RESEARCH ARTICLE

DESIGNING THE TRAFFIC PHASES USING GRAPH COLOURING AND BOOK THICKNESS TO REDUCE THE TRAFFIC FOR A SELECTED JUNCTION IN COLOMBO CITY.

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Abstract
Traffic signaling systems can be optimized by reducing the cycle time byway of minimizing the traffic phases. In this study the concept of graph colouring and the book thickness were used to develop an algorithm in order to minimize the number of traffic phases at Borella junction in the Colombo city and optimize the traffic signaling system. The proposed algorithm facilitates a mechanism to reduce the Cycle Time of that junction by reducing the number of traffic phases in a traffic signaling system. As a result, the number of traffic phases of Borella junction was reduced from five to four.

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Introduction:
Road traffic is a critical problem in Sri Lanka especially in the Colombo city and suburbs. According to the ComTrans report [5], a huge speed drop due to the heavy traffic is clearly visible during the peak hours. During the peak hours, lengthy vehicle queues are there in many junctions even though they are controlled through the traffic light signaling system. Most of the junctions with traffic light systems are operated manually by the traffic police during the peak hours due to heavy traffic.

Moreover, a direct connection between the Cycle Time of a traffic signaling system and the traffic queue formation can be witnessed, since the other traffic movements should be stopped when one traffic movement is allowed. Then the queues will form in the traffic movements which were stopped within the time period of inter green. Similarly when the unnecessary excess amount of green time was allocated for certain traffic movements, all the other vehicles in other traffic movements should be stopped and it will be result in the queue formation in those traffic movements. In that situation if the green time given for a certain traffic movement is not adequate to flow the length of the newly formed queue, the traffic congestion in that area will increase further.

Since the Cycle Time contains with the green time of each phase with inter green time, the reduction of the Cycle Time needs to be considered when forming a system to reduce the traffic. The Cycle Time of a certain traffic signaling system depends on the number of traffic phases in the relevant junction. Therefore obtaining the minimum number of traffic phases will lead to optimize the traffic signaling system.

Graph theoretical concepts such as planarity, book thickness together with graph colouring can be used to avoid intersecting edges. Also there can be seen a method of managing the signaling process using graph theory with the concept of grouping the traffic movement (Baruah and Baruah 2012). Moreover the concept of circular arcs in Graph theory has been used when installing the traffic lights system (Darshankumar and Jhala 2014), (Hosseini and Orooji 2009).

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The graph colouring is a method which has used to avoid edge intersection. Therefore the formulation of the traffic phases is possible when the concepts in graph colouring are used. The concept of graph colouring has used to find the different types of accident zones in traffic flows (Myna 2015). However they have used the edge colouring to obtain the accident zones. Similarly the vertex colouring concept was used in many applications including air traffic flow management (Barnier and Brisse 2002), scheduling radio transmission networks (Ramanathan and Lloyd 1993), time table scheduling and Birectortasks (Marx 2004).

Borella is a junction where currently traffic signaling system is operated but considered as a junction where huge traffic congestion occurs during the peak hours. Thus graph theoretical concepts such as planarity, book thickness and graph colouring were used in this study to come up with more suitable model and optimize the performance of the model.

**Methodology:**

There is a need for the identification of a graph for the selected junction. Planar graphs from the identified graph should be obtained while minimize the number of planar graphs. The junction with traffic signaling system should be converted to a graph by properly identifying the relevant edges and vertices. Since the objective of this study is to optimize traffic signaling through the reduction of number of traffic phases, a proper attention must be paid to the traffic movements. The graph will reflect a representation of the traffic movements. A junction can be converted to a graph by taking the entering or exit point as vertices and the road segments connected by them as edges. Moreover the corresponding traffic movements can be represented through directed edges, since those road segments will represent the traffic movements from entering point to the junction and from junction to exit point.

After converting the traffic movements of the selected junction to a graph, the existence of planar graphs must be examined. Since each planar graph include the traffic movements which doesn’t intersect with each other, it can be regarded as traffic phases to the relevant junction. The process of obtaining the traffic phases can be considered through obtaining planar graphs using graph colouring.

For this purpose (to apply graph colouring), an edge of the graph should be formed between two vertices if those traffic movements are intersecting. Then according to the concept of graph colouring, the different colours will be assigned to the vertices which connected through edges. Then two vertices which have same colour will represent two traffic movements which doesn’t intersect while the two vertices with different colour will provide the traffic movements which intersect with each other. As a result, the chromatic number of the graph is obtained. Hence minimization of planar graphs can be obtained through this process which will provide the set of traffic movements (edges in the former graph) which doesn’t intersect with each other. And the chromatic number will provide the minimum number of colours that can be used when generating the planar graphs which is referred as the book thickness.

The program which constructed to implement the above algorithm will be facilitated to input the traffic movements which are intersecting with each other as a set of edges to obtain the set of traffic phases as the output.

**Algorithm:**

Step 1: Construct a graph \( G \) for the selected junction in terms of the edges \( E \) and vertices \( V \).
Step 2: Select any vertex \( v \in V \) and create two empty sets \( S_1 \) & \( S_2 \).
Step 3: Assign a colour to \( v \) and add it to the set \( S_1 \).
Step 4: For \( v_j \in V \) when \( v_i \neq v_j \) and if \( v_i \), \( v_j \) is not connected then assign the same colour to \( v_j \) and add to \( S_2 \), otherwise assign a different colour and save in a different set \( S \).
Step 5: Take another \( v_k \) from \( S \) and repeat the step 4.
Step 6: Repeat the process till all the vertices has assigned colours such that no two vertices which are connected has same colours.
Step 7: Return the chromatic number.
Step 8: Build a book (book embedding) using all planar graphs which were obtained from the chromatic number.

The traffic phases are designed by converting the planar graphs to the traffic movements in a manner that each vertex represents an edge in the graph. There are the vertices that will be used to represent the entering and exit points while the edges represent traffic movements. Hence each planar graph in above process can be identified as a representation of a traffic phase.
A suitable method of calculating the Cycle Time was used to model the signaling system after the traffic phases were designed. Here the following equation was used to obtain the green time for each traffic phase after obtaining the traffic phases. It was developed by considering the time which a car (not a vehicle since the calculations were done using passenger car equivalence) with average speed will be needed to pass the entering and exit points in a certain traffic movement along with the number of vehicles in a certain traffic length. The traffic length represents the optimum number of vehicles which needed to be flowed to reduce the possibility of occurring the traffic congestion in the nearest junction which connected to the entering point.

\[ T_i = t + (N - 1) \cdot \frac{v \cdot 1000}{3600 \cdot c}, \]

where
- \(T_i\) – Green time for the relevant traffic movement,
- \(t\) – Time needed for first vehicle to pass the junction,
- \(N\) – Number of vehicles in the traffic length in passenger car equivalence,
- \(v\) – Average vehicle speed and
- \(c\) – Average length of a car.

Then the maximum \(T_i\) for a relevant traffic phase can be considered as the corresponding green time for that phase and using all the green times for the traffic phases in the junction and the inter green time, the cycle time can be calculated. Thereafter the results from the new algorithm and manual results can be compared to validate the new method.

**Results:**
Formation of the graph was done by considering the traffic movements in the junction where entering/exit point of each road connected to the junction was labelled as A, B, C, D and E (See the Fig. 1)

![Fig. 1:- The map of Borella Junction.](image)

The complete graph model \(G = (V, E)\) is given by Fig. 2 where \(V\) represents the set of all intersecting traffic movements and \(E\) represents set of all edges to represent the relation of intersection between two traffic movements. For example the traffic movements BD and AC intersect with each other. Then both traffic movements were included in \(V\) and \([BD, AC]\) was taken as an edge.
Above algorithm was used to construct a procedure in Maple 12. Then the minimum colours that needed in the vertex colouring was provided by the program along with the relevant planar graphs as output. Since here the number of planar graphs which was used as the traffic phases was minimized, the minimum number of colours will be the book thickness.

The program for the above mentioned algorithm had provided the output of the above graph (in Fig. 2) as follows; \{AC, AD, AE\}, \{BA, BD, BE\}, \{CA, CB, CE\} and \{DA, DB, DC\} in different sets. The traffic movements that not included here do not intersect with any other movement and hence the flow of those movement can be allowed in any time.

The above four sets represent four planar graphs which can be obtained through a simple transformation. Then the above sets of vertices were transformed to a more comprehensive form where the book embedding was easily obtained. There each vertex which represent a traffic movement was transformed to an edge by taking the entering and exit points as vertices. For example if the vertex AC in set one was considered, it will transformed to the edge \[A, C^{'}) in the planar graph \[G_1\]. Here \(C^{'}) represents an exit point while \(A\) represent entering point. Similarly from converting each vertex in the set one, the planar graph \(G_1\) was formed. Then \(G_i = (V_i, E_i)\) where \(V_i = \{A, C, D, E\}\) and \(E_i = \{[A, C], [A, D], [A, E]\}\). In a similar process all the planar graphs were obtained. Afterward the book embedding was obtained considering whether the number of pages were minimized. There it was compared that the chromatic number obtained through the above procedure was equivalent to the book thickness. The book embedding which was obtained using above planar graphs are given in Fig. 3.

Since the traffic movements which doesn’t intersect with each other will not affect the planarity in each graph which obtained earlier, the edges was added to each page without violating the planarity. Each page of the book embedding
was clearly given as the pages in Fig. 4 along with the edges for the traffic movements which doesn’t intersect each other (Those traffic movements were given in light green colour). Here the number of pages were optimized and the book thickness is four.

Moreover each edge of the above book embedding represent a traffic movement and therefore the pages were taken as the traffic phases. For example, phase 1 was taken as 

\[ [A, C], [A, D], [A, E] \]

The Cycle Time for the junction was calculated using above mentioned process. Here the Cycle Time was obtained for the current process and for the traffic phases obtained by the book embedding. Then the green time for each phase has been around 9.1 seconds and inter green time was taken as 4.8 seconds.

There is five traffic phases in the current traffic signaling system in this junction, which has the Cycle Time of 69.5 seconds. For the traffic phases obtained from book embedding, the Cycle Time was 55.6 seconds. The number of traffic phases were reduced to four through this approach and as a result of that approximately 14 seconds was reduced from the Cycle Time.

Discussion:-
The output of the program of above mentioned algorithm only reflects the traffic movement which at least intersecting once. Then the traffic movements which doesn’t intersect with any other traffic movement can be allowed without any restriction and it will not be reflected in the Cycle Time. Because those traffic movements always will have free flow since it will not be encountered any restriction.

However there should be a proper knowledge of the road capacity of the road segment which the flow projects in, because allowing the flow of that road segment might cause the traffic congestion when other flows are projected in to the same road segment concurrently.

Conclusion:-
When the concept of graph colouring was used to design the traffic phases, the number of traffic phases were reduced ultimately reducing the Cycle Time.

Furthermore the Cycle Time calculation which was developed was considered the vehicle density in a certain traffic length when calculating the green time and therefore it was considered as a proper indication of the traffic flow. Here both current and the proposed processes were compared and the proposed process is better than the current process when considering the number of phases and Cycle Time.
References:


