RESEARCH ARTICLE

EFFECTS OF CONFIDENCE AND SOCIAL PRESSURE ON DECISION MAKING PROCESSES.

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Abstract

A new opinion dynamics model in which the individuals are identified by two variables, namely parameterized with their binary opinion and confidence levels is introduced. In this model the exchange of opinions are realized through pair wise interactions. The dynamics is governed by the pressure of the neighboring sites and personal convictions of the individuals living on a regular lattice. The model successfully to describe opinion exchange mechanisms of the societies where individuals with various levels of confidence exchange opinion through pair wise interactions under the influence of nearest neighbors.

Introduction:

Societies consist of individuals who try to optimize their welfare. Each individual make some local changes with an expectation to improve its welfare state which leads to underlying dynamics of the global changes of the whole society. In this sense social systems and physical systems, such as spins systems, exhibit close similarities: Both want to optimize their statue in the system. These similarities are basic motivation for using statistical physics in many situations that occur in social systems. In applying statistical physics to the social systems the essential assumption is that, the individual preferences do not play crucial role in the global changes of the societies. Simulation techniques play an important role in statistics which are successfully applied in various areas of the social sciences. Rumour propagation, epidemics, opinion dynamics are the major areas of social sciences that simulation techniques have been applied with considerable success [1].

To modeling social systems, it is necessary to quantify the outcome of the individual behavior of the by suitable variables. These variables can take continuous or discrete values. Together with determining the variables, also the local the interactions between the individuals must be formulated. Basic difference between the physical systems and the social systems appear at this point. In physical systems the constituents have well defined interaction rules based on the energy minimization, while for the social systems different rules for dynamics are proposed for the evolution of the system. These rules are based on which behavior of the individual is observed.

One of the major issues in simulation studies of the social behavior is how opinions evolve in a social system, namely opinion dynamics. In opinion dynamics variables are opinions of the individuals which may take continuous [2]-[4] or discrete [5]-[6] values. Starting from randomly distributed initial opinions, dynamics may leads to consensus or fraction. Consensus is unanimously agreement on one of the opinions. While fraction is more than one opinion are supported by various group of the society. In magnetic systems all spins can be aligned (consensus) or sum of the spins may point one direction while the others point opposite (fraction).
There exist a great number of opinion dynamics models which describe the formation of opinions in a social system. Schelling model [7] is known as the first one of the simulation model. Since then various models and their modifications are improved [8]. Some of these models introduce detailed opinion exchange mechanisms such as bounded confidence model [9] and they are applied to various social events successfully [10]-[12]. Bounded confidence model assumes continuous opinion and pair wise interaction. In this model the interacting individual may change opinion only if two interacting individuals have opinion differences less than a threshold value. In Hegselmann and Krause [13] model a randomly selected individuals accepts an average opinion of the neighbors which is different from his opinion by a confidence parameter $\epsilon$. Some others take simple opinion exchange rules. In these simple models exchange of opinions can simply be accepting opinion of randomly chosen neighbor [10]-[12] or the individual may be influence by its neighbors and take the opinion of majority [14]-[15]. Sznajd model among these simple models is an exception. This model assumes that every individual try to influence its neighbors. If two neighbors share the same opinion neighbors of the pair accept their opinion.

As a sociological fact, opinion formation is strongly related to social pressure and confidence among many other effects. Some people easily accept the opinion of the neighbors while some others are reluctant to accept opinions coming from the others. Hence, even simple model of opinion formation requires taking into consideration social pressure on the individuals. The majority rule, in which agents agree with the local majority opinion at every time step is widely used in opinion dynamics studies [17]-[18]. This type of behaviour called herding behaviour, it is assumed that the minority definitely follows the majority. It is obvious that together with the effects of the neighbors, some personal merits plays important role in persuasion process.

Recently a variation the bounded confidence model has been introduced [19]. In this model agents are placed at the vertices of a regular lattice. Interactions are possible within a certain range the agents to interact with those which are located beyond their first neighbors. Each agent is identified by two parameters, its opinion and degree of conviction. The degree of conviction of an agent is affected during the interaction. If the interaction is with a neighbors of higher conviction, the agent lose some of his/her conviction. If the opponent has lover conviction in the light of the above discussion gain conviction. When one of the agents reaches a certain lower bound of conviction then its opinion changes to that of the opponent. Therefore, in this work, the difficulty in persuading someone who has a strong conviction was taken into consideration.

In this work, a new model which aims to study the relation between the social pressure and the confidence is proposed. In this model the individuals interact only with their nearest neighbors and it is assumed that they may exchange opinion through pair wise interactions. The model is essentially a variation of the majority rule model: two randomly selected individuals exchange opinion under the pressure of the neighboring sites and their own personal convictions which is added to the model as a new parameter for each individual. If an individual interact with one of its neighbors, a) may convince the neighbor to change opinion, b) may change its opinion. If an individual is in local agreement with its surrounding neighbors and have high confidence may hardly change opinion. In this model a function which measures the satisfaction level of the individuals is introduced. This function sets numerical value for the local agreement with the surrounding neighbors and the confidence of the individual living at site $i$. When two neighbors exchange opinion the relative values of their satisfaction functions play the decisive role and exchanging opinion can be realized according to the rules give in Section II.

The main aim of this study is to determine the role of confidence of the individuals and the pressure created by the neighboring agents in opinion exchange mechanisms. This work differs from the existing literature by the opinion exchange dynamics.

This work is organized as follows. Section II is devoted to a detailed description of the model. The results and discussions on the results are presented in Section III whereas concluding remarks are drawn at Section IV.

**Computer Simulation Model:-**

The society consists of $N$ interacting individuals living on a regular 2-dimensional lattice with only nearest neighbor interactions. The dynamics of the opinion exchange is assumed to take place through discrete time steps. The agents are identified by two parameters: their opinion and confidence level. Opinion can only take discrete values while the confidences of individuals are parameterized by real numbers.
Agents possess binary opinions: ±1 for supporting and for opposition the proposition. Initially, the opinions are set randomly ±1 with probabilities \( p \) and \( q = 1 - p \). The confidence parameters are set according to the initial distribution of the opinion of the individuals. In this work, all individuals may carry i) the same confidence level, ii) two different confidence levels for +1 and −1 opinion holders, iii) randomly distributed confidence levels for each individual. Since both, confidence levels and the opinions depend on the probability, \( p \) is a crucial control parameter of the simulation. During this work the probability is kept as \( p = 1/2 \).

Opinion dynamics models such as Sznajd Model \[19\] assume interaction between the agents sharing the same opinion. When such a pair interacts, they influence all of their nearest neighbors. The Sznajd model is an exception in this sense. There exist also some other models where the majority opinion is the decisive factor for changing opinion. The proposed model is based on mutual interaction of pair of individuals occupying two neighboring sites. In this sense, the interaction type is similar to Sznajd model with the difference that the Sznajd model considers only two sites with the same opinion whereas in this model pairs can carry any opinion. The selected pair can influence each other according to their level of satisfaction. In such a societies satisfaction can be described as the level of cooperation with the neighbors: if the majority of the neighbors share the same opinion the individual feels comfortable and satisfied with its position. The satisfaction, in this model, has another component: the confidence. An individual with high confidence level feels satisfied even when some neighbors are not in accord. If the majority of the neighbors carry the opposite opinion then this individual either have to change its opinion to reduce the pressure exerted by the neighbors or has to move to another place. This is common in physical systems where under pressure constituents of the system change state to reduce their energy or exerted pressure. Hence in a society, satisfied members are the highly confident individuals which feel comfortable in interactions with the like individuals.

The satisfaction of the individual living at site \( i \) can be formulated as,

\[
E_i = O_i \left( \sum_{j \in NN} O_j + C_l \right)
\]

(1)

where \( E_i, O_i \) and \( C_l \) are the satisfaction, opinion and the confidence parameters of the individual living at site \( i \). \( j \) and \( O_j \) indicate the location and opinion of the neighboring sites.

The model is based on interaction of two neighboring individuals. One of the individual may convert the other through their mutual interaction. This interaction and the opinion changing mechanism depends on the satisfaction parameters of the individuals.

As far as the satisfaction parameters are concerned three different cases must be considered.

1. Both neighbors are unsatisfied 
   \( E_i < 0 \), \( E_j < 0 \).
2. Either one of the neighbors is satisfied while the other is unsatisfied 
   \( E_i < 0 \), \( E_j > 0 \) or \( E_i > 0 \), \( E_j < 0 \).
3. Both neighbors are satisfied 
   \( E_i > 0 \), \( E_j > 0 \).

These three cases can be used to formulate the local dynamic of the opinion changing mechanism.

1. Case 1: if both parties are unsatisfied, changing opinion for both will create satisfied individuals. Hence in this situation both interacting individuals change opinion. 
   \( E_i < 0 \), \( E_j < 0 \) \( O_i, O_j = O_i \).
2. Case 2: In the case one of the neighbors are unsatisfied while the other is satisfied. 
   \( E_i < 0 \), \( E_j > 0 \) \( O_i, O_j = O_j \).
   \( E_i > 0 \), \( E_j < 0 \) \( O_i, O_j = O_j \).

Unsatisfied individual of the pair will accept opinion of the other.
3. Case 3: If both neighbors are satisfied, $E_i > 0, E_j > 0$, and they both share the same opinion. Opinion change is not possible. But if the opinions are different, $O_i \neq O_j$, relative satisfaction determines which one of the interacting neighbors will change opinion:

$$E_i < E_j O_i = O_j, O_j = O_i$$

$$E_i > E_j O_i = O_j, O_i = O_j$$

Here in this opinion updating process opinion changes are realized according to the satisfaction level of the individuals.

These updating rules are the local opinion exchange moves. After setting the opinion distribution and confidence levels, the observation of the dynamic changes of the opinions of the individuals or the society as the whole constitute the basic steps of the simulation.

A configuration is considered as a snapshot of the society at a given time $t$. Initial configuration is the distribution of opinions at the time $t_0$, the beginning of the debate. Debate continues with pair wise interactions of the individuals until the society reaches the final distribution of opinions. Reaching this final state takes some time. The final state can be a consensus or polarization. In order to observe the final state of the society it is best to analyze the configuration average of the opinions distributed in the society at any given time. Configuration average is calculated as follows,

$$\overline{O}(t) = \frac{1}{N} \sum_{i=1}^{N} O_i(t)$$  \hspace{1cm} (2)

Where $O_i(t)$ and $\overline{O}(t)$ are the number of individuals, the opinion of the $i^{th}$ individual at time and $\overline{O}(t)$ is the configuration average of the opinion held by the society respectively. $\overline{O}(t) = \pm 1$ indicate consensus on one or the other opinion.

Since the initial configuration is an arbitrary distribution of opinions with a given fixed value of probabilities p and q, one must take average over many statistically independent configurations. The number of statistically independent configurations are $N_{samples}$. The simulation should be repeated and averages must be taken over the statistically independent configurations.

$$\langle \overline{O} \rangle (t) = \frac{1}{N_{samples}} \sum_{i=1}^{N_{samples}} \overline{O}_i(t)$$  \hspace{1cm} (3)

Given any starting configuration the dynamics determined by the defined interactions change the average opinion held by the society to the most stable configuration. This process requires the transition time, $t_{transition}$. After this transition period the society reaches a final state which may fluctuate around an average value or reaches to consensus over one of the states. The time duration from a given initial state, to the state where stationary state are observed $t_{max} \gg t_{transition}$ varies with the values of free parameters and the interactions between the individuals. For a given set of parameters, configuration averages calculated at $t_{max}$ fluctuate around an average $\langle O \rangle$.

$$\langle \overline{O} \rangle = \frac{1}{N_{sample}} \sum_{i=1}^{N_{samples}} \overline{O}_i(t_{max})$$  \hspace{1cm} (4)

where, $N_{sample}$ is the number of different statistically independent initial configurations which contribute to the average.
If both opinions live together, the average opinion represents the opinion of the society. In some cases, in the long term, the societies end up with consensus on one or the other opinion. Depending on the initial configuration either one of the opinions can win. In the case of equal distribution of opinions consensus may be reached on one or the other opinion values. Hence, taking the average gives misleading result. Together with the average one also must check whether this average is truly representing the opinion of the society. Best method is to calculate the distribution of the configuration averages. If distribution is normalized to the number of configurations one can obtain the percentages of the configurations which end with consensus on the pro or con opinions.

Results:
The simulations are performed on $(50 \times 50)$ square lattice. Initial configurations are prepared by setting two parameters which identify the opinion and confidence level of each individual. A configuration is created by filling the sites randomly with opinion values $\pm 1$ with equal probability. Confidence parameters are set according to two scenarios: i) All individuals have fixed confidence values, ii) Supporters of one of the opinion have higher confidence level. After preparing the initial configuration the time development of the opinion change of the society is observed for $10000$ discrete time steps. During the simulation a new configuration is created by updating randomly chosen lattice sites. Updating each site is repeated volume times $(50 \times 50)$. Since pairwise interactions are considered, this updating procedure corresponds to approximately updating each lattice site twice. Averages are calculated over $500$ statistically independent configurations.

During the simulations, opinion averages are calculated by taking the configuration averages, Eq. (2) at each time step. The average of the configuration averages are calculated for each time slice over the $500$ statistically independent runs, Eq. (3). The configuration averages are also used to calculate configuration distribution at every time step. Distributions are very illuminating since opinion averages may be misleading for the situations where consensus may be reached on either one of the opinions.

Fig. 1 shows the time evolution of the average opinion formation for various confidence parameters. From bottom to top, first two curves corresponds to equal confidence parameter values which are $C_1 = C_2 = 0$ and $C_1 = C_2 = 0.1$. For the case where vanishing confidence parameter is considered, all configurations exhibit very low average opinion value. This is an indication that in an equally divided society, with individuals ready to change opinion under any pressure, both opinions will always survive. Even small confidence parameters change this situation. In the case where supporters of both opinions have equal, small confidences, an immediate change in the behavior appears. This changing behavior can be summarized in two stages. The first stage corresponds to a very rapid opinion changing regime. After this rapid change the system gets into a more stable phase where more and more individuals share the same opinion. The effect of the unbalanced confidences is shown in top curve. Considering supporters of either one of the opinions with slightly larger confidence, $C_1 = 0.1$, $C_2 = 0.2$ change the picture completely. The initial rapid opinion changes are much faster in this case. System reaches consensus after a very rapid persuasion process.

![Fig. 1: Time evolution of the average opinion of an initially equally divided society.](image)

Fig. 2-5 show the time evolution of the distribution of configuration averages of the opinion formation. Each figure exhibits the effects of a different set of confidence parameter. In Fig. 2 vanishing confidence level is considered for supporters of either one of the opinions. As it is seen from the time development of average opinion formation Fig. 1 the distribution also exhibit that with vanishing confidence, individuals change opinion under the influence of
neighbors only. This influence is a random effect. Hence the configuration averages are distributed around the vanishing average value.

**Fig. 2:** Distribution of average opinion for a society with vanishing confidence $C_1 = 0, C_2 = 0$.

Fig. 3, also shows a similar behavior to that of Fig. 2: distributions are centered around the vanishing average opinion value. Similarity between the Fig. 2 and Fig. 3 ends here. The reason of having distributions centered around vanishing average opinion value, in this case, is very different. The strong confidence does not allow the individuals to change opinion under the influence of the neighbors. Since the original initial configuration consist of equal number of both pro and con individuals, configuration average remains very small under the influence of the high and equal confidence levels.

**Fig. 3:** Distribution of average opinion for a society with high and equal confidence $C_1 = 2.0, C_2 = 2.0$.

Fig. 4 shows time development of the distribution of configuration average of opinions. Here both opinions are supported by the individuals with non-vanishing and equal confidence. Supporters of both opinions try to persuade the individuals with opposite opinion. Depending on the initial configurations, one or the other opinion succeed to reach consensus. The peaks at the extreme ends, $< \bar{O} > = \pm 1$ indicate that consensus is reached considerable times. In the case where both parties have equal confidence, it is almost impossible to predict which initial configurations will lead to consensus. There are also many configurations with average opinion values lies in between $-1$ and $+1$ which give a distribution around 0. This behavior can be better understood by comparing the Fig. 1 and Fig. 4.

**Fig. 4:** Distribution of average opinion for a society with non-vanishing and equal confidence $C_1 = 0.1, C_2 = 0.1$
This situation completely different when even a small difference appear in confidences. Fig. 5 is devoted to the discussions of the effects of difference in the consensus parameter. Individuals with strong confidence are difficult to persuade. Moreover they have advantage that they are more satisfied in their local environment.

![Graph](image)

**Fig. 5:** Distribution of average opinion for a society with high and equal confidence \( C_1 = 2.0, C_2 = 2.0 \)

This high satisfaction enables them to persuade the opponents in interactions. This is clearly seen in Fig. 1, and Fig. 3. In Fig. 5 distribution of the configuration averaged opinion values peak around zero at the initial state of the simulation. But immediately after few hundred iterations every configuration ends with consensus on the opinion which is supported by high confidence supporters. Here in this simulation work the consensus is reached on the opinion with the value \(-1\), whose supporters carry confidence level of 0.2.

**Conclusions:**

The proposed model is a simple, deterministic model of opinion formation with confidence parameter which is similar to the external field in the physical systems. In this simple form, the model is capable of predicting the conditions for partition or consensus. Numerical results give the indication that even a small non vanishing confidence parameter is essential for reaching consensus. If supporters of both opinions have equal confidence, the consensus is just coincidental.

The model in this simple form can be considered as a laboratory to discuss the decision making mechanisms. Moreover it is open to extensions. The simple extension can be that the confidences may change in time with pairwise interactions.

**References:**