performance of MinMse is the best estimator for (n=30, 50, 100).
2. Table (2), shows that, the performance of Bayes estimator under Exponential prior with second polynomials (G2) when (λ=0.8) is the best estimator with small sample sizes (n=5, 10), while the performance of the same estimates type G1 is the best estimator for (n=30, 50, 100).
3. From table (3), we notice the performance of MinMSE is the best estimator for all sample sizes.
4. Tables (4), shows that Bayes estimator under Exponential prior type G2 with (λ=0.8) is the best estimator for (n=10), otherwise, the performance of Bayes estimator under Exponential prior type G1 with (λ=0.8) is the best estimator.
5. The performance of Bayes estimator under Generalized square error loss function with Exponential prior is better than the corresponding estimators with Jefferys prior for all cases.
6. Bayes estimates under Generalized square error loss function, based on Exponential prior information with (**λ=0.8**), were generally better than the corresponding Bayes estimates with (**λ=0.8**) for all sample sizes
7. For all parameter values, an obvious reduction in (MSE) is observed with the increase in sample sizes Finally, It is observed that, (MSE) of all estimators of the scale parameter is increasing with the increase of the scale parameter value with all sample sizes.

Generally, the results showed that the best estimator for the one parameter Inverse Rayleigh distribution is the Bayesian estimation under Generalized squared error loss function when $a_0$ is much bigger than $a_1$, and $a_1$ is bigger than $a_2$, with Exponential prior when the scale **λ** parameter of Exponential prior is less than 1.

**Table 1: Expected values and MSE's of the parameter of the Inverse Rayleigh distribution**

**When θ = 1, A0 = 300, A1 = 50, A2 = 10**

| n | Estimator Criteria | Min MSE | Jefferys prior | | Exponential prior λ=0.8 | | Exponential prior λ=1.8 | |
|---|---|---|---|---|---|---|---|---|
| | | | G1 | G2 | G1 | G2 | G1 | G2 |
| 5 | Exp.( θ) | 0.74151 | 1.28547 | 1.31971 | 1.11297 | 1.12900 | 1.30862 | 1.33479 |
| | MSE | 0.23135 | 0.61744 | 0.72924 | **0.18818** | 0.20626 | 0.44907 | 0.50682 |
| 10 | Exp.( θ) | 0.88528 | 1.12529 | 1.13482 | 1.070858 | 1.077955 | 1.15671 | 1.16567 |
| | MSE | 0.11357 | 0.18344 | 0.19496 | **0.111225** | 0.11649 | 0.17367 | 0.18359 |
| 30 | Exp.( θ) | 0.96545 | 1.03962 | 1.04172 | 1.02810 | 1.03020 | 1.05326 | 1.05536 |
| | MSE | **0.03480** | 0.04090 | 0.04150 | 0.03580 | 0.03630 | 0.04150 | 0.04220 |
| 50 | Exp.( θ) | 0.97961 | 1.02344 | 1.02460 | 1.01733 | 1.01845 | 1.03192 | 1.03308 |
| | MSE | **0.02100** | 0.02310 | 0.02330 | 0.02140 | 0.02158 | 0.02339 | 0.02360 |
| 100 | Exp.( θ) | 0.99050 | 1.01218 | 1.01273 | 1.00940 | 1.00994 | 1.01653 | 1.01708 |
| | MSE | **0.01020** | 0.01070 | 0.01070 | 0.01030 | 0.01030 | 0.01080 | 0.01080 |

**Table 2: Expected values and MSE's of the parameter of the Inverse Rayleigh distribution**

**When θ = 3, A0 = 300, A1 = 50, A2 = 10**

| n | Estimator Criteria | Min MSE | Jefferys prior | | Exponential prior $\lambda$=0.8 | | Exponential prior $\lambda$=1.8 | |
|---|---|---|---|---|---|---|---|---|
| | | | G1 | G2 | G1 | G2 | G1 | G2 |
| 5 | Exp.( θ) | 2.22453 | 4.01575 | 4.36482 | 2.27856 | 2.35981 | 3.16953 | 3.34302 |
| | MSE | 2.08211 | 6.38791 | 9.16071 | 0.84601 | **0.79013** | 1.24986 | 1.63436 |
| 10 | Exp.( θ) | 2.65585 | 3.44355 | 3.56712 | 2.58962 | 2.64785 | 3.13084 | 3.22079 |
| | MSE | 1.02215 | 1.79489 | 2.17803 | 0.54659 | **0.54256** | 0.85281 | 0.99396 |
| 30 | Exp.( θ) | 2.89636 | 3.13904 | 3.17093 | 2.85892 | 2.88383 | 3.06003 | 3.08912 |
| | MSE | 0.31311 | 0.38000 | 0.40642 | **0.24931** | 0.25212 | 0.30657 | 0.32397 |
| 50 | Exp.( θ) | 2.93882 | 3.08215 | 3.10028 | 2.91509 | 2.93073 | 3.03771 | 3.05489 |
| | MSE | 0.18867 | 0.21202 | 0.22079 | **0.16414** | 0.16559 | 0.18704 | 0.19340 |
| 100 | Exp.( θ) | 2.97150 | 3.04237 | 3.05107 | 2.95923 | 2.96732 | 3.02123 | 3.02971 |
| | MSE | 0.09146 | 0.09724 | 0.09929 | **0.08532** | 0.08577 | 0.09140 | 0.09298 |

**Table 3: Expected values and MSE's of the parameter of the Inverse Rayleigh distribution**

**When θ = 1, A0 = 10, A1 = 50, A2 = 100**

| n | Estimator Criteria | Min MSE | Jefferys prior | | Exponential prior $\lambda$=0.8 | | Exponential prior $\lambda$=1.8 | |
|---|---|---|---|---|---|---|---|---|
| | | | G1 | G2 | G1 | G2 | G1 | G2 |
| 5 | Exp.( θ) | 0.74151 | 1.44954 | 1.65266 | 1.23564 | 1.37816 | 1.45033 | 1.62334 |
| | MSE | **0.23135** | 0.85693 | 1.31659 | 0.26950 | 0.42178 | 0.62698 | 0.94188 |
| 10 | Exp.( θ) | 0.88528 | 1.20057 | 1.28680 | 1.13645 | 1.21004 | 1.22727 | 1.30830 |
| | MSE | **0.11357** | 0.22938 | 0.30683 | 0.13773 | 0.18351 | 0.21818 | 0.29005 |
| 30 | Exp.( θ) | 0.96545 | 1.06333 | 1.08909 | 1.05091 | 1.07547 | 1.07652 | 1.10187 |
| | MSE | **0.03480** | 0.04510 | 0.05170 | 0.03920 | 0.04450 | 0.04630 | 0.04630 |
| 50 | Exp.( θ) | 0.97961 | 1.03750 | 1.05260 | 1.03104 | 1.04573 | 1.04582 | 1.06079 |
| | MSE | **0.02100** | 0.02460 | 0.02680 | 0.02260 | 0.02460 | 0.02510 | 0.02750 |
| 100 | Exp.( θ) | 0.99050 | 1.01916 | 1.02658 | 1.01629 | 1.02361 | 1.02346 | 1.03086 |
| | MSE | **0.01020** | 0.01110 | 0.01160 | 0.01060 | 0.01110 | 0.01120 | 0.01180 |

**Table 4: Expected values and MSE's of the parameter of the Inverse Rayleigh distribution**

**When θ = 3, A0 = 10, A1 = 50, A2 = 100**

| n | Estimator Criteria | Min MSE | Jefferys prior | | Exponential prior λ=0.8 | | Exponential prior λ=1.8 | |
|---|---|---|---|---|---|---|---|---|
| | | | G1 | G2 | G1 | G2 | G1 | G2 |
| 5 | Exp.( θ) | 2.22453 | 4.41154 | 5.10840 | 2.511551 | 2.835672 | 3.46364 | 3.92390 |
| | MSE | 2.08211 | 7.91131 | 12.50396 | 0.61875 | **0.52363** | 1.59425 | 2.65557 |
| 10 | Exp.( θ) | 2.65585 | 3.63304 | 3.93945 | 2.73242 | 2.93804 | 3.29397 | 3.54636 |
| | MSE | 1.02215 | 2.10846 | 2.91486 | **0.48143** | 0.48473 | 0.98177 | 1.34923 |
| 30 | Exp.( θ) | 2.89636 | 3.20043 | 3.29453 | 2.91456 | 2.99679 | 3.11844 | 3.20703 |
| | MSE | 0.31311 | 0.41073 | 0.48155 | **0.24325** | 0.25091 | 0.32522 | 0.37376 |
| 50 | Exp.( θ) | 2.93882 | 3.11876 | 3.17426 | 2.94958 | 3.00077 | 3.07324 | 3.12680 |
| | MSE | 0.18867 | 0.22284 | 0.24735 | **0.16218** | 0.16582 | 0.19409 | 0.21212 |
| 100 | Exp.( θ) | 2.97150 | 3.06059 | 3.08802 | 2.97692 | 3.00325 | 3.03919 | 3.06613 |
| | MSE | 0.09150 | 0.09990 | 0.10593 | **0.08490** | 0.08610 | 0.09330 | 0.09791 |

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