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RESEARCH ARTICLE

COMPARATIVE STUDY OF A NON-DIMENSIONAL NUMBER USED IN SEEPAGE FLOW

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Abstract

For flows where viscous resistance is dominant, Reynolds number is used to classify the flow into laminar, transitional or turbulent regime based on the prescribed limits, to apply corresponding equations to compute point velocity, resistance, discharge etc., Reynolds number, ratio of inertia to viscous forces, is a function of both solid and fluid media properties such as, characteristic length, characteristic velocity, viscosity and density. For pipe flow, it is easy to compute Reynolds number as all the parameters are well defined. Though, porous media flow is simulated to flow through bundle of pipes, defining and computing Reynolds number is not a simple task, owing to multiple definitions for the characteristic length and characteristic velocity. This in turn results in numerous definitions for Reynolds number, as proposed by Kozney-Carman, Collins, Ward, Kovacs , Thirriot, Zampaglione and Kovacs-Valentine. This paper presents the results of a comparative study on these definitions of Reynolds number. As a reference, Reynolds number defined based on bulk velocity of flow and volumetric diameter (as size) of the particle is used. Based on statistical analysis involving Standard Deviation and Efficiency Coefficient (EC), the definition of Reynolds number with EC value very near to 1 is proposed. A specially designed permeameter with water as fluid medium and seven sizes of coarse gravel as solid media is used in the experimentation. The results are expected to reduce the confusion pertaining to the definition of Reynolds number.

INTRODUCTION

A glance at the earlier works on flow through porous media generally leads to an impression that it is very similar to the flow through bundle of pipes, which in turn results in a simple picture of flow through porous media. A deeper and a curious look into the flow phenomenon, presents the state of indefiniteness even in defining parameters such as shape of the pore , characteristic length and characteristic velocity etc., Flow is possible under the action of driving and resisting forces, the relative influence of which is expressed in terms of a set of dimensionless numbers, such as Reynolds number, Froude number, Mach number etc,. Depending on the nature of resisting force, the suitable number is selected to analyze the flow behavior. In the case of flow through porous media, due to low velocity (or less inertial effects), Reynolds number is used to analyze the flow behavior.(2,3,4,5,9,13,15)

Mathematically, Reynolds number is defined as the ratio of inertia force to viscous force, which is expressed as

$$\text{Reynolds number} = Re = \frac{V_c\ L_c}{\nu} \quad (1)$$

where V_c = Characteristic velocity, L_c = Characteristic linear dimension and ν = Kinematic Viscosity of the fluid.

In the case of flow through a pipe, average velocity of flow is taken as V_c and pipe diameter is taken as L_c both of them being well defined and can be determined to reasonable accuracy. In the case of porous media flow, characteristic velocity is the velocity of flow trough a pore and characteristic linear dimension is the size of the pore[1,6,7,8]. As these two parameters suffer from lack of clear procedures to determine them, Reynolds number could not be defined and determined exactly. Present work aims at comparing the different definitions of Reynolds number[10] proposed by Kozney-Carman , Collins, Ward, Kovacs, Thirriot , Zampaglione and Kovacs-Valentine (Table 1) with a reference Reynolds number with bulk velocity as characteristic velocity and size of the particle as characteristic linear dimension.

The terms in Table-1 are V_p = Pore velocity (considering only pore area) V_b = Bulk velocity (considering gross area of flow i.e., cross sectional area of the permeameter) , k_i = Intrinsic permeability, n = Porosity , α = Shape factor and υ = Kinematic viscosity of the fluid.

In order to achieve the objective, it is necessary to quantitatively determine all the Reynolds numbers listed in Table 1, in addition to reference Re. An experimental programme is planned and carried out, the details of which are presented in the succeeding section.

Experimentation and media

Experimental set up, a specially designed permeameter of a circular G.I. Column of 6.20 m high and 0.15 m inner diameter facilitates collection of wide range of data. It is provided with three sets of piezometric tapping points for measuring head loss over three different lengths of travel (1.0 m, 3.0 m, 5.03 m) (Fig.1). This is in contrast with the short permeameters used in earlier studies,[1,2,7,14] resulting in possible errors of measurement. Water is supplied to the porous medium packed in the permeameter from an over head tank through balancing tank. Water level is maintained constant in the balancing tank during each run. Perforated horizontal pipe ensures that water will not fall in the form of thick jet at the inlet. A 3.5 mm thick aluminum screen with 85% perforations placed at the exit allowed the retention of media in the permeameter. Clearance of 0.5 m from the entry and exit points of the permeameter is given to top and bottom piezometers. Proper care is exercised to file away all the burrs and projections at the taping points. The short copper tubings facilitate connection of polythene tubes to manometer board to measure manometric heads. Two valves of 75 mm diameter are fitted at the inlet and exit to regulate the flow through the permeameter. Further, a bye pass valve at the exit ensures fine regulation of flow and steady conditions. Discharge was determined by volumetric method. Before taking the piezometric readings, it is ensured that all the entrapped air is removed. As the water level in the piezometer fluctuates, three pairs of maximum and minimum readings are taken and average of which is taken for computing piezometric head. Further, three sets of piezometers, using which three values of hydraulic gradient are computed, reduce the error in calculation of hydraulic gradient due to non-uniformity in packing of the porous medium.

Seven sizes of coarse gravel as porous media and water as fluid medium are used in the study. Volume diameter, which is the diameter of a hypothetical sphere having same volume as that of the coarse particle, is taken as size of the solid medium. Water displacement method is used to determine porosity. The details of the media used are presented in Table 2.Velocity of flow is computed from the known size of the permeameter and discharge. Average value of the three hydraulic gradients is used in the analysis of the experimental data. Temperatures were recorded for every run, from which viscosity of out flowing water is determined. A value of 8.8 is taken as the shape factor for coarse gravel used in the present study. (This is from the experiments carried out by the first author[11,12] on the determination of relation between specific surface (i.e., surface area of the particle per its volume) and size of the coarse particles)

Analysis of Experimental Data:

Now, with the known values of volume diameter of the particle, porosity, discharge, velocity of flow, hydraulic gradient and shape factor, further analysis is carried out by computing values of Reynolds numbers as listed in Table 1.

Reynolds number based on volume diameter and bulk velocity of flow is calculated using Eq.(2) as

$$Re_b = \frac{V_b\ d}{\nu} \quad (2)$$

where d = volume diameter of the particle.

Kozney-Carman proposed the following expression for Reynolds number,

$$Re_{KC} = \sqrt{\frac{5k_i}{n^3}}\frac{V_b}{\nu}\frac{(1-n)}{n}\alpha \tag{3}$$

with characteristic length as $\sqrt{5\frac{k_I}{n^3}}(1-n)\alpha$ and characteristic velocity as $V_P = \frac{V_b}{n}$. It may be noted that this definition involves the usage of shape factor and intrinsic permeability in addition to porosity and bulk velocity of flow. While the later two terms are already known, shape factor being taken as 8.8, the only parameter whose value is required is k_i (intrinsic permeability). It is found as follows: From the experimental data of V_b and i (for a given size), a plot is drawn between these two values with V_b as ordinate and i as abscissa. The slope of the fit up to linear trend of the data distribution, is taken as coefficient of permeability (k) (from Darcy's law , V_b = k i)[6,8]. But, k = k_i (γ/μ) where γ = specific weight of the fluid (water) and μ = viscosity of the fluid (water). Thus from the values of k, γ and μ, the value of k_i is computed and is used in calculating Re_{KC} and other Re values where k_i is used. This is done for all the discharges through the seven sizes of coarse media and then values of Re_b and Re_{KC} are computed.

Figure 2 shows the variation of Re_{KC} with Re_b with Re_{KC} plotted along y-axis and Re_b on x- axis. While the range of values of Re_{KC} varies from 0.00252 to 393.1831, that of Re_b is from 0.005686 to 1662.787. The highest value of Re_{KC} is 393.1831 and that of Re_b is 1662.787. This may be due to difference in definitions of Reynolds numbers, in that respect that, while Re_b is based on only volume diameter of the particle and bulk velocity, the other one considers the effect of porosity and shape also.

Reynolds number proposed Collins is

$$Re_c = \frac{V_b}{\nu}\sqrt{\frac{k_i}{n}} \tag{4}$$

with characteristic length as $\sqrt{(k_i/n)}$ and characteristic velocity as V_b. Using the value of V_b, ν, k_i and n , the values of Re_C and Re_b are computed. A plot between these two parameters is obtained as shown in Fig. 3. In this case, range of values of Re_C is from 0.000118 to 14.46933, that of Re_b is from 0.005686 to 1662.787. The highest value of Re_C is 14.46933 and that of Re_b is 1662.787.

As per Ward's version, the value of Reynolds number is computed using,

$$Re_w = \frac{V_b}{\nu}\sqrt{k_i} \tag{5}$$

with $\sqrt{k_i}$ and V_b as characteristic length and characteristic velocity respectively. As is done earlier, values of Re_W and Re_b are computed for all the discharges through all the seven sizes of the porous media and the variation is depicted as shown in Fig.4. While the range of Re_W is from 0.0000819 to 9.377199, that of Re_b is from 0.005686 to 1662.787. Though the lower values of Re_W appear to be unconvincing, these may be due to the format of the definition proposed by Ward.

Kovacs suggested the following expression for Reynolds number,

$$Re_k = \sqrt{80}\frac{V_b}{\nu}\sqrt{\frac{k_i}{n^3}} \tag{6}$$

with characteristic length as $\sqrt{80\frac{k_i}{n}}$ and characteristic velocity as $V_P = \frac{V_b}{n}$. It may be noted that this definition involves the usage of intrinsic permeability in addition to porosity and bulk velocity of flow. With the known values of these parameters, values of Re_K are computed for all the discharges. Variation of Re_K with Re_b with Re_K along ordinate axis and Re_b along abscissa is shown in Fig.5. While the range of values of Re_K varies from 0.002202 to 308.137 that of Re_b is from 0.005686 to 1662.787. The highest value of Re_K is 308.137 and that of Re_b is 1662.787. The reason may be due to difference in definitions of both the Reynolds number.

Thirriot's version of Reynolds number is,

$$Re_T = \sqrt{32}\frac{V_b}{\nu}\sqrt{\frac{k_i}{n^3}} \tag{7}$$

with $\sqrt{32\frac{k_i}{n}}$ and V_b/n as characteristic length and characteristic velocity respectively. With the known values of k_i, V_b, n and ν, values of Re_T are computed for all the discharges along with corresponding Re_b values and the relation between them is depicted as shown in Fig.6. While the range of Re_T is from 0.001393 to 194.883, that of Re_b is from 0.005686 to 1662.787.

As per Zampiglione, the value of Reynolds number is computed using,

$$Re_Z = \frac{V_b}{\nu}\sqrt{\frac{k_i}{n^2}} \tag{8}$$

with $\sqrt{\frac{k_i}{n^{\frac{2}{3}}}}$ and $\frac{V_b}{n^{\frac{2}{3}}}$ as characteristic length and characteristic velocity respectively. As is done earlier, values of Re_Z and Re_b are computed for all the discharges through all the seven sizes of the porous media and the variation of Re_z with Re_b is depicted as shown in Fig.7. While the range of Re_z is from 0.000171 to 22.3266, that of Re_b is from 0.005686 to 1662.787.

Using the definition of Reynolds number proposed by Kovacs- Valentine as

$$Re_{KV} = \frac{5V_b}{\nu}\sqrt{nk_i} \tag{9}$$

with as $\sqrt{nk_i}$ characteristic length and V_b characteristic velocity respectively, values of Re_{KV} and Re_b are computed. Figure 8 presents the relationship between these two types of Reynolds numbers. In this case it may be observed that the Re_{KV} lies between 0.000284 and 30.3856.

From the above presentation it may be summarized that different definitions of Re have different ranges of values between lower and higher limits for the same experimental data and under same conditions of experimentation for the same media. In order to compare all of them with $Re_{b,}$ it is necessary to bring them to a common higher limit through normalization. Therefore, all the experimental data are normalized by recalculating them to have a common higher value of 1662.787 (higher limit of Re_b). All the data are subjected statistical analysis, such as computing Standard Deviation (SD) and Efficiency Coefficient (EC).

$$\text{Efficiency coefficient =E.C. =} \quad 1 - \frac{\sum_{i=1}^{N}(Re_b^i - Re^i)^2}{\sum_{i=1}^{N}(Re_b^i - \overline{Re}_b)^2} \tag{10}$$

where $Re_b^i = (V_b^i\, d)/\nu^i$ (11)

Re_b^i = i th value of Re_b

$\overline{Re_b}$ = mean of Re_b values

V_b^i = i th value of bulk velocity

Re^i = i th value of Reynolds number, as defined by different proposers

ν^i = i th value of viscosity

Efficiency coefficient varies from -∞ to 1. EC=1 corresponds to a perfect match of Re^i to the bulk Reynolds number Re_b^i . An efficiency of 0 (EC=0) indicates that the values of Re^i are as accurate as Re_b . Further, EC<0 indicates that Re_b is better than Re^i. Therefore, the closer the value of E.C to 1 is the better definition for Re. Efficiency coefficient (E.C) values are calculated for the above seven definitions and values are given in Table 3.

From these values it may be observed that the expression for Reynolds number proposed by Kozney-Carman has higher EC value and least SD value. This infers that this expression for Reynolds number compared to those proposed by others has a reasonably good agreement with Re_b values, which is considered as reference Reynolds number. Therefore, the expression for Reynolds number proposed by Kozney- Carman is a better representative Reynolds number for use in porous media flow.

Results and Conclusions: Though porous media flow is simulated to flow through bundle of pipes, defining and computing Reynolds number is not a simple task. This is due to multiple definitions for the characteristic length and characteristic velocity. This led to different definitions for Reynolds number. Seven different definitions for Reynolds number proposed by Kozney-Carman, Collins, Ward, Kovacs, Thirriot, Zampiglione, Kovacs-Valintine were considered and were compared with a reference Reynolds number. These seven equations use characteristic length in terms of intrinsic permeability; and characteristic velocity as a function of bulk velocity and porosity. Reference Reynolds number is expressed in terms of volume diameter of the media and bulk velocity. Experiments were conducted using seven sizes of coarse granular media with water as fluid medium. Specially designed permeameter was used to simulate a porous medium. With the known values of discharge, velocity of flow, hydraulic gradient, intrinsic permeability, volume diameter and porosity from experimentation, all the Reynolds numbers were computed along with those of reference Reynolds number. Relationship between individual Res with reference Re was analyzed graphically. On subjecting the trend and relationship between these individual Reynolds numbers and reference Re to statistical analysis, it is concluded that the expression proposed by Kozney-Carman is a better representative Reynolds number for use in porous media flow.

Table : 1 Definitions for Reynolds Number

S.No	Proposed by	Characteristic velocity	Characteristic length	Reynolds number
1.	Kozney - Carman	$\frac{V_b}{n}$	$\sqrt{5\frac{k_I}{n^3}}(1-n)\alpha$	$Re_{KC} = \sqrt{\frac{5k_i}{n^3}}\frac{V_b}{\nu}\frac{(1-n)}{n}\alpha$
2.	Collins	V_b	$\sqrt{\frac{k_i}{n}}$	$Re_c = \frac{V_b}{\nu}\sqrt{\frac{k_i}{n}}$
3.	Ward	V_b	$\sqrt{k_i}$	$Re_w = \frac{V_b}{\nu}\sqrt{k_i}$
4.	Kovacs	$\frac{V_b}{n}$	$\sqrt{80\frac{k_i}{n}}$	$Re_k = \sqrt{80}\frac{V_b}{\nu}\sqrt{\frac{k_i}{n^3}}$
5.	Thirriot	$\frac{V_b}{n}$	$\sqrt{32\frac{k_i}{n}}$	$Re_T = \sqrt{32}\frac{V_b}{\nu}\sqrt{\frac{k_i}{n^3}}$
6.	Zampaglione	$\frac{V_b}{n^{\frac{2}{3}}}$	$\sqrt{\frac{k_i}{n^{\frac{2}{3}}}}$	$Re_Z = \frac{V_b}{\nu}\sqrt{\frac{k_i}{n^2}}$
7.	Kovacs Valentine	V_b	$\sqrt{nk_i}$	$Re_{KV} = \frac{5V_b}{\nu}\sqrt{nk_i}$

Table 2: Characteristics of Media Used in the Study

Sl. No.	Media	Volume Diameter (mm)	Porosity (%)
1	Coarse gravel	1.42	48.0
2	Coarse gravel	3.85	49.2
3	Coarse gravel	5.74	49.0
4	Coarse gravel	7.63	51.0
5	Coarse gravel	8.74	46.0
6	Coarse gravel	14.38	44.0
7	Coarse gravel	17.77	42.0

Table 3: Statistical Parameters of the study

S.No	Statistical parameter	Kozney Carman	Collins	Ward	Kovacs	Thirriot	Zampaglione	Kovacs Valentine
1	Std.Dev (σ)	368.137	379.807	384.550	372.378	372.378	375.792	390.164
2	E.C.)	0.84832	0.75015	0.70449	0.81556	0.81556	0.78664	0.64778

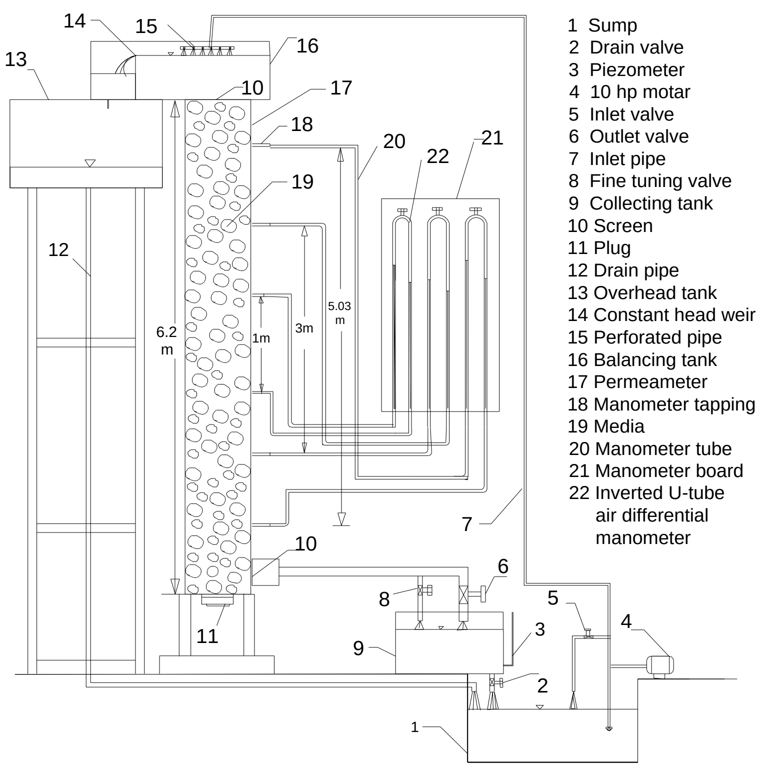

Fig .1 Experimental setup

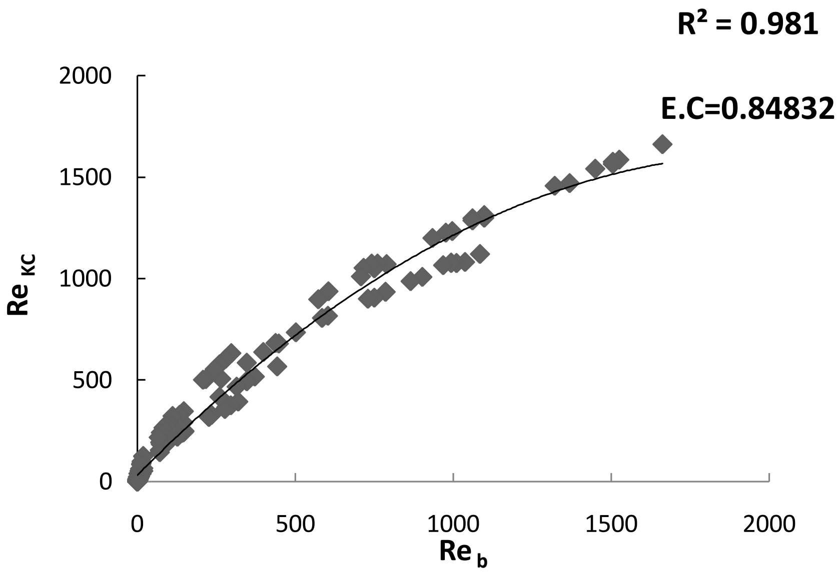

Fig . 2. Variation of Re_{KC} with Re_b

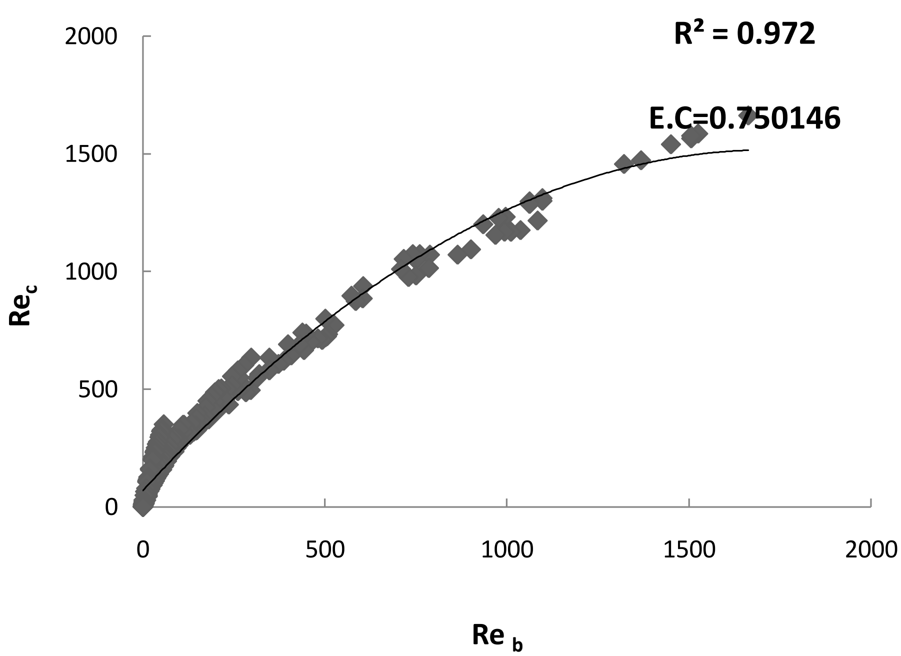

Fig . 3. Variation of Re_c with Re_b

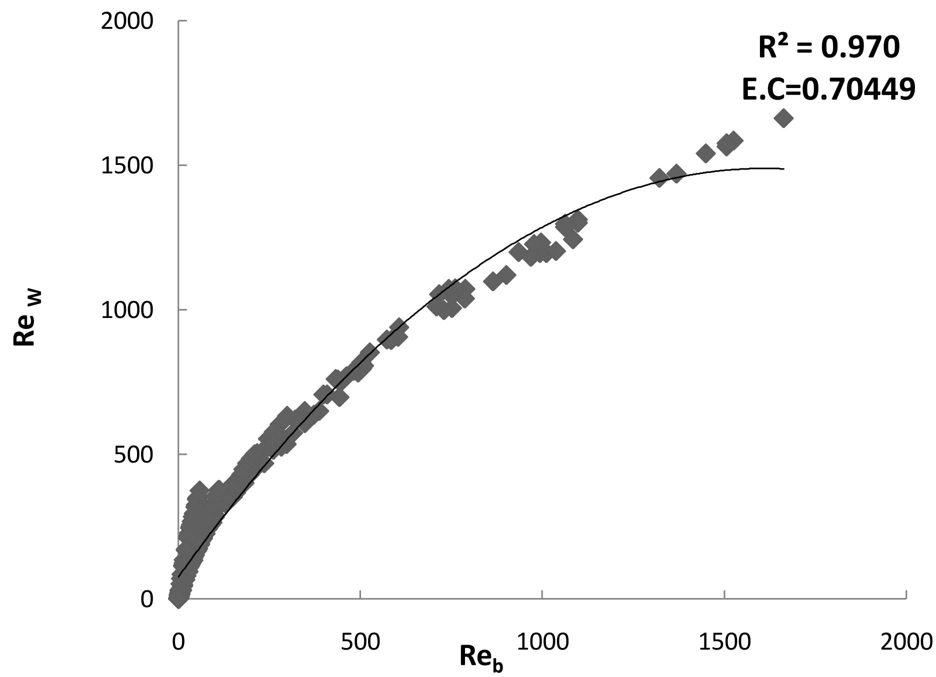

Fig .4. Variation of Re_w with Re_b

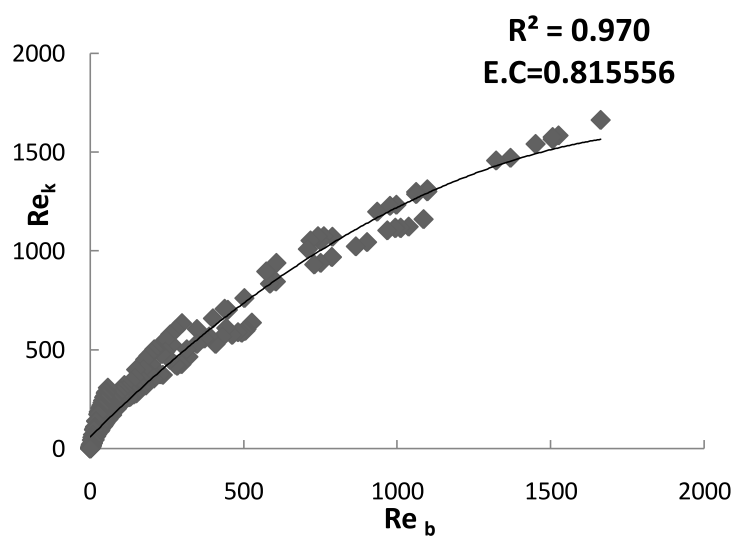

Fig . 5. Variation of Re_k with Re_b

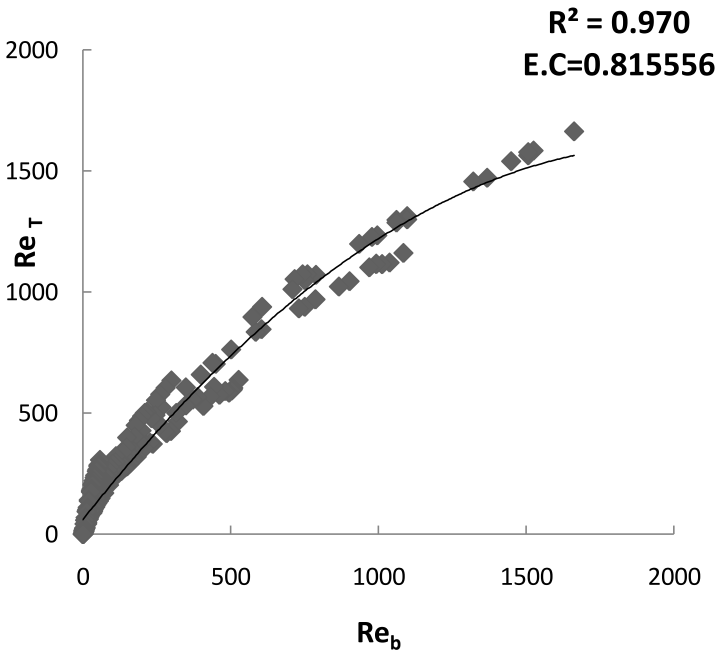

Fig . 6. Variation of Re_T with Re_b

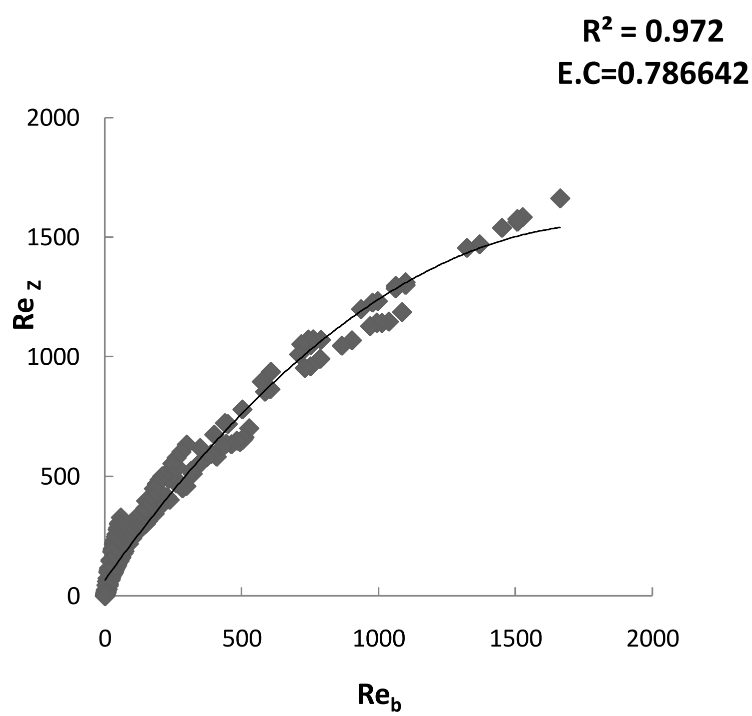

Fig . 7. Variation of Re_Z with Re_b

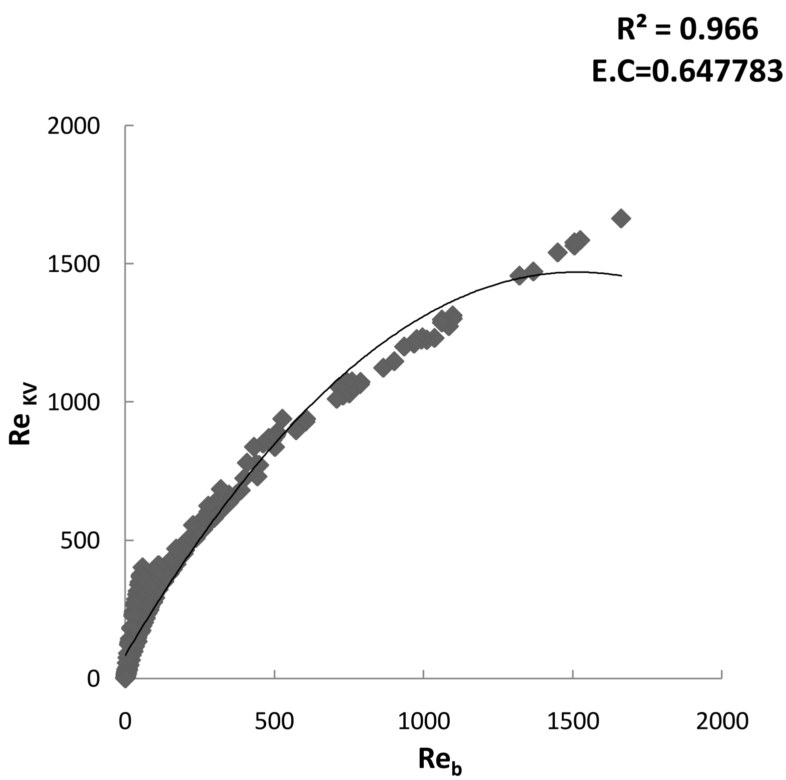

Fig .8. Variation of Re_{KV} with Re_b

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Notation

d = Volume diameter of the particle
K_i = Intrinsic permeability
Lc = Characteristic linear dimension
n = Porosity
Re = Reynolds number
Re_b = Definition for Reynolds number with bulk velocity and volume diameter of particle as characteristic length
Re_w = Definition for Reynolds number after Ward
Re_c = Definition for Reynolds number after Collins
Re_T = Definition for Reynolds number after Thirriot
Re_K = Definition for Reynolds number after Kovacs
Re_Z = Definition for Reynolds number after Zampaglione
Re_{KV} = Definition for Reynolds number after Kovacs Valentine
Re_{KC} = Definition for Reynolds number after Kozney-Carman
V_c = Characteristic velocity

V_b = Bulk velocity of flow
V_p = Pore velocity
ν = Kinematic viscosity of the fluid
α = Shape factor of the particle
μ = Mean
σ = Standard deviation

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