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RESEARCH ARTICLE

ON R#-CONTINUOUS AND R#-IRRESOLUTE MAPS IN TOPOLOGICAL SPACES.

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Abstract

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In this paper, a new class of continuous functions called R#-continuous maps in topological spaces are introduced and studied. Also some of their properties have been investigated. We also introduce R#-irresolute maps, strongly R#-continuous maps, perfectly R#-continuous maps and discussed some of their properties

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Introduction:-

In general topology continuous functions play a very vital role. The regular continuous and completely continuous functions are introduced and studied by Arya S P [2]. Later, R S walli et all [33] introduced and investigated αrw-continuous functions in topological space. Recently, Basavaraj M Ittanagi et all [5] introduced and studied the basic properties of $R^{\#}$-closed sets in topological space. The aim of this paper is to introduce $R^{\#}$-continuous and irresolute maps in topological space..

Preliminaries:-

In this paper X or (X,τ) and Y or (Y,σ) denote topological spaces on which no separation axioms are assumed. For a subset A of a topological space X, cl(A), int(A), X-A or A^c represent closure of A, interior of A and complement of A in X respectively.

Definition 2.1: A subset A of a topological space (X,τ) is called a

i. Regular open set [26] if A=int(cl(A)) and regular closed if A=cl(int(A))
ii. Regular semi open set [9] if there exists a regular open set U such that $U \subseteq A \subseteq cl(U)$
iii. Generalized closed set (g-closed) [18] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
iv. $R^{\#}$-closed set [5] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is R* open in (X, τ).

The complement of the closed sets mentioned above are their open sets respectively and vice versa.

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Definition 2.2: A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is called a

i. Continuous if f^{-1}(V) is closed in X for every closed subset V of Y.
ii. Regular continuous [2] if f^{-1}(V) is r-closed in X for every closed subset V of Y.
iii. Completely continuous [2] if f^{-1}(V) is regular closed in X for every closed subset V of Y.
iv. α-continuous [14] if f^{-1}(V) is α-closed in X for every closed subset V of Y.
v. Semi continuous [15] if f^{-1}(V) is semi closed in X for every closed subset V of Y.
vi. Semi pre continuous [1] if f^{-1}(V) is semi pre closed in X for every closed subset V of Y.
vii. Strongly Continuous [24] if f^{-1}(V) is clopen in X for every subset V of Y.
viii. g-continuous [4] if f^{-1}(V) is g closed in X for every closed subset V of Y.
ix. w-continuous [28] if f^{-1}(V) is w closed in X for every closed subset V of Y.
x. gr-continuous [22] if f^{-1}(V) is gr closed in X for every closed subset V of Y.
xi. g*-continuous [30] if f^{-1}(V) is g* closed in X for every closed subset V of Y.
xii. swg*-continuous [19] if f^{-1}(V) is swg* closed in X for every closed subset V of Y.
xiii. βwg*-continuous [11] if f^{-1}(V) is βwg* closed in X for every closed subset V of Y.
xiv. r^g-continuous [21] if f^{-1}(V) is r^g closed in X for every closed subset V of Y.
xv. rwg-continuous [20] if f^{-1}(V) is rwg closed in X for every closed subset V of Y.
xvi. βwg**-continuous [25] if f^{-1}(V) is βwg** closed in X for every closed subset V of Y.
xvii. g α-continuous [10] if f^{-1}(V) is gα closed in X for every closed subset V of Y.
xviii. swg-continuous [20] if f^{-1}(V) is swg closed in X for every closed subset V of Y.
xix. αg-continuous [17] if f^{-1}(V) is αg closed in X for every closed subset V of Y.
xx. gp-continuous [18] if f^{-1}(V) is gp closed in X for every closed subset V of Y.
xxi. wg-continuous [20] if f^{-1}(V) is wg closed in X for every closed subset V of Y.
xxii. g*p-continuous [29] if f^{-1}(V) is g*p closed in X for every closed subset V of Y.
xxiii. w α-continuous [7] if ff^{-1}(V) is wα closed in X for every closed subset V of Y.
xxiv. αrw-continuous [31] if f^{-1}(V) is αrw closed in X for every closed subset V of Y.
xxv. ρ-continuous [9] if f^{-1}(V) is ρ closed in X for every closed subset V of Y.
xxvi. sg-continuous [26] if f^{-1}(V) is sg-closed in X for every closed subset V of Y.
xxvii. gs-continuous [3] if f^{-1}(V) is gs closed in X for every closed subset V of Y.
xxviii. rps-continuous [23] if f^{-1}(V) is rps closed in X for every closed subset V of Y.
xxix. gsp-continuous [12] if f^{-1}(V) is gsp closed in X for every closed subset V of Y.

Definition 2.3:

A map f: $(X,\tau)\rightarrow(Y, \sigma)$ is called a

i. Irresolute if f^{-1}(V) is semi closed in X for every semi closed subset V of Y.
ii. w-Irresolute [28] if f^{-1}(V) is w-closed in X for every w-closed subset V of Y.
iii. gc-Irresolute [27] if f^{-1}(V) is g-closed in X for every g-closed subset V of Y.
iv. Contra w Irresolute [28] if f^{-1}(V) is w open in X for every w-closed subset V of Y.
v. Contra Irresolute [14] if f^{-1}(V) is semi open in X for every semi closed subset V of Y.
vi. Contra r-irresolute [2] if f^{-1}(V) is regular open in X for every regular closed subset V of Y.
vii. Contra continuous [13] if f^{-1}(V) is open in X for every closed subset V of Y.

Results 2.4[5]:

i. Every closed (respectively regular closed, g-closed, w-closed, $\hat{g}$-closed set) set is $R^{\#}$-closed set in X.
ii. Every $R^{\#}$-closed set in X is rg(respectively gpr-closed, rwg-closed, gspr-closed, r^g-closed, rgβ-closed) set in X.

Results 2.5[5]:

Let A be a subset of a topological space (X,τ)

i. If A is regular open and rg-closed set in (X,τ) then A is $R^{\#}$-closed set in (X,τ).
ii. If A is g-open and rg-closed set in (X,τ) then A is $R^{\#}$-closed set in (X,τ).
iii. If A is a regular-open and rwg-closed set in (X,τ) then A is $R^{\#}$-closed in (X,τ).
iv. If A is a regular-open and gpr-closed set in (X,τ) then A is $R^{\#}$-closed in (X,τ).
v. If A is regular open and r^g-closed set in (X,τ) then A is $R^{\#}$-closed set in (X,τ).
vi. If A is regular open and βwg^{**}-closed set in (X,τ) then A is $R^{\#}$-closed set in (X,τ).

$R^{\#}$-Continuous Functions:-

Definition 3.1:
A function f from a topological space X in to a topological space Y is called a $R^{\#}$-continuous if inverse image of every closed set in Y is a $R^{\#}$-closed set in X.

Example 3.2: Let X=Y={a,b,c}.Let τ={∅,X,{a},{b},{a, b},{a, c}}be a topology on X and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$ } be a topology on Y. $R^{\#}$-C(X)={X,∅,{b},{c},{a,c},{b,c}} and closed set of Y are σ={Y,∅,{c},{a,c},{b,c}}. Let f: X→Y defined by f(a)=a, f(b)=c, f(c)=c is $R^{\#}$-continuous.

Theorem 3.3: Every continuous function is $R^{\#}$-continuous but not conversely.

Proof: Let f: X→Y be continuous and F be any closed set in Y. Then f^{-1}(F) is closed set in X.Since every closed set in X is $R^{\#}$-closed then f^{-1}(F) is $R^{\#}$-closed set in X. Therefore f is $R^{\#}$-continuous.

Example 3.4: Let X = Y = {a, b, c}. Let τ={∅, X, {a}, {b, c}} be a topology on X and σ = {∅, Y, {a},{b}, {a, b}} be a topology on Y, closed set of X are τ={X, ∅, {a}, {b, c}}, closed set of Y are σ={Y, ∅, {c}, {a, c}, {b, c}}, $R^{\#}$-C(X)={X, ∅ , {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}. Let f: X→Y defined by identity function, then is $R^{\#}$-continuous but not continuous function, as the closed set {c} in Y, then $f^{-1}(\{c\}) = c$ is not a closed set in X.

Theorem 3.5:

i. Every g-continuous is $R^{\#}$-continuous but not conversely.
ii. Every w-continuous is $R^{\#}$-continuous but not conversely.
iii. Every $\hat{g}$-continuous is $R^{\#}$-continuous but not conversely.
iv. Every r-continuous is $R^{\#}$-continuous but not conversely.

Proof: The proof follows from the fact that every g-closed (resp. w-closed, $\hat{g}$-closed and r-closed) set is $R^{\#}$- closed set.

Similarly we can prove ii,iii,iv

Example 3.6: Let X = Y = {a, b, c, d}, let τ = {∅, X, {a}, {b}, {a, b}, {a, b, c}} be a topology on X and σ = { ∅, Y, {a}, {a, b}, {a, b, c}} be a topology on Y. Closed sets of X={X, ∅, {d}, {c, d}, {a, c, d},{b, c, d}}, closed sets of Y={Y, ∅, {d}, {c, d}, {b, c, d}}, $R^{\#}$-C(X)={X, ∅,{d},{a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d},{a,c, d}, {b, c, d}}, g-C(X)={X, ∅, {d}, {a, d}, {b, d}, {c, d}, {a, c, d}, {b, c, d}, {a, b, d}}. Let f : X → Y defined by f(a)=d, f(b)=d, f(c)=d, f(d)=b is $R^{\#}$-continuous function, as the closed set {d} in Y, then $f^{-1}(\{d\}) = \{a.b.c\}$ is not g-closed set in X.

Theorem 3.7: Every $R^{\#}$-continuous function is rg-continuous but not conversely.

Proof: Let f: X→Y be $R^{\#}$-continuous and F be a closed set in Y, by definition $f^{-1}(F)$ is $R^{\#}$closed set in X. Since every $R^{\#}$-closed set is rg-closed, then $f^{-1}(F)$ is rg closed in X. hence f is rg -continuous.

Theorem 3.8:

i. Every $R^{\#}$-continuous function is $\widehat{rg}$ continuous but not conversely
ii. Every $R^{\#}$-continuous function is $gspr$ continuous but not conversely
iii. Every $R^{\#}$-continuous function is gpr continuous but not conversely
iv. Every $R^{\#}$-continuous function is $rg\beta$ continuous but not conversely
v. Every $R^{\#}$-continuous function is rwg continuous but not conversely
vi. Every $R^{\#}$-continuous function is $wgr\alpha$continuous but not conversely.

Proof: The proof follows from the fact that every $R^{\#}$-closed set is $\widehat{rg}$ -closed (resp. $gspr$-closed, gpr-closed, $rg\beta$-closed, rwg-closed, $wgr\alpha$-closed) set in X.

Similarly we can prove (ii), (iii), (iv), (v) and (vi).

Example 3.9: Let X=Y={a, b, c}. Let τ={∅,X,{a},{b},{a,b}} be a topology on X and σ={∅, Y, {a}, {b}, {a, b}, {a, c}} be a topology on Y. Closed sets of X={X, ∅,{c},{a,c},{b,c}}, Y={Y,∅,{b},{c},{a,c}, {b, c}}, $R^{\#}$-C(X)={X,∅,{c},{a,c},{b,c}}. Let f : X→Y defined by f(a)=a, f(b)=a, f(c)=b is rg continuous, r^g-continuous,gspr-continuous,gpr-continuous,rgβ-continuous,rwg-continuous, wgrα-continuous but not $R^{\#}$-continuous function, as the closed set F={a,c} in Y then $f^{-1}(F) = \{a, b\}$ is not $R^{\#}$closed set in X.

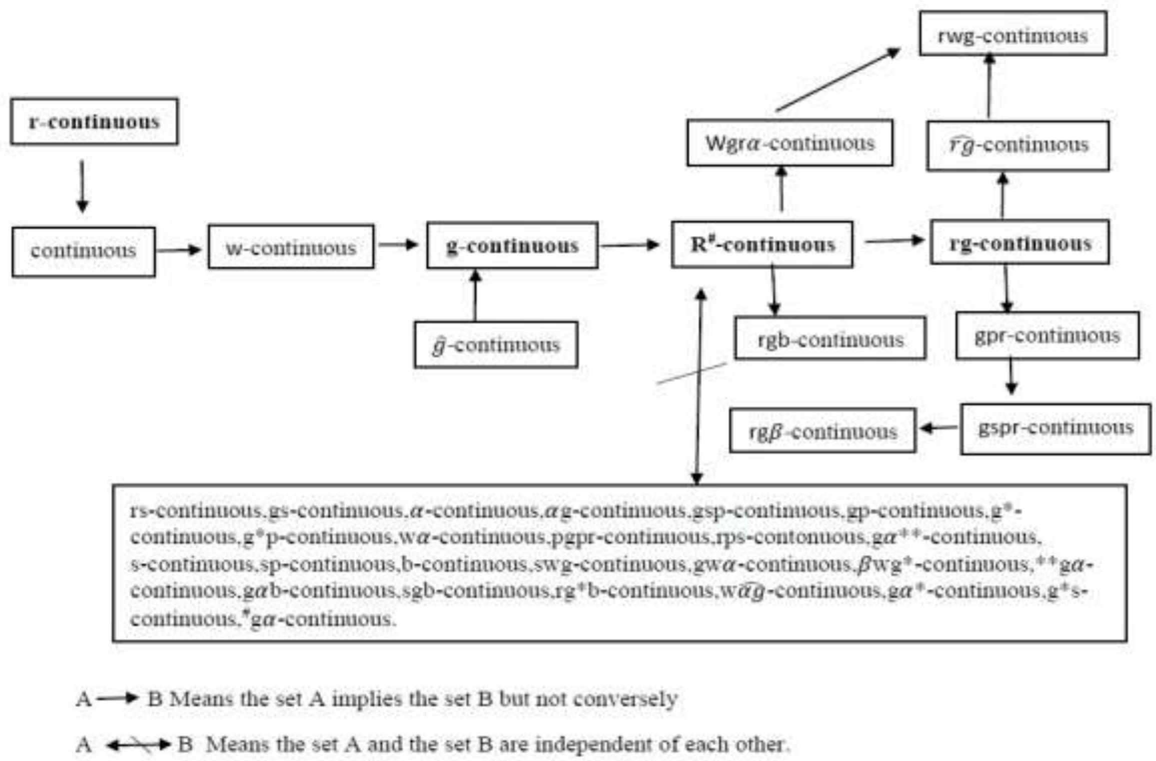
Remark 3.10: The following example shows that $R^{\#}$-continuous is independent with some existing continuous functions in topological spaces

Example 3.11: Let X=Y={a, b, c, d}, let τ={∅, X, {a}, {b}, {a, b}, {a, b, c}} be a topology on X and σ = { ∅, Y, {a}, {a, b}, {a, b, c}} be a topology on Y. Closed sets of X={X, ∅, {d},{c, d}, {a, c, d}, {b, c, d}} and closed sets of Y={Y, ∅ , {d}, {c, d}, {b, c, d}}, $R^{\#}$-C(X)={X, ∅, {d}, {a, c}, {a, d},{b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}. Let f: X→Y defined by f(a)=d, f(b)=d, f(c)=d, f(d)= b is $R^{\#}$-continuous function but not a rs-continuous, gs-continuous, αg-continuous,gsp-continuous, gp-continuous, g*-continuous, g*p-continuous, wα-continuous, pgpr-continuous, rps-continuous and gα**-continuousin X, as $f^{-1}(d) = \{a, b, c\}$ is not a rs-closed set, gs-closed set, αg- closed set, gsp- closed set, gp- closed set, g*- closed set, g*p- closed set, wα- closed set, pgpr-closed set, rps- closed set and gα**- closed set in X.

Example 3.12: Let X=Y={a, b, c, d}, let τ={∅,X,{a},{b},{a,b},{a,b,c}} be a topology on X and σ = {∅, Y,{a},{a, b},{a, b, c}}be a topology on Y. Closed sets of X={X,∅,{d},{c,d},{a,c,d},{b,c,d}} and closed sets of Y={Y,∅,{d},{c,d},{b,c,d}}, $R^{\#}$-C(X)={X, ∅,{d},{a,d},{b,d},{c,d},{a, b, d},{a, c, d},{b, c, d}}.

Let f:X→Y defined by f(a)=d,f(b)=d,f(c)=b,f(d)=d is pre continuous. Semi continuous, sp continuous,b-continuous, swg-continuous, gwα-continuous, sgb-continuous, rg*b-continuous, wαg-continuous, gα*-continuous, g*s-continuous and #gα-continuous,but not $R^{\#}$-continuous in X as $f^{-1}(a,b) = \{c\}$ is pre closed set, Semi pre closed set, sp pre closed set, b pre closed set, swg- pre closed set, gwα- pre closed set, sgb- pre closed set, rg*b- pre closed set, w$\alpha\hat{g}$- pre closed set, gα*- pre closed set, g*s- pre closed set and #gα- closed set in X but not $R^{\#}$-closed set in X.

Remark 3.13: From the above discussions and known facts, the relation between $R^{\#}$-continuous and some existing continuous functions in topological space is shown in the following figure.



Theorem 3.14: Let f: X→Y be a map. Then the following statements are equivalent

i. f is $R^{\#}$-continuous
ii. The inverse image of each open set in Y is $R^{\#}$-open in X.

Proof:

i. Let f: X→Y be a $R^{\#}$-continuous, let U be an open set in Y. Then U^c is closed in Y. Since f is $R^{\#}$-continuous, $f^{-1}(u^c)$ is $R^{\#}$-closed in X. But $f^{-1}(u^c) = X - f^{-1}(u)$. Thus $f^{-1}(u)$ is $R^{\#}$ open set in X.
ii. Suppose that inverse image of each open set in Y is $R^{\#}$-open in X. Let V be any closed set in Y. By assumption $f^{-1}(v^c)$ is $R^{\#}$-open set in X. But $f^{-1}(v^c) = X - f^{-1}(v)$. Thus [X - $f^{-1}(v)$] is $R^{\#}$-closed in X. Thus f is $R^{\#}$-continuous. Hence the proof.

Theorem 3.15: If f : (X,τ) →(Y,σ) is a map then the following holds

i. f is contra r- irresolute and rg-continuous map then f is $R^{\#}$-continuous.

ii. f is contra r- irresolute and rwg-continuous map then f is $R^{\#}$-continuous.
iii. f is contra r- irresolute and gpr-continuous map then f is $R^{\#}$-continuous.
iv. f is contra r- irresolute and r^g-continuous map then f is $R^{\#}$-continuous.

Proof:

i. Let V be any regular closed set of Y. Since every regular closed set is closed, V is closed set in Y. Since F is rg-continuous and contra r-irresolute map, f^{-1}(V) is rg-closed and regular open in X, by results 2.5(i), f^{-1}(V) is $R^{\#}$-closed in X. Thus f is $R^{\#}$-continuous.

ii. Let V be any regular closed set of Y. Since every regular closed set is closed, V is closed set in Y. Since F is rg-continuous and contra r-irresolute map, f^{-1}(V) is rwg-closed and regular open in X. Now by results 2.5 [5] f^{-1}(V) is $R^{\#}$-closed in X. Thus f is $R^{\#}$-continuous.

Similarly we can prove iii,iv

Theorem 3.16: If f : X→Y is $R^{\#}$-continuous then f($R^{\#}$cl(A)⊆cl(f(A)) for every subset A of X.

Proof: Let f: X→Y be $R^{\#}$-continuous. Let A be a subset of X. Then cl(f(A)) is closed in Y, this implies f^{-1}[cl(f(A))] is $R^{\#}$-closed in X. Also f(A) ⊆ cl(f(A)) and A ⊆ f^{-1}[cl(f(A))]. Hence $R^{\#}$cl(A)⊆ f^{-1}[cl(f(A))]. Therefore f($R^{\#}$cl(A)) ⊆ cl(f(A)).

Theorem 3.17: Let f: $(X,\tau)\to(Y,\sigma)$ be a map. Then the following statements are equivalent

i. For each point x ∈ X and each open set V in Y with f(x) ∈ V, there is a $R^{\#}$-open set U in X such that x ∈ U and f(U) ⊆ V.
ii. For each subset A of X, f($R^{\#}$cl(A)) ⊆ cl(f(A))
iii. For each subset B of Y, $R^{\#}$cl(f^{-1}(B)) ⊆ f^{-1}(cl(B))

Proof:

(i) →(ii) : Suppose (i) holds and let y ∈ f($R^{\#}$cl(A)) and V be an open set containing Y. From (i), there exists x ∈$R^{\#}$cl(A) such that f(x)=y and $R^{\#}$-open set U containing x such that f(U) ⊆V and x ∈ $R^{\#}$cl(A). Then we know that for a subset A of a topological space X. Then x∈$R^{\#}$cl(A) if and only if U∩A≠ ∅ for every $R^{\#}$-open set U containing x.That is ∅ ≠ f(U∩A) ⊆ f(U) ∩f(A) ⊆ V∩f(A). Therefore f($R^{\#}$cl(A))⊆cl(f(A)).

(ii) →(i) : Suppose (ii) holds and V be an open set in Y containing f(x). Let A ∈ f^{-1}(V^c). This implies that x∉A. Since f($R^{\#}$cl(A))⊆cl(f(A)) ⊆V^c. This implies that $R^{\#}$cl(A) ⊆ f^{-1}(V^c)=A. Since x∉A implies that x∉ $R^{\#}$cl(A) and we know that for a subset A of a topological space X. Then x∈$R^{\#}$cl(A) if and only if U∩A≠ ∅ for every $R^{\#}$-open set U containing x, there exists a $R^{\#}$-open set U containing x such that U∩A= ∅ then U⊆A^c and hence f(U) ⊆f(A^c) ⊆V.

(ii) →(iii): Suppose(ii) holds. Let B be any subset of Y. Replacing A by f^{-1}(B) in (ii) we get f($R^{\#}$cl(f^{-1}(B)))⊆cl(f(f^{-1}(B))) ⊆cl(B). Hence $R^{\#}$cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

(iii) →(ii): Suppose (iii) holds. Let B=f(A) where A is a subset of X. Then from (iii) we get $R^{\#}$cl(f^{-1}(f(A))) ⊆ f^{-1}(cl(f(A))). That is $R^{\#}$cl(A) ⊆ f^{-1}(cl(f(A))). Therefore $R^{\#}$cl(f^{-1}(B)) ⊆ f^{-1}(cl(B)).

Definition 3.18: Let (X, τ) be a topological space and $\tau_{R^{\#}}$={V⊆X / $R^{\#}$cl(V^c)=V^c} is a topology on X.

Definition 3.19: A topological space (X,τ) is called a $T_R^{\#}$ space if every $R^{\#}$-closed is closed.

Definition 3.20: A topological space (X,τ) is called a ${}_R{}^{\#}T_g$ space if every $R^{\#}$-closed is g-closed in X.

Remark 3.21: The composition of two $R^{\#}$-continuous maps need not be continuous.

Example 3.22: Let X=Y=Z={a, b, c, d}. Let τ={∅, X, {a}, {b}, {a, b},{a, b, c}}be a topology on X. σ={∅, Y, {a}, {a, b}, {a, b, c}} be a topology on Y and η={∅, Z,{a, b}, {c, d}} be a topology on Z. Let f:(X, τ)→(Y, σ) and g: (Y, σ)→(Z, η) are identity functions, then f and g are $R^{\#}$-continuous but gof: (X, τ)→(Z, η) is not a $R^{\#}$-continuous map as the closed set F={a, b} in Z, $(gof)^{-1}$(F)={a,b} is not $R^{\#}$-closed set in X.

Theorem 3.23: Let f: X→Y is $R^{\#}$-continuous and g: Y→Z is continuous then gof: X→Z is $R^{\#}$-continuous.

Proof: Let V be any open set in Z. Since g is continuous, g^{-1}(V) is open in Y. Since f is $R^{\#}$-continuous, f^{-1}(g^{-1}(V))=$(gof)^{-1}$(V) is $R^{\#}$-open in X. Hence gof is $R^{\#}$-continuous.

Theorem 3.24: Let f: X→Y and g: Y→Z be $R^{\#}$-continuous functions and Y be $T_R^{\#}$ space then gof: X→Z is $R^{\#}$-continuous.

Proof: Let V be any open set in Z. Since g is $R^{\#}$-continuous, g^{-1}(V) is $R^{\#}$-open in Y and Y is $T_R^{\#}$ space, then g^{-1}(V) is open in Y. Since f is $R^{\#}$-continuous f^{-1}(g^{-1}(V))=$(gof)^{-1}$(V) is $R^{\#}$-open in X. Hence gof is $R^{\#}$-continuous.

Definition 3.25: A function f: X→Y is called a perfectly $R^{\#}$-continuous if f^{-1}(V) is clopen (open and closed) set in X for every $R^{\#}$-open set V in Y.

Theorem 3.26: If f: X→Y is continuous then the following holds.

i. If f is perfectly $R^{\#}$-continuous then it is $R^{\#}$-continuous

ii. If f is perfectly $R^{\#}$-continuous then it is rg-continuous (resp. r^g-continuous, gpr-continuous, gspr-continuous, rgβ-continuous, rwg-continuous, wgrα-continuous)

Proof:

i. Let U be open set in Y. Since f is perfectly continuous then f^{-1}(U) is both open and closed in X. Since every open is $R^{\#}$-open, f^{-1}(U) is $R^{\#}$-open in X. Hence f is $R^{\#}$-continuous.

ii. Let U be open set in Y. Since f is perfectly continuous then f^{-1}(U) is both open and closed in X. Since every open is rg- open (resp. r^g- open, gpr- open, gspr- open, rgβ- open, rwg- open, wgrα- open) set in X. Hence f is rg-continuous (resp. r^g-continuous, gpr-continuous, gspr-continuous, rgβ-continuous, rwg-continuous, wgrα-continuous).

Definition 3.27: A function f: X→Y is called $R^{\#}$*-continuous if f^{-1}(V) is $R^{\#}$-closed set in X for every g-closed set V in Y.

Theorem 3.28: If f: X→Y is $R^{\#}$*-continuous then it is $R^{\#}$-continuous but converse is not true.

Proof: Let f: X→Y be $R^{\#}$*-continuous. Let F be any closed set in Y. Since f is $R^{\#}$*-continuous, f^{-1}(F) is $R^{\#}$-closed set in X. Since every closed set is g-closed set in Y, then the inverse image f^{-1}(F) is $R^{\#}$-closed set in X. Hence f is $R^{\#}$-continuous.

Example 3.29: Let X=Y={a, b, c}, let τ={∅, X, {a}, {b}, {a, b}} be a topology on X and σ={∅,Y,{a}, {b,c}} be a topology on Y. $R^{\#}$-C(X)={X, ∅, {c}, {a, c}, {b, c}}, $R^{\#}$-C(Y)={Y, ∅, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}}. Let f: (X, τ)→(Y, σ) be a function defined by f(a)=a, f(b)=b, f(c)=c is $R^{\#}$-continuous but not a $R^{\#}$*-continuous function as the g-closed set F={a} in Y, f^{-1}(F)={a} is not a $R^{\#}$-closed set in X.

$R^{\#}$-irresolute and strongly $R^{\#}$-continuous functions

Definition 3.30: A function f: X→Y is called a $R^{\#}$-irresolute map if f^{-1}(V) is $R^{\#}$-closed set in X for every $R^{\#}$-closed set V in Y.

Definition 3.31: A function f: X→Y is called a strongly $R^{\#}$-continuous map if f^{-1}(V) is closed set in X for every $R^{\#}$-closed set V in Y.

Theorem 3.32: If f: (X,τ)→(Y, σ) is $R^{\#}$-irresolute then it is $R^{\#}$-continuous but not conversely.

Proof: Let f: X→Y be $R^{\#}$-irresolute. Let F be any closed set in Y and hence $R^{\#}$-closed in Y. Since f is $R^{\#}$-irresolute, f^{-1}(V) is $R^{\#}$-closed set in X. Therefore f is $R^{\#}$-continuous.

Example 3.33: Let X=Y={a, b, c}. Let τ={∅, X, {a}, {b},{a, b}} be a topology on X and σ={∅, Y, {a}, {b, c}} be a topology onY.$R^{\#}$-C(X)={X,∅,{c},{a,c},{b,c}}, $R^{\#}$-C(Y)={Y,∅,{a},{b},{c},{a,b},{a,c},{b,c}}. Let f: (X,τ)→(Y,σ) be a function defined by f(a)=a, f(b)=b, f(c)=c is $R^{\#}$-continuous but not a $R^{\#}$-irresolute map as the $R^{\#}$-closed set F={a} in Y, f^{-1}(F)={a} is not a $R^{\#}$-closed set in X.

Theorem 3.34: If f: (X,τ)→(Y, σ) is $R^{\#}$-irresolute if and only if f^{-1}(V) is $R^{\#}$-open set in X for every open set V in Y.

Proof: Suppose that f: X→Y is $R^{\#}$-irresolute and U be $R^{\#}$-open set in Y. Then U^c is $R^{\#}$-closed in Y. By the definition of $R^{\#}$-irresolute, f^{-1}(U^c) is $R^{\#}$-closed in X. But f^{-1}(U^c)=X- f^{-1}(U). Thus f(U) is $R^{\#}$-open in X.

Conversely, suppose that f^{-1}(F) is $R^{\#}$-open set in X for every $R^{\#}$-open set F in Y. Let F be any $R^{\#}$-closed set in Y. By the definition, f^{-1}(F^c) is $R^{\#}$-open in X. But f^{-1}(F^c)=X=f^{-1}(F). Thus X-f^{-1}(F) is $R^{\#}$-open in X and hence f^{-1}(F) is $R^{\#}$-closed in X. Therefore f is $R^{\#}$-irresolute.

Theorem 3.35: If f: (X,τ)→(Y,σ) is $R^{\#}$-irresolute then it is $R^{\#}$*-continuous but not conversely.

Proof: Let f: X→Y be $R^{\#}$-irresolute. Let F be any g-closed set in Y and hence f is $R^{\#}$-closed in Y. By the definition of $R^{\#}$-irresolute, f^{-1}(F) is $R^{\#}$-closed set in X. Therefore f is $R^{\#}$*-continuous.

Example 3.36: Let X=Y={a, b, c}. Let τ={∅, X, {a},{b},{a, b}} and σ={∅,Y,{a},{b},{a, b}, {a, c}}. $R^{\#}$-C(X)={X,∅,{c},{a,c},{b, c}},$R^{\#}$-C(Y)={Y,∅,{a},{b},{c},{a,b},{a,c},{b,c}}. Let f:(X,τ)→(Y,σ) be a function defined by f(a)=a, f(b)=b, f(c)=c is $R^{\#}$*-continuous but not a $R^{\#}$-irresolute map as the $R^{\#}$-closed set F={a} in Y, f^{-1}(F)={a} is not a $R^{\#}$-closed set in X.

Theorem 3.37: Let f: (X,τ)→(Y,σ) is $R^{\#}$-irresolute then f($R^{\#}$cl(A))⊆gcl(f(A)) for every subset A of X.

Proof: Let A⊆X and gcl(f(A)) is $R^{\#}$-closed in Y. Since f is $R^{\#}$-irresolute, f^{-1}($R^{\#}$cl(A)) is $R^{\#}$-closed in X. Further A⊆ f^{-1}(f(A)) ⊆ f^{-1}(gcl(f(A))). By the definition of $R^{\#}$-closure, $R^{\#}$cl(A) ⊆ f^{-1}(gcl(A)). Hence f($R^{\#}$cl(A))⊆gcl(f(A)).

Theorem 3.38: Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y, \sigma)\rightarrow(Z, \eta)$ be any two functions. Then

i. gof: $(X, \tau)\rightarrow(Z, \eta)$ is R#-irresolute if g is R#-irresolute and f is R#-irresolute
ii. gof: $(X, \tau)\rightarrow(Z, \eta)$ is R#-continuous if g is R#-continuous and f is R#-irresolute.

Proof: (i) Let F be any R#-closed set in (Z, η). Since g is R#-irresolute then g^{-1}(F) is R#-closed set in (Y, σ). Since f is R#-irresolute $f^{-1}(g^{-1}$(F)) is R#-closed set in (X,τ). But $(gof)^{-1}$(F)=$f^{-1}(g^{-1}$(F)) and hence gof is R#-irresolute.

(ii) Let F be any R#-closed set in (Z, η). Since g is R#-continuous then g^{-1}(F) is R#-closed set in (Y, σ). Since f is R#-irresolute $f^{-1}(g^{-1}$(F)) is R#-closed set in (X,τ). But $(gof)^{-1}$(F)=$f^{-1}(g^{-1}$(F)) and hence gof is R#-continuous.

Theorem 3.39: If f: $(X,\tau)\rightarrow(Y, \sigma)$ is strongly R#-continuous then f is continuous but converse is not true.

Proof: Let f: X→Y be strongly R#-continuous. Let F be any closed set in Y. Since every closed set is R#-closed and hence F is R#-closed set in Y. Since f is strongly R#-continuous then f^{-1}(F) is closed set in X. Therefore f is continuous.

Example 3.40: Let X=Y={a, b, c}. Let τ={∅, X, {a}, {b}, {a, b}}be the topology on X, and σ={∅, Y, {a}, {b,c}} be the topology on Y. Closed sets of X={X,∅,{c},{a,c},{b,c}}, R#-C(Y)={Y,∅,{a},{b},{c}, {a, b},{a, c},{b, c}}. Let f: $(X, \tau)\rightarrow(Y, \sigma)$ be a function defined by f(a)=a,f(b)=b, f(c)=c is continuous but not strongly R#-continuous as the R#-closed set F={a} in Y, f^{-1}(F)={a} is not a closed set in X.

Theorem 3.41: Every strongly R#-continuous is strongly g-continuous but not conversely.

Proof: Let f: X→Y be strongly R#-continuous. Let F be any g-closed set in Y. Since every g-closed set is R#-closed and hence F is R#-closed set in Y. Since f is strongly R#-continuous then f^{-1}(F) is closed set in X and hence g-closed set in X. Therefore f is g-continuous.

Example 3.42: In example 3.40, f is strongly g-continuous but not a strongly R#-continuous as the R#-closed set F={a} in Y, f^{-1}(F)={a} is not a closed set in X.

Theorem 3.43: If a mapping f: $(X, \tau)\rightarrow(Y, \sigma)$ is strongly R#-continuous if and only if f^{-1}(U) is open set in X for every R#-open set U in Y.

Proof: Suppose that f: X→Y is strongly R#-continuous. Let U be any R#-open set in Y and hence U^c is R#-closed set in Y. Since f is strongly R#-continuous, f^{-1}(U) is closed set in X. But $f^{-1}(U^c)$=X- f^{-1}(U). Thus f^{-1}(U) is open in X.

Conversely, suppose that f^{-1}(U) is open set in X for every R#-open set U in Y. Let F be any R#-closed set in Y and hence F^c is R#-open in X. But $f^{-1}(F^c)$=X- f^{-1}(F). Thus X- f^{-1}(F) is open in X and so f^{-1}(F) is closed in X. Therefore f is strongly R#-continuous.

Theorem 3.44: Every strongly continuous is strongly R#-continuous but not conversely.

Proof: Let f: X→Y is strongly continuous. Let G be any R#-open set in Y and also any subset of Y. Since f is strongly continuous then f^{-1}(G) is both open and closed in X, say f^{-1}(G) is open in X. Therefore f is strongly R#-continuous.

Example 3.45: Let X=Y={a, b, c}. Let τ={∅, X,{a},{b, c}and σ={∅,Y,{a},{b},{a,b}}. Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a function defined by f(a)=a, f(b)=b, f(c)=c is strongly R#- continuous but not a strongly continuous as the set F={c} in Y, f^{-1}(F)={c} is not a clopen set in X

Theorem 3.46: Every strongly R#-continuous is R#-continuous but not conversely.

Proof: Let f: X→Y be strongly R#-continuous. Let F be any closed set in Y and hence R#-closed in Y. Since f is strongly R#-continuous, then f^{-1}(F) is closed set in X and hence R#-closed set in X. Therefore f is R#-continuous.

Example 3.47: In example 3.40, f is R#-continuous but not strongly R#-continuous as the R#-closed set F={a} in Y, f^{-1}(F)={a} is not a closed set in X.

Theorem 3.48: In discrete topological space, every strongly R#-continuous is strongly continuous.

Proof: Let f: X→Y be strongly R#-continuous in a discrete topological space. Let F be any subset of Y. Since F is both open and closed subset of Y in discrete space. We have the following two cases.

Case (i): Let F be any closed subset of Y and hence R$^{\#}$-closed in Y. Since f is strongly R$^{\#}$-continuous then f^{-1}(F) is closed in X.
Case (ii): Let F be any open subset of Y and hence R$^{\#}$-open in Y. Since f is strongly R$^{\#}$-continuous then f^{-1}(F) is open in X.
Therefore f^{-1}(F) is both open and closed in X. Hence f is strongly continuous.

Theorem 3.49 Let f: X→Y and g: Y→Z be any two functions. Then

i. gof: X→Z is strongly R$^{\#}$-continuous if both f and g are R$^{\#}$-continuous.
ii. gof: X→Z is strongly R$^{\#}$-continuous if g is strongly R$^{\#}$-continuous and f is continuous.
iii. gof: X→Z is R$^{\#}$-irresolute if g is strongly R$^{\#}$-continuous and f is R$^{\#}$-continuous.
iv. gof: X→Z is continuous if g is R$^{\#}$-continuous and f is strongly R$^{\#}$-continuous.

Proof:

i. Let G be R$^{\#}$-closed set in (Z,η). Since g is strongly R$^{\#}$-continuous then g^{-1}(G) is closed set in (Y,σ) and hence R$^{\#}$-closed set in (Y,σ). Since f is also strongly R$^{\#}$-continuous then $f^{-1}(g^{-1}$(G)) closed set in (X,τ). But $(gof)^{-1}$(G)=$f^{-1}(g^{-1}$(G)) and hence gof is strongly R$^{\#}$-continuous.
ii. Let G be R$^{\#}$-closed set in (Z,η). Since g is strongly R$^{\#}$-continuous then g^{-1}(G) is closed set in (Y,σ). Since f is continuous then $f^{-1}(g^{-1}$(G)) closed set in (X,τ). But $(gof)^{-1}$(G)=$f^{-1}(g^{-1}$(G)) and hence gof is strongly R$^{\#}$-continuous.
iii. Let G be any R$^{\#}$-closed set in (Z,η). Since g is strongly R$^{\#}$-continuous then g^{-1}(G) is closed set in (Y,σ). Since f is R$^{\#}$-continuous then $f^{-1}(g^{-1}$(G)) is R$^{\#}$-closed set in (X,τ). But $(gof)^{-1}$(G)=$f^{-1}(g^{-1}$(G)). Hence gof is R$^{\#}$-irresolute.
iv. Let G be any closed set in (Z,η). Since g is R$^{\#}$-continuous then g^{-1}(G) is R$^{\#}$-closed set in (Y, σ). Since f is strongly R$^{\#}$ continuous then $f^{-1}(g^{-1}$(G)) closed set in (X,τ). But $(gof)^{-1}$(G)=$f^{-1}(g^{-1}$(G)). Hence gof is continuous.

Theorem 3.50: Let f: X→Y and g: Y→Z be any two functions. Then

i. gof: X→Z is strongly R$^{\#}$-continuous if g is perfectly R$^{\#}$-continuous and f is continuous.
ii. gof: X→Z is perfectly R$^{\#}$-continuous if g is strongly R$^{\#}$-continuous and f is perfectly R$^{\#}$-continuous.

Proof:

i. Let G be any R$^{\#}$-open set in (Z,η). Since g is perfectly R$^{\#}$-continuous then g^{-1}(G) is clopen set in (Y,σ), say g^{-1}(G) is open set in (Y,σ). Since f is continuous then $f^{-1}(g^{-1}$(G)) open set in (X,τ). Thus $(gof)^{-1}$(G)=$f^{-1}(g^{-1}$(G)). Hence gof is strongly R$^{\#}$-continuous.
ii. Let G be a R$^{\#}$-open set in (Z,η). Since g is strongly R$^{\#}$-continuous then g^{-1}(G) is open set in (Y,σ).Since f is perfectly R$^{\#}$-continuous then $f^{-1}(g^{-1}$(G)) clopen set in (X,τ). But $(gof)^{-1}$(G)=$f^{-1}(g^{-1}$(G)). Hence gof is perfectly R$^{\#}$-continuous.

Theorem 3.51: Let (X,τ) be a discrete topological space and (Y,σ) be any topological space. Let f:(X,τ)→(Y,σ) be a function. Then the following statements are equivalent.

i. f is strongly R$^{\#}$-continuous
ii. f is perfectly R$^{\#}$-continuous.

Proof:

(i) →(ii): Let G be any open set in (Y,σ). Since f is strongly R$^{\#}$-continuous then f^{-1}(G) is open set in (X,τ). But in discrete space, f^{-1}(G) is closed set in (X,τ). Thus f^{-1}(G) is both open and closed in (X,τ). Hence f is perfectly R$^{\#}$-continuous.
(ii) →(i): Let U be any R$^{\#}$-open set in (Y,σ). Since f is perfectly continuous then f^{-1}(G) is both open and closed set in (X,τ). Hence f is strongly R$^{\#}$-continuous.

Theorem 3.52: Let (X,τ) be any topological space and (Y,σ) be $T_R{}^{\#}$ space and f: (X,τ)→(Y,σ) be a map. Then the following are equivalent.

i. f is strongly R$^{\#}$-continuous
ii. f is continuous

Proof:

(i) →(ii): Let F be any closed set in (Y,σ). Since every closed set is R$^{\#}$-closed and hence F is R$^{\#}$-closed in (Y,σ). Since f is strongly R$^{\#}$ continuous then f^{-1}(F) is closed set in (X,τ). Hence f is continuous.

(i) →(ii): Let G be any $R^{\#}$-closed set in (Y,σ). Since (Y,σ) is $T_R^{\#}$ space, F is closed set in (Y,σ). Since f is continuous then f^{-1}(F) is closed set in (X,τ). Hence f is strongly $R^{\#}$-continuous.

Theorem 3.53: Let f: (X,τ)→(Y,σ) be a map. Both (X,τ) and (Y,σ) are $T_R^{\#}$ space. Then the following are equivalent.

i. f is $R^{\#}$-irresolute
ii. f is strongly $R^{\#}$-continuous
iii. f is continuous
iv. f is $R^{\#}$-continuous

The proof is obvious.

Theorem 3.54: Let X and Y be ${}_R{}^{\#}T_g$ spaces. Then for the function f: (X,τ)→(Y,σ) the following are equivalent.

i) f is gc-irresolute
ii) f is $R^{\#}$-irresolute

Proof:

(i) →(ii): Let f: X→Y be gc-irresolute. Let F be a g-closed set in Y and hence $R^{\#}$-closed in Y. Since f is gc-irresolute then f^{-1}(F) is g-closed set in X and hence $R^{\#}$-closed set in X. Therefore f is $R^{\#}$-irresolute.

(i) →(ii): Let f:X→Y be $R^{\#}$-irresolute. Let F be a g-closed set in Y and hence $R^{\#}$-closed in Y. Since f is $R^{\#}$-irresolute then f^{-1}(F) is $R^{\#}$-closed set in X. But X is ${}_R{}^{\#}T_g$ space and hence f^{-1}(F) is g-closed set in X. Therefore f is g-irresolute.

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