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RESEARCH ARTICLE

STOCHASTTIC BEHAVIOUR OF A SYSTEM WITH PATIENCE TIME FOR REGULAR REPAIRMAN

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Abstract

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The present paper presents analysis of a two unit redundant system with the concept of regular repairman and patience time. As regards to repairing of the system, it has to be wait for repair due to unavailability of repair facility after common cause failure. The analysis is carried out using the supplementary variable technique and Laplace transformation for evaluating reliability measures such as availability, reliability, mean time failure, mean time to repair and expected number of visits by the repair facility.

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Introduction:-

Agnihotri, Satsangi and Agarwal (1995), Chandrashekar (1996), Kumar and Garg (1991), Wang (2002) and many authors engaged in the field of reliability theory analyzed many engineering systems with the assumptions that only available regular repair facility completes the repair of the failed unit without considering the time factor taken by the repair facility to complete its repair. Ram, Singh and Varshney (2013) investigated the reliability of a standby system under human failure but it is quite reasonable to fix an amount of time known as patience time i.e. if the regular repair facility is able to complete the repair of the failed unit within the patience time then it is okay otherwise an urgent call should be send for expert repairman who is very costly and specialist for repair. The expert repairman takes a random amount of time to become available and repair the failed unit with the help of regular repairman. Keeping this view, the present paper analyse a two unit redundant system with the concept of regular repairman and patience time. Using regenerative point techniques with Markov renewal process the following reliability characteristics of interest which are useful to system designers are obtained.

- Transition and steady state transition probabilities
- Mean Sojourn times in various states
- Mean time to system failure (MTSF)
- Point wise and steady state availability of the system
- Expected busy period of the repairman in (0, t]
- Expected number of visits by the repairman in (0, t]

Model Description and Assumptions:-

1. The system consists of only two non-identical units in which first is operative and the second unit is kept as warm standby.

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2. First unit gets priority over second unit for both operation and repair.
3. A single repair facility known as regular repairman is available in the system for both the units.
4. There is a patience time for regular repairman i.e. if the regular repairman is unable to repair the failed priority unit within patience time then an urgent call is send to expert repairman. Expert repairman is very costly which takes a random amount of time to become available. The expert repairman repairs the failed unit with the collaboration of regular repairman. The concept of patience time is applicability only for priority unit.
5. The failure time distribution of both the units are negative exponential. Also rate of completing patience time for regular repairman and rate of availability of experty repairman are negative exponential.
6. The repair time distributions of the failed unit by the regular and expert repairman are general.

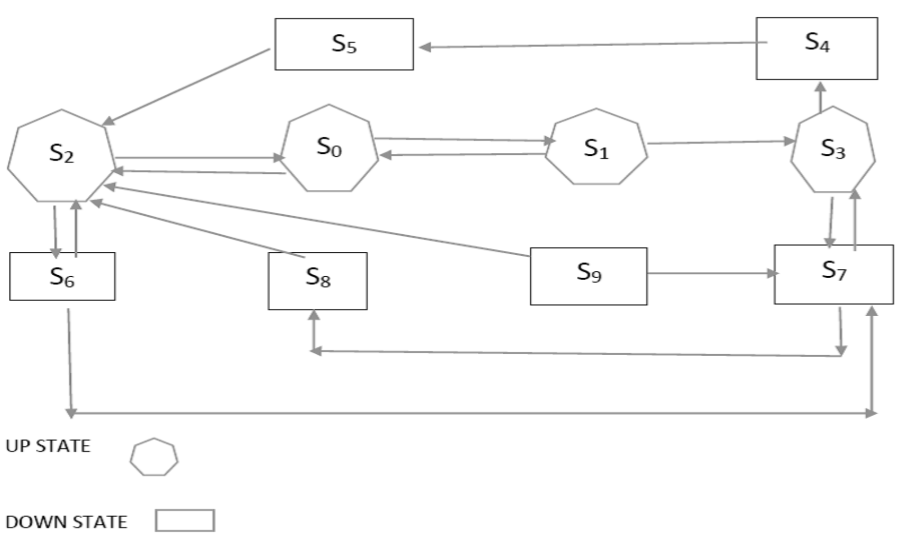
Notation and Symbols:-

N_0	:	Normal unit as operative
N_5	:	Normal unit kept as warm standby
F_{wr}	:	Failed unit waiting for repair by regular repairman
F_{rr}	:	Failed unit under repair by regular repairman
F_{RR} state	:	Repair of failed unit by regular repairman is continued from earlier
E_{UC}	:	Expert repairman is under urgent call
F_{re}	:	Failed unit under repair by expert repairman
F_{RE} state	:	Repair of failed unit by expert repairman is continued from earlier
α	:	Constant rate of first unit
β	:	Constant rate of second unit
γ	:	Constant rate of completing patience time
δ	:	Constant rate of time to available expert repairman
f(.), F(.) :	:	pdf and cdf of time to repair of first unit by regular repairman
g(.), G(.):	:	pdf and cdf of time to repair of second unit by regular repairman
h(.), H(.):	:	pdf and cdf of time to repair of first unit failed by expert repairman
m_1, m_2, m_3	:	Mean time for repair

The possible states of the system are here under using the notations and symbols above:

Up States : $S_0 \equiv (N_0, N_s)$ $S_1 \equiv (F_{rr}, N_0)$ $S_2 \equiv (N_0, F_{rr})$
$S_3 \equiv (E_{UC}, N_0)$ $S_4 \equiv (F_{re}, N_0)$

Down States : $S_5 \equiv (F_{RE}, F_{wrr})$ $S_6 \equiv (F_{rr}, F_{wrr})$ $S_7 \equiv (E_{UC}, F_{rr})$
$S_8 \equiv (F_{re}, F_{wrr})$ $S_9 \equiv (F_{RR}, F_{wr})$



Transition Probabilities:-

Let T_0 (= 0), T_1, T_2,………be the epochs at which enters the states S_i ε E. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = Pr[X_{n+1} = S_k, T_{n+1}-T_n \le t \mid X_n = S_i] \quad (4.1)$$

is semi Markov over E. The stochastic matrix of embedded Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty) \quad (4.2)$$

By simple probabilistic consideration, the non-zero elements of $Q_{ik}(t)$ are:

$Q_{01}(t) = \int_0^t \propto e^{-(\alpha+\beta)u}\,du = \frac{\alpha}{\alpha+\beta}[1-e^{-(\alpha+\beta)t}]$

$Q_{02}(t) = \int_0^t \gamma e^{-(\alpha+\beta)u}\,du = \frac{\beta}{\alpha+\beta}[1-e^{-(\alpha+\beta)t}]$

and similarly, other elements are Q_{10}, Q_{13}, Q_{19}, Q_{20}, Q_{26}, Q_{34}, Q_{37}, Q_{40}, Q_{45}, Q_{62}, Q_{67}, Q_{73}, Q_{78}, Q_{82}, Q_{92}, Q_{97} and

$Q^{(9)}{}_{12}(t) = \int_0^t \beta e^{-\beta u} e^{-\gamma u}\bar{F}(u).du \int_u^t dH(x)/\bar{F}(u) = \frac{\beta}{\beta+\gamma}[\int_0^t dF(x) - \int_0^t e^{-(\beta+\gamma)x} DF(x)$

$Q^{(5)}{}_{42}(t) = \int_0^t \beta e^{-\beta u}\bar{H}(u).du \int_u^t dH(x)/\bar{H}(u) = \int_0^t dH(x) - \int_0^t e^{-\beta x}dH(x)$ (4.3)

Taking limit as $t\to\infty$, the steady state transition p_{ij} can be obtain from equations in (4.3)

i.e. $p_{ik} = \lim_{t\to\infty} Q_{ik}(t)$ (4.4)

Thus,

$p_{01} = \frac{\alpha}{\alpha+\gamma}$, $p_{02} = \frac{\beta}{\alpha+\beta}$, $p_{10} = f^*(\alpha+\beta)$, $p_{13} = \frac{\gamma}{\beta+\gamma}[1-f^*(\beta+\gamma)]$,

$p_{19} = \frac{\beta}{\beta+\gamma}[1-f^*(\beta+\gamma)]$, $p_{20} = g^*(\alpha)$, $p_{26} = 1-g^*(\alpha)$, $p_{34} = \frac{\delta}{\beta+\delta}$, $p_{37} = \frac{\beta}{\beta+\delta}$,

$p_{40} = h^*(\beta)$, $p_{45} = 1-h^*(\beta)$, $p_{62} = f^*(\gamma)$, $p_{67} = 1-f^*(\gamma)$, $p_{73} = g^*(\delta)$,

$p_{78} = 1-g^*(\delta)$, $p_{82} = 1$, $p_{92} = f^*(\gamma)$ and $p_{97} = 1-f^*(\gamma)$, (4.5)

From these probabilities, we can have following relations:

$p_{01} + p_{02} = 1 = p_{10} + p_{13} + p_{19}$, $p_{20} + p_{26} = p_{34} + p_{37} = 1$, $p_{40} + p_{45} = 1 = p_{62} + p_{67}$ and

$p_{73} + p_{78} + = p_{82} = p_{92} + p_{97} = 1$ (4.6)

Mean Sojourn Times:-

The mean time taken by the system in a particular state S_i before transiting to any other state is known as mean sojourn time and is defined by

$\mu_i = \int_0^\infty P[T > t]dt$ (5.1)

where T is time of stay in state S_i by the system.

We assume tat so long as the system is in state S_i, it will not transit to any other state. Therefore mean sojourn time μ_i in state S_i are:

$\mu_0 = \int_0^\infty e^{-(\alpha+\beta)t}dt = \frac{1}{\alpha+\beta}$, $\mu_1 = \int_0^\infty e^{-(\beta+\gamma)t}\bar{F}(t)dt = [1-f^*(\beta+\gamma)]$ and similarly,

$\mu_2 = \frac{1}{\alpha}[1-g^*(\alpha)]$, $\mu_3 = \frac{1}{\beta+\delta}$, $\mu_4 = \frac{1}{\beta}[1-h^*(\beta)]$, $\mu_6 = \frac{1}{\gamma}[1-f^*(\gamma)]$, $\mu_7 = \frac{1}{\delta}[1-g^*(\delta)]$,

$\mu_8 = \int_0^\infty t.h(t)dt$ and $\mu_9 = \frac{1}{\gamma}[1-f^*(\gamma)]$ (5.2)

The mean sojourn time in state $S_i \in$ E in the occurrence of non-generative state can also be contributed as:

$m_{ij} = \int_0^\infty t.q_{ij}(t)dt = q^*_{ij}(0)$ (5.3)

Therefore,

$m_{01} = \frac{\alpha}{(\alpha+\beta)^2}$, $m_{02} = \frac{\beta}{(\alpha+\beta)^2}$, $m_{10} = \int_0^\infty t.e^{-(\beta+\gamma)t}f(t)dt$, $m_{13} = \gamma.t.e^{-(\beta+\gamma)t}\bar{F}(t)dt$

$m_{19} = \int_0^\infty \beta.t.e^{-(\beta+\gamma)t}\bar{F}(t)dt$, $m_{20} = \int_0^\infty t.e^{-\alpha t}g(t)dt$, …………, m_{26}, m_{34}, m_{37}, m_{40}, m_{45}, m_{62}, m_{67}, m_{67}, m_{73}, m_{78}, m_{82}, m_{92}, m_{97} and finally

$$m^{(9)}_{12} = \frac{\beta}{\beta+\gamma}[\int_0^\infty t.f(t)dt - \int_0^\infty t.e^{-(\beta+\gamma)t}f(t)dt]$$

$m^{(5)}_{42} = \int_0^\infty t.h(t)dt - \int_0^t t.e^{-\beta t}f(t)dt]$ (5.4)

Hence,

$m_{01} + m_{02} = \frac{1}{\alpha+\beta} = \mu_0$, $m_{10} + m_{13} = \mu_1$, $m_{20} + m_{26} = \mu_2$, $m_{34} + m_{37} = \mu_3$,

$m_{40} + m_{45} = \mu_4$, $m_{62} + m_{67} = \mu_6$, $m_{73} + m_{78} = \mu_7$, $m_{82} = \mu_8$, $m_{92} + m_{97} = \mu_9$ (5.5)

Mean Time to System Failure (MTSF):-

The mean time to system failure (MTSF) can be obtained by E (T) given below by using Laplace Stieltjes transform of the relations for the distribution function $\pi_i(t)$of the time to system failure with starting time S_0

$$E(T) = \frac{d}{ds}\pi_0(s)|_{s=0} = \frac{D_1'(0)-N_1'(0)}{D_1(0)} \quad (6.1)$$

Where

$$N_1 = \mu_0 + m_1 p_{01} + m_2 p_{01}\, p_{13}\, p_{34} + p_{01}\, p_{13}\, \mu_3 + \mu_2(p_{02} p_{12}^{(9)} + p_{01}\, p_{13}\, p_{34}) \quad (6.2)$$

and

$$D_1 = 1 - p_{01}p_{10} - p_{01}\, p_{13}\, p_{34}\, p_{42}^{(5)} - p_{01}p_{12}^{(9)}p_{20} - p_{02}p_{20} - p_{01}\, p_{13}\, p_{34}\, p_{20} \quad (6.3)$$

Availability analysis:-

System availability is defined as

$A_i(t) = P_r$ [Starting from state S_i the system is available at epoch t without passing through any regenerative state]

$M_i(t) = P_r$ [Starting from up state S_i the system remains up till epoch t without passing through any regenerative state]

Hence, obtaining $A_i(t)$ by using elementary probability argument, we get

$$A_0(t) = M_0(t) + q_{01}©A_1(t) + q_{02}©A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}©A_0(t) + q_{12}^{(9)}©A_2(t) + q_{13}©A_3(t) + q_{17}^{(9)}©A_7(t)$$
$$A_2(t) = M_2(t) + q_{20}©A_0(t) + q_{26}©A_6(t)$$
$$A_3(t) = M_3(t) + q_{34}©A_4(t) + q_{37}©A_7(t)$$
$$A_4(t) = M_4(t) + q_{40}©A_0(t) + q_{42}^{(5)}©A_2(t)$$
$$A_6(t) = q_{62}(t)©A_2(t) + q_{67}©A_7(t)$$
$$A_7(t) = q_{73}(t)©A_3(t) + q_{78}©A_8(t)$$
$$A_8(t) = q_{82}(t)©A_2(t) \quad (7.1)$$

Where $M_0(t) = e^{-(\alpha+\beta)t}$, $M_1(t) = e^{-(\beta+\gamma)t}$, $M_2(t) = e^{-(\alpha)t}.\bar{G}(t)$, $M_3(t) = e^{-(\beta+\gamma)t}$,
$M_4(t) = e^{-(\beta)t}.\bar{H}(t)$ (7.2)

Taking Laplace transform of the equations (7.1) and solving for point wise availability by omitting the arguments 's' for brevity, the steady state functioning availability of the system, when the system starts operation from the state S_i, we get

$$A_0(\infty) = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} A_0^*(s) = \frac{N_2(0)}{D_0'(0)} = \frac{N_2}{D_2}$$
(7.3)

where in terms of $M_0^*(0) = \mu_0$, $M_1^*(0) = \mu_1$, $M_2^*(0) = \mu_2$, $M_3^*(0) = \mu_3$, $M_4^*(0) = \mu_4$ (7.4)

N_2 and D_2 can be easily obtained.

Busy Period Analysis:-

Let $W_i(t)$ be the probability that the system is under repair by repair facility in the state $S_i \in E$ at time t without transiting to any regenerative state. Therefore,

$$W_1(t) = \bar{F}(t) = W_6(t)$$
$$W_2(t) = e^{-\alpha t}\bar{G}(t)$$
$$W_3(t) = e^{-(\beta+\delta)t}$$
$$W_7(t) = e^{-\delta t}\bar{G}(t) \quad (8.1)$$

Let $B_i(t)$ be the probability that the system is under repair at time t. We obtain the following recursive relations among $B_i(t)$'s:

$B_0(t) = q_{01}(t)©B_1(t) + q_{02}(t)©B_2(t)$ and similarly for $B_1(t)$, $B_2(t)$, $B_3(t)$, $B_4(t)$, $B_6(t)$, $B_7(t)$ and $B_8(t)$.

Taking Laplace transform of the equations (8.1) and solving the equations by omitting the argument for brevity we get the fraction of time for which the repair facility is busy in repair as

$$B_0(t) = \lim_{t\to\infty} B_0(t) = \lim_{s\to\infty} B_0^*(s) = N_3(0)/D_3'(0) = N_3/D_3$$
(8.2)

where $N_3 = \mu_0 + p_{24}\mu_4$ and D_3 is same as D_2 in (4.3).

Expected number of visits by repair facility:-

Let $V_i(t)$ be the expected number of visits by the repair facility in (0, t] given that the system initially started from regenerative state S_i at t = 0. The following recurrence relations among $V_i(t)$'s can be obtained as:

$$V_0(t) = Q_{01}(t)\$[1 + V_1(t)] + Q_{02}(t)\$[1 + V_2(t)]$$
$$V_1(t) = Q_{10}(t)V_0(t) + Q_{12}^{(9)}\$V_2(t) + Q_{13}(t)\$V_3(t) + Q_{17}^{(9)}(t)\$V_9(t)$$
$$V_2(t) = Q_{20}(t)\$V_0(t) + Q_{26}(t)\$V_6(t)$$
$$V_3(t) = Q_{34}(t)\$V_4(t) + Q_{37}(t)\$V_7(t)$$
$$V_4(t) = Q_{40}(t)\$V_0(t) + Q_{42}^{(5)}(t)\$V_2(t)$$
$$V_6(t) = Q_{62}(t)\$V_2(t) + Q_{27}(t)\$V_7(t)$$
$$V_7(t) = Q_{73}(t)\$V_2(t) + Q_{78}(t)\$V_8(t)$$
$$V_8(t) = Q_{82}(t)\$V_2(t) \tag{9.1}$$

Using Laplace Stieltjes transform of the above equations and omitting the argument 's' for brevity, we can get the number of visits per unit of time when the system starts after entrance into state S_0 as:

$$V_0 = \lim_{t\to\infty}[V_0(t)/t] = \lim_{s\to 0} s\,\widetilde{V_0}(s) = N_5/D_5 \tag{9.2}$$

Where $N_5 = (1 - p_{26}p_{62})(1 - p_{37}p_{73}) - p_{26}p_{67}p_{72} - p_{26}p_{67}p_{73}p_{34}p_{42}^{(5)}$ and D_5 is same as in (4.3).

With the help of this study we concluded that the performance of the manufacturing system can be improved by improving the procedures on considering patience time, proper training of employees and proper maintenance of the system. The results derived in this paper are valuable in a study of improving the reliability of the systems and additionally they can be extensively used in many engineering disciplines.

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