Journal Homepage: - www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI: 10.21474/IJAR01/4086
DOI URL: http://dx.doi.org/10.21474/IJAR01/4086

RESEARCH ARTICLE

OPERATIONS ON M-STRONG FUZZY GRAPHS.

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Manuscript Info

Manuscript History

Received: 08 March 2017
Final Accepted: 01 April 2017
Published: May 2017

Key words:-
Deleted lexicographical product,Disjunction,Fuzzy graph, Symmetric difference, Strong fuzzy graph.

Abstract

In current paper, deleted lexicographical product, disjunction and symmetric difference on fuzzy graphs are defined and some of their properties are discussed. Moreover, the concept of M-strong fuzzy graphs are investigated for mentioned operations. These results also are illustrated with some examples.

Introduction:-

In 1965, Lotfi A. Zade established Fuzzyset for representing uncertainty [**6**].Fuzzy set has numerous applicationsin different branches ofmodern sciencesconsisting operations research, transportation, information theory and neural networks[7, 8].

A graph$G$ is anordered pair(V, E), where $V = V(G)$ is the set of vertices of G and $E = E(G)$is the set of edges of G where $E \subseteq V \times V$.Graph theory has been used to study modern science such as operations research, transportationand cluster analysis.

In 1975, Rosenfeld introduced fuzzy graphs [**5**] based on fuzzy set.Fuzzy graph theory plays essential roles in various disciplinesincluding information theory, neural networks, clustering problems and control theory, etc.Fuzzy models is more compatible to the systemin compare with classical models [**9**, 10].

Bhutani and Rosenfeld introduced the notion of M-strong fuzzy graphs and studied some of their properties. [1, 4]
Many interesting graphs are obtained fromcomposing simpler graphs via several operations. For more information on graph operations see [3].

In this paper,we definedeleted lexicographical product,disjunction and symmetric difference oftwo fuzzy graphsand prove that new graphs constructed from mentioned operations are fuzzy graph. Also we show that deleted lexicographical product,disjunction and symmetric difference of two M-strong fuzzy graphs are also M-strong fuzzy graph.Finally we prove that if $G_1 \overline{\times} G_2$, $G_1 \vee G_2$ and$G_1 \oplus G_2$are M-strong fuzzy graphs, then at least one factor must be M-strong fuzzy graph.All properties are illustrated with examples.

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Preliminaries:-

In this section, we list some necessary definitions as follows:

Definition 2.1[6]. A fuzzy set A is a set of ordered pairs $\{(x, \mu_A(x)|x \in X\}$ where X is universal set. $\mu_A(x)$ is a map from X to $[0, 1]$ which is called a membership function or degree of membership of x in A.

Definition 2.2[5].A fuzzy graph G is a pair of functions (μ, ρ) where μ is a fuzzy subset of X and $\rho: X \times X \to [0,1]$ is symmetric fuzzy relation on μ such that $\rho(x, y) \le min\{\mu(x), \mu(y)\}$.

Throughout the paper, we use xy instead of $(x, y)$for an element of $E(G)$.

Definition 2.3[2].The *deleted lexicographical product* of two graphs G_1 and G_2 is defined as a graph $G_1 \overline{\times} G_2$with the vertex set$V(G_1) \times V(G_2)$andvertex$(x_1, x_2)$isadjacent with vertex$(y_1, y_2)$whenever $x_1$is adjacent with y_1 in G_1 and $x_2 \ne y_2$ or $x_1 = y_1$and x_2 is adjacent with y_2 in G_2.

Definition 2.4[3].The *disjunction* of two fuzzy graphs $G_1$and G_2 is defined as a fuzzy graph $G_1 \vee G_2$withthevertexset$V(G_1) \times V(G_2)$andvertex(x_1, x_2)isadjacentwithvertex(y_1, y_2)whenever $x_1$is adjacent with y_1 or x_2 is adjacent with y_2 in G_2or both of them.

Definition 2.5[3].The *symmetric difference* of two fuzzy graphs $G_1$and G_2 is defined as a fuzzy graph $G_1 \oplus G_2$withthevertexset $V(G_1) \times V(G_2)$andvertex(x_1, x_2)isadjacent with vertex(y_1, y_2)whenever x_1 is adjacent with y_1 in G_1 or x_2 is adjacent with y_2 inG_2 but not both.

Definition 2.6[1]. A fuzzy subgraph (μ, ρ) of G is called a M-strong fuzzy *sub graph* if $\rho(xy) = min\{\mu(x), \mu(y)\}$ for all $xy \in E(G)$**.**

Some operations on fuzzy graphs:-

In this section, we present new definitions for some fuzzy graph operationsand prove that new graphs are also fuzzy graphs.

Definition3.1. The *deleted lexicographical product* of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G_1 \overline{\times} G_2$with the vertex set$V(G_1) \times V(G_2)$and vertex$(x_1, x_2)$is adjacent with vertex$(y_1, y_2)$whenever $x_1$is adjacent withy_1 in G_1 and $x_2 \ne y_2$or $x_1 = y_1$and x_2 is adjacent with y_2 in G_2 with

$(\mu_1 \overline{\times} \mu_2)(x_1, x_2) = min\{\mu_1(x_1), \mu_2(x_2)\}$ for all $(x_1, x_2) \in V(G_1 \overline{\times} G_2)$

and

$(\rho_1 \overline{\times} \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = min\{\rho_2(x_2y_2), \mu_1(x_1)|\ x_1 = y_1 \in V(G_1),\ x_2y_2 \in E(G_2)\}$

$(\rho_1 \overline{\times} \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = min\{\rho_1(x_1y_1), \mu_2(x_2), \mu_2(y_2)|\ x_1y_1 \in E(G_1),\ x_2, y_2 \in V(G_2)\}$

Theorem 3.1. Let G beadeleted lexicographical product of two fuzzy graphs G_1 and $G_2$and (μ_i, ρ_i)be a fuzzy subgraph of $G_i$where$i \in \{1,2\}$, then $\big((\mu_1 \overline{\times} \mu_2), (\rho_1 \overline{\times} \rho_2)\big)$be a fuzzy subgraph of G**.**

Proof. Let $G = G_1 \overline{\times} G_2$, we have:

$(\rho_1 \overline{\times} \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = min\{\mu_1(x_1), \rho_2(x_2y_2)\}$

$\le min\{\mu_1(x_1), min\{\mu_2(x_2), \mu_2(y_2)\}\}$

$= min\{min\{\mu_1(x_1), \mu_2(x_2)\}, min\{\mu_1(x_1), \mu_2(y_2)\}\}$

$= min\{min\{\mu_1(x_1), \mu_2(x_2)\}, min\{\mu_1(y_1), \mu_2(y_2)\}\}$

$= min\{(\mu_1 \overline{\times} \mu_2)(x_1, x_2), (\mu_1 \overline{\times} \mu_2)(y_1, y_2)\}$.

and also,

$(\rho_1 \overline{\times} \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = min\{\rho_1(x_1y_1), \mu_2(x_2), \mu_2(y_2)\}$

$\le min\{min\{\mu_1(x_1), \mu_1(y_1)\}, \mu_2(x_2), \mu_2(y_2)\}$

$= min\{min\{\mu_1(x_1), \mu_2(x_2)\}, min\{\mu_1(y_1), \mu_2(y_2)\}\}$

$= min\{(\mu_1 \overline{\times} \mu_2)(x_1, x_2), (\mu_1 \overline{\times} \mu_2)(y_1, y_2)\}$

Definition 3.2. The *disjunction* of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G_1 \vee G_2$with the vertex set$V(G_1) \times V(G_2)$andvertex$(x_1, x_2)$is adjacent with vertex(y_1, y_2)whenever $x_1$is adjacent with y_1 or x_2 is adjacent with y_2 in G_2or both of them with

$(\mu_1 \vee \mu_2)(x_1, x_2) = \min\{\mu_1(x_1), \mu_2(x_2)\}$ for all $(x_1, x_2) \in V(G_1 \vee G_2)$

and

$(\rho_1 \vee \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = \min\{\rho_1(x_1y_1), \mu_2(x_2), \mu_2(y_2)|\ x_1y_1 \in E(G_1),\ x_2, y_2 \in V(G_2)\}$

$(\rho_1 \vee \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = \min\{\rho_2(x_2y_2), \mu_1(x_1), \mu_1(y_1)|\ x_2y_2 \in E(G_2),\ x_1, y_1 \in V(G_1)\}$

$(\rho_1 \vee \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = \min\{\rho_1(x_1y_1), \rho_2(x_2y_2)|\ x_1y_1 \in E(G_1),\ x_2y_2 \in E(G_2)\}$

Theorem 3.2. Let G be a disjunctionof two fuzzy graphs G_1 and $G_2$and (μ_i, ρ_i)be a fuzzy sub graph of G_i where$i \in \{1,2\}$, then $\big((\mu_1 \vee \mu_2), (\rho_1 \vee \rho_2)\big)$ be a fuzzy subgraph of G .

Proof. Let $G = G_1 \vee G_2$, we have:

$(\rho_1 \vee \rho_2)\big((x_1,x_2)(y_1,y_2)\big) = \min\{\rho_1(x_1y_1), \mu_2(x_2), \mu_2(y_2)\}$
$\leq \min\{\min\{\mu_1(x_1), \mu_1(y_1)\}, \mu_2(x_2), \mu_2(y_2)\}$
$= \min\{\min\{\mu_1(x_1), \mu_2(x_2)\}, \min\{\mu_1(y_1), \mu_2(y_2)\}\}$
$= \min\{(\mu_1 \vee \mu_2)(x_1,x_2), (\mu_1 \vee \mu_2)(y_1,y_2)\}$
and also
$(\rho_1 \vee \rho_2)\big((x_1,x_2)(y_1,y_2)\big) = \min\{\rho_1(x_1y_1), \rho_2(x_2y_2)\}$
$\leq \min\{\min\{\mu_1(x_1), \mu_1(y_1)\}, \min\{\mu_2(x_2), \mu_2(y_2)\}\}$
$= \min\{\min\{\mu_1(x_1), \mu_2(x_2)\}, \min\{\mu_1(y_1), \mu_2(y_2)\}\}$
$= \min\{(\mu_1 \vee \mu_2)(x_1,x_2), (\mu_1 \vee \mu_2)(y_1,y_2)\}$

Definition 3.3. The *symmetric difference* of two fuzzy graphs G_1 and G_2 is defined as a fuzzy graph $G_1 \oplus G_2$ with the vertex set $V(G_1) \times V(G_2)$and vertex$(x_1,x_2)$is adjacent with vertex(y_1,y_2)whenever x_1 is adjacent with y_1 in G_1 or x_2 is adjacent with y_2 in G_2 but not both with
$(\mu_1 \oplus \mu_2)(x_1,x_2) = \min\{\mu_1(x_1), \mu_2(x_2)\}$ for all $(x_1,x_2) \in V(G_1 \oplus G_2)$
and
$(\rho_1 \oplus \rho_2)\big((x_1,x_2)(y_1,y_2)\big) = \min\{\rho_1(x_1y_1), \mu_2(x_2), \mu_2(y_2) \mid x_1y_1 \in E(G_1),\ x_2, y_2 \in V(G_2)\}$
$(\rho_1 \oplus \rho_2)\big((x_1,x_2)(y_1,y_2)\big) = \min\{\rho_2(x_2y_2), \mu_1(x_1), \mu_1(y_1) \mid x_2y_2 \in E(G_2),\ x_1, y_1 \in V(G_1)\}$

Theorem 3.3. Let Gbe a symmetric differenceof two fuzzy graphs G_1 and $G_2$and (μ_i, ρ_i)be a fuzzy subgraph of $G_i$where$i \in \{1,2\}$, then $\big((\mu_1 \oplus \mu_2), (\rho_1 \oplus \rho_2)\big)$ be a fuzzy subgraph of G .

Proof. Using Theorem 3.2, we can get the desired result.

M-strong fuzzy graphs:-

In this section, we prove some theorems to showthat if G_1 and G_2 are M-strong fuzzy graphs, then new fuzzy graph constructed from them areM-strong fuzzy graph too.

Also,if $G_1 \,\overline{\times}\, G_2, G_1 \vee G_2andG_1 \oplus G_2$are M-strong fuzzy graphs, then at least one factor must be M-strong fuzzy graph. All computations are illustrated with examples.

Theorem 4.1. Let G be a deleted lexicographical product of two M-strong fuzzy graphs $G_1$and G_2, then G is a M-strong fuzzy graph***.***

Proof. The first part is taken over all edges $(x_1,x_2)(y_1,y_2) \in E(G)$ such that$x_2y_2 \in E(G_2)$and $x_1 = y_1$. Using the fact that G_2 is a M-strong fuzzy graph, we have
$(\rho_1 \,\overline{\times}\, \rho_2)\big((x_1,x_2)(y_1,y_2)\big) = min\{\mu_1(x_1), \rho_2(x_2y_2)\}$
$= min\{\mu_1(x_1), min\{\mu_2(x_2), \mu_2(y_2)\}\}$
$= min\{min\{\mu_1(x_1), \mu_2(x_2)\}, min\{\mu_1(x_1), \mu_2(y_2)\}\}$
$= min\{min\{\mu_1(x_1), \mu_2(x_2)\}, min\{\mu_1(y_1), \mu_2(y_2)\}\}$
$= min\{(\mu_1 \,\overline{\times}\, \mu_2)(x_1,x_2), (\mu_1 \,\overline{\times}\, \mu_2)(y_1,y_2)\}$.

The second part is taken over alledges $(x_1,x_2)(y_1,y_2) \in E(G)$ such that $\square_1\square_1 \in \square(\square_1)$ and $\square_2, \square_2 \in \square(\square_2)$.Using the fact that G_1 is a M-strong fuzzy graph, we have
$(\square_1 \,\overline{\times}\, \square_2)\big((\square_1,\square_2)(\square_1,\square_2)\big) = \square\square\square\{\square_l(\square_1\square_1), \mu_2(\square_2), \mu_2(\square_2)\}$
$= \square\square\square\{\square\square\square\{\mu_1(\square_1), \mu_1(\square_1)\}, \mu_2(\square_2), \mu_2(\square_2)\}$
$= \square\square\square\{\square\square\square\{\mu_1(\square_1), \mu_2(\square_2)\}, \square\square\square\{\mu_1(\square_1), \mu_2(\square_2)\}\}$
$= \square\square\square\{(\mu_1 \,\overline{\times}\, \mu_2)(\square_1,\square_2), (\mu_1 \,\overline{\times}\, \mu_2)(\square_1,\square_2)\}$.

Example 4.1. Let $\square_1$and $\square_2$be two fuzzy graphsillustrated in figure1:

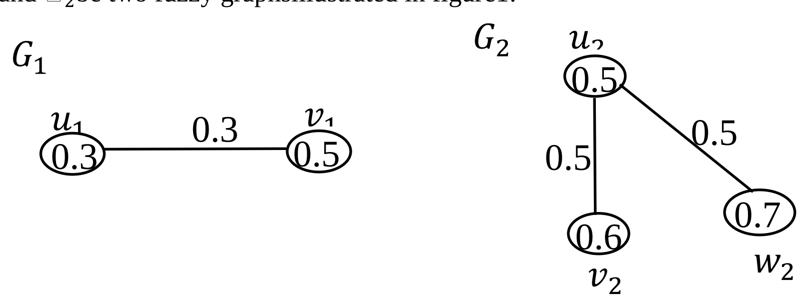

Figure 1:- Two M-strongfuzzy graphs $\square_1$ and $\square_2$.

According todefinition 3.1, deleted lexicographical product of two fuzzy graphs $\square_1$ and $\square_2$is presented in figure2:

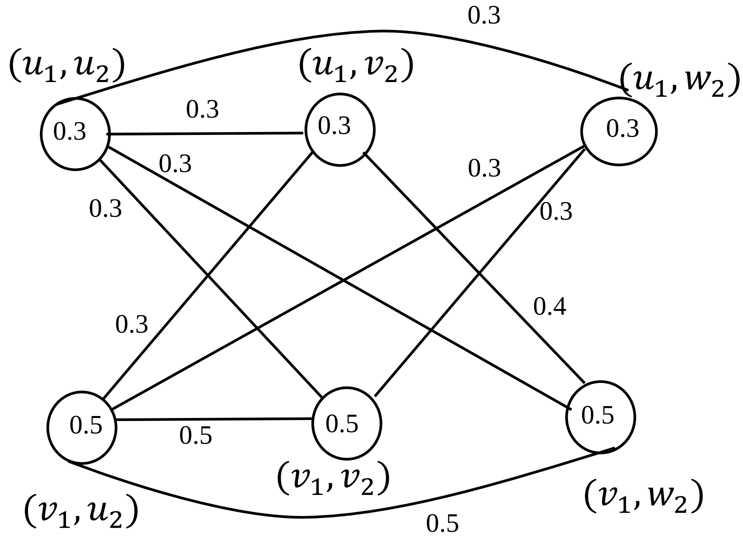

Figure 2:-Thedeleted lexicographical product of two graphs $\square_1$ and G_2 and it is easy to see that it is a M-strong fuzzy graph.

Remark. The opposite is not necessarilytrue.

Example 4.2. Two fuzzy graphs G_1andG_2anddeleted lexicographical product of them areconsidered in figure 3 and 4, respectively.

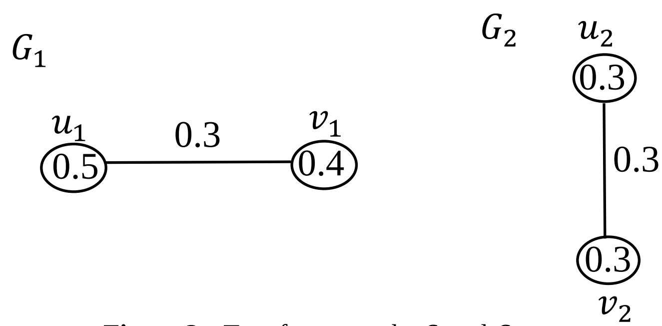

Figure 3:- Two fuzzy graphs $G_1$and G_2

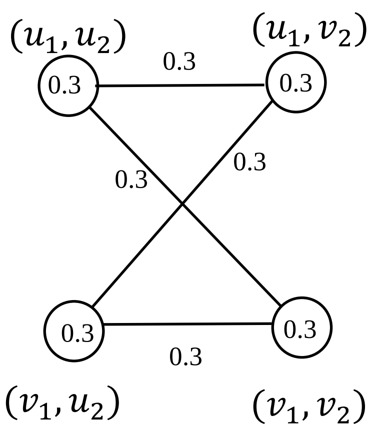

Figure 4:-The graph$G_1 \overline{\times} G_2$ formed by G_1 and G_2

It is easy to see that $G_1 \overline{\times} G_2$ and G_2 are strong fuzzy graphs but G_1 is not.

Theorem 4.2. If $G = G_1 \overline{\times} G_2$ is M-strong fuzzy graph, then at least G_1 or G_2 must be M-strong fuzzy graph**.**

Proof. Suppose that both G_1 and G_2 are not M-strong fuzzy graphs.
Then there exist $x_1y_1 \in E(G_1)$ and $x_2y_2 \in E(G_2)$ such that
$□_1(x_1y_1) < □□□\{\mu_1(x_1), \mu_1(y_1)\}$ and $□_2(x_2y_2) < □□□\{\mu_2(x_2), \mu_2(y_2)\}$
According to the definition 3.1,
$(\rho_1 \overline{\times} \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = □□□\{□_2(x_2y_2), \mu_1(x_1) \mid x_1 = y_1 \in V(G_1) \,\&\, x_2y_2 \in E(G_2)\}$
and
$(\mu_1 \overline{\times} \mu_2)(x_1, x_2) = □□□\{\mu_1(x_1), \mu_2(x_2)\}$ and $(\mu_1 \overline{\times} \mu_2)(y_1, x_2) = □□□\{\mu_1(y_1), \mu_2(x_2)\}$
Since
$(\rho_1 \overline{\times} \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = □□□\{□_2(x_2y_2), \mu_1(x_1)\} < □□□\{\mu_2(x_2), \mu_2(y_2), \mu_1(x_1)\}$
and

$$min\{(\mu_1 \overline{\times} \mu_2)(x_1, x_2), (\mu_1 \overline{\times} \mu_2)(x_1, y_2)\} = □□□\{\mu_1(x_1), \mu_2(x_2), \mu_2(y_2)\}$$

So

$$(\rho_1 \overline{\times} \rho_2)\big((x_1, x_2)(x_1, y_2)\big) < □□□\{(\mu_1 \overline{\times} \mu_2)(x_1, x_2), (\mu_1 \overline{\times} \mu_2)(y_1, x_2)\}$$

Which is in contradiction to $G = G_1 \overline{\times} G_2$ as M-strong fuzzy graph. Hence if $G = G_1 \overline{\times} G_2$ is M-strong fuzzy graph, then at least G_1 or G_2 must be M-strong fuzzy graph**.**
Usingdefinition 3.2,wecan easily arrive at:

Theorem 4.3. Let G be a disjunction of two M-strong fuzzy graphs G_1 and G_2, then G is a M-strong fuzzy graph.

Example 4.3. Consider two fuzzy graphs G_1 and G_2 in figure5:

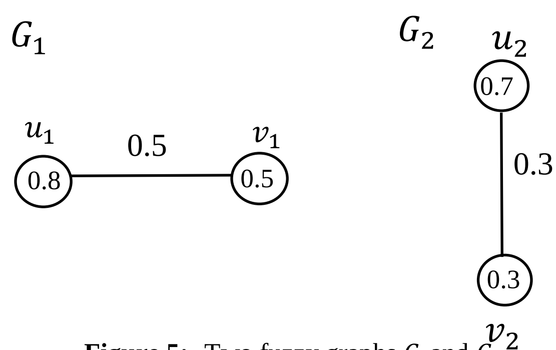

Figure 5:-. Two fuzzy graphs $G_1$and G_2

it is obvious that G_1 and G_2 are strong fuzzy graphs.
Based ondefinition 3.2,the graph $G_1 \vee G_2$ is illustrated in figure 6 as follows:

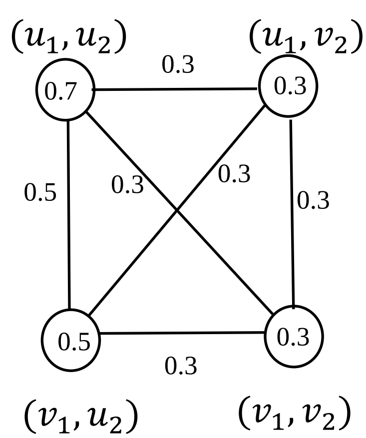

Figure 6:-Thedisjunction of two fuzzy graphs G_1 and G_2.

It is easy to see that the disjunction of G_1 and G_2 is a strong fuzzy graph.

Theorem 4.4. If $G = G_1 \vee G_2$ is M-strong fuzzy graph, then at least G_1or G_2 must be M-strong fuzzy graph**.**

Proof. Suppose that both G_1 and G_2 are not M-strong fuzzy graphs, then there exist $x_1y_1 \in E(G_1)$and $x_2y_2 \in E(G_2)$such that
$\square_1(x_1y_1) < \square\square\square\{\mu_1(x_1), \mu_1(y_1)\}and\square_2(x_2y_2) < \square\square\square\{\mu_2(x_2), \mu_2(y_2)\}$
According to the definition3.2,
$(\rho_1 \vee \rho_2)\big((x_1, x_2)(y_1, y_2)\big) = \square\square\square\{\square_1(x_1y_1), \mu_2(x_2), \mu_2(y_2)|\ x_1y_1 \in E(G_1),\ x_2, y_2 \in V(G_2)\}$
and
$(\mu_1 \vee \mu_2)(x_1, x_2) = \square\square\square\{\mu_1(x_1), \mu_2(x_2)\}and(\mu_1 \vee \mu_2)(y_1, x_2) = \square\square\square\{\mu_1(y_1), \mu_2(x_2)\}$
Since
$(\rho_1 \vee \rho_2)\big((x_1, x_2)(y_1, y_2)\big) =$

$$\square\square\square\{\square_1(x_1y_1), \mu_2(x_2), \mu_2(y_2)\} < \square\square\square\{\mu_1(x_1), \mu_1(y_1), \mu_2(x_2), \mu_2(y_2)\}$$

and

$$min\{(\mu_1 \vee \mu_2)(x_1, x_2), (\mu_1 \vee \mu_2)(y_1, y_2)\} = \square\square\square\{\mu_1(x_1), \mu_1(y_1), \mu_2(x_2), \mu_2(y_2)\}$$

therefore

$$(\rho_1 \vee \rho_2)\big((x_1, x_2)(y_1, y_2)\big) < \square\square\square\{(\mu_1 \barwedge \mu_2)(x_1, x_2), (\mu_1 \barwedge \mu_2)(y_1, x_2)\}$$

Which is in contradiction to$G = G_1 \vee G_2$ as M-strong fuzzy graph. Hence if $G = G_1 \vee G_2$ is M-strong fuzzy graph, then at least G_1 or G_2must be M-strong fuzzy graph**.**

Theorem 4.5. If $G = G_1 \oplus G_2$ be M-strong fuzzy graph, then at least G_1 or G_2must be M-strong fuzzy graph**.**

Proof. It is straightforward.

Theorem 4.6. Let G be a symmetric difference of two M-strong fuzzy graphs $G_1$and G_2, then G is a M-strong fuzzy graph***.***

Proof. It is straightforward.
In next example, we will illustrate that the opposite is not necessarily true.

Example 4.4. Two fuzzy graphs G_1 and $G_2$are illustrated in figure 7 and symmetric difference of G_1 and G_2 is presented in figure8:

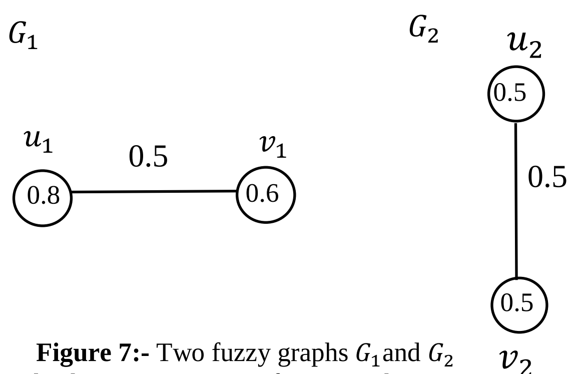

Figure 7:- Two fuzzy graphs $G_1$and G_2

$G_1 \oplus G_2$ and G_2 are strong fuzzy graphs, but $G_1$is not strongfuzzy graph.

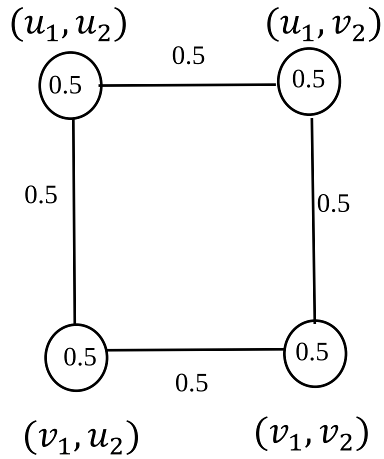

Figure 8:- The symmetric difference of two graphs G_1 and G_2

Conclusion:-

Inpresent paper, specific operations on fuzzy graphs have been introduced and some theorems are discussed. Some properties of M-strongfuzzy graphs are investigated.

Fuzzy graph theory is highly utilized in various areas. In future work, we can focus on Intuitionistic, bipolar and hyperfuzzy graphs and attempt to investigate many properties on them.

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