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RESEARCH ARTICLE

$\psi^*\alpha$-continuous maps.

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Abstract

In this paper we introduce a new class of maps called $\psi^*\alpha$-continuous maps by using $\psi^*\alpha$-closed sets. We investigate its implications and independent relationship with other types of continuous maps. Also we analyze the association of $\psi^*\alpha$-continuous maps with various kinds of continuous maps via separation axioms.

Introduction:-

Levine [9] introduced the idea of continuous functions in 1970.Mashhour [11] introduced and studied α-continuous functions in topological spaces. The generalized continuous (briefly g-continuous) functions was introduced and studied by Balachandran et.al [1].Veerakumar [18] introduced and studied ψ-continuous functions in topological spaces. Ramya and Parvathi [15] introduced ψg-continuous functions. The notion of $\psi^*\alpha$-closed sets was defined and investigated by Balamani and Parvathi [2]. The purpose of this paper is to introduce and study the concept of a new class of maps called $\psi^*\alpha$-continuous maps in topological spaces.

Preliminaries:-

Throughout this paper (X, τ), (Y, σ) and (Z,η) represent non-empty topological space on which no separation axioms are assumed, unless otherwise mentioned. The interior and closure of a subset A of a space (X, τ) are denoted by int(A) and cl(A) respectively.

Definition 2.1 A subset A of a topological space (X, τ) is called
1) generalized closed set (briefly g-closed) [9] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
2) semi-generalized closed set (briefly sg-closed)[4] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi- open in (X, τ).
3) ψ-closed set [18] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ).
4) ψg-closed set [14] if $\psi cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ).
5) $\psi^*\alpha$ -closed set [2] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ψg -open in (X, τ).
6) The closure operator of $\psi^*\alpha$-closed set is defined as $\psi^*\alpha cl(A) = \cap\{F \subseteq X: A \subseteq F$ and F is $\psi^*\alpha$ -closed in $(X, \tau)\}$[2]

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(i) ${}_{\psi^*\alpha}T_c$-space if every $\psi^*\alpha$ closed subset of (X, τ) is closed in (X, τ).[3]
(ii) ${}_{\psi^*\alpha}T_\alpha$-space if every $\psi^*\alpha$ closed subset of (X, τ) is α- closed in (X, τ).[3]
(iii) ${}_{g\alpha}T_{\psi^*\alpha}$ - space if every gα- closed subset of (X, τ) is $\psi^*\alpha$-closed in (X, τ).[3]
(iv) ${}_{\alpha g}T_{\psi^*\alpha}$ -space if every αg- closed subset of (X, τ) is $\psi^*\alpha$-closed in (X, τ).[3]
(v) ${}_{\psi g}T_{\psi^*\alpha}$-space if every ψg- closed subset of (X, τ) is $\psi^*\alpha$-closed in (X, τ).[3]

Definition 2.3 A map f : (X, τ) → (Y, σ) is called
(i) Continuous [9] if f^{-1}(V) is closed in (X, τ) for each closed set V of (Y, σ).
(ii) Semi continuous [8] if f^{-1}(V) is semi closed in (X, τ) for each closed set V of (Y, σ).
(iii) α-continuous [11] if f^{-1}(V) is α-closed in (X, τ) for every closed set V of (Y, σ).
(iv) g-continuous [1] if f^{-1}(V) is g-closed in (X, τ) for every closed set V of (Y,σ).
(v) gα-continuous [5] if f^{-1}(V) is gα-closed in (X, τ) for every closed set V of (Y, σ).
(vi) αg-continuous [5]if f^{-1}(V) is αg-closed in (X, τ) for every closed set V of (Y, σ).
(vii) g^*-continuous [19] if f^{-1}(V) is g^*-closed in (X, τ) for every closed set V of (Y, σ).
(viii) $\hat{g}$- continuous [20] if f^{-1}(V) is $\hat{g}$- closed in (X, τ) for each closed set V of (Y, σ).
(ix) $g^{\#}$-continuous [21] if f^{-1}(V) is $g^{\#}$-closed in (X, τ) for every closed set V of (Y, σ).
(x) ${}^*g$-continuous [22] if f^{-1}(V) is ${}^*g$-closed in (X, τ) for every closed set V of (Y, σ).
(xi) $\tilde{g}$- continuous[13] if f^{-1}(V) is a $\tilde{g}$-closed in (X, τ) for every closed set V of (Y, σ).
(xii) $\alpha\hat{g}$-continuous[16] if f^{-1}(V) is $\alpha\hat{g}$-closed in (X, τ) for every closed set V of (Y, σ).
(xiii) $\tilde{g}_\alpha$- continuous[17] if f^{-1}(V) is $\tilde{g}_\alpha$-closed in (X, τ) for every closed set V of(Y, σ).
(xiv) $g^{\#}p^{\#}$-continuous [12] if f^{-1}(V) is $g^{\#}p^{\#}$-closed in (X, τ) for every closed set V of (Y, σ).
(xv) ψ- continuous[18] if f^{-1}(V) is ψ-closed in (X, τ) for every closed set V of (Y, σ).
(xvi) ψg- continuous [15] if f^{-1}(V) is ψg-closed in (X, τ) for every closed set V of (Y, σ).
(xvii) $\psi\hat{g}$- continuous [15] if f^{-1}(V) is $\psi\hat{g}$- closed in (X, τ) for every closed set V of (Y, σ).

Definition 2.4 A map f : (X, τ) → (Y, σ) is called
(i)strongly continuous [7] if f^{-1}(V) is both open and closed in (X, τ)for every subset V of (Y, σ).
(ii)totally continuous [6] if f^{-1}(V) is a clopen subset of (X, τ) for every open set V of (Y, σ).
(iii)α- irresolute [10] if f^{-1}(V) is α-closed in (X, τ) for every α-closed set V of (Y, σ).

$\psi^*\alpha$ - continuous maps

Definition 3.1 A map f : (X, τ) → (Y, σ) is called **ψ *star alpha continuous*** (briefly, **$\psi^*\alpha$ - continuous**) if f^{-1}(V) is $\psi^*\alpha$ - closed in (X, τ) for each closed set V in (Y, σ).
Example 3.2 Let X = Y = {a, b, c} with the topologies τ = {ϕ, {a},{b},{a, b}, X} and σ= {ϕ, {a, b}, Y}. Let f : (X, τ) → (Y, σ) be a map defined by f(a) = b, f(b) = a, f(c) = c. Then f is $\psi^*\alpha$ -continuous.
Proposition 3.3 Every continuous (resp.α-continuous) map is $\psi^*\alpha$-continuous but not conversely.
Proof: Let V be a closed set in (Y, σ).Since f : (X, τ) → (Y, σ) is continuous (resp.α-continuous) map, f^{-1}(V) is closed (resp.α-closed) in (X, τ). Since every closed (resp.α-closed) set is $\psi^*\alpha$ - closed, f^{-1}(V) is $\psi^*\alpha$ - closed in (X, τ). Hence f is $\psi^*\alpha$-continuous.
Example 3.4 Let X = Y = {a, b, c} with the topologies τ = {ϕ, {a, b}, X} and σ= {ϕ, {a}, Y}. Let f : (X, τ) → (Y, σ) be a map defined by f(a) = b, f(b) = a, f(c) = c. Then f is $\psi^*\alpha$ - continuous but not continuous and not α - continuous, since {b, c} is closed in (Y, σ) but f^{-1}({b, c}) = {a, c} is not closed and not α-closed in (X, τ)
Proposition 3.5 Every $\psi^*\alpha$ -continuous map is gα - continuous, αg-continuous, $\alpha\hat{g}$-continuous, $\tilde{g}_\alpha$ -continuous, ψ-continuous, ψg - continuous and $\psi\hat{g}$-continuous but not conversely.
Proof: As every $\psi^*\alpha$-closed set is gα - closed, αg-closed, $\alpha\hat{g}$- closed, $\tilde{g}_\alpha$ -closed, ψ- closed, ψg- closed and $\psi\hat{g}$-closed [2], the result follows.
Example 3.6 Let X = Y = {a, b, c, d} with the topologies τ= {ϕ, {d},{a, b},{a, b, d},X} and σ= {ϕ, {a}, Y}. Let f : (X, τ) → (Y, σ) be a map defined by f(a) = b, f(b) = a, f(c) = c, f(d)=d. Then f is gα-continuous, αg-continuous, $\alpha\hat{g}$-continuous,$\tilde{g}_\alpha$-continuous, ψg- continuous and $\psi\hat{g}$- continuous but not $\psi^*\alpha$-continuous, since for the closed set {b, c, d} in (Y, σ), f^{-1}({b, c, d}) = {a, c, d} is gα - closed, αg-closed, $\alpha\hat{g}$- closed, $\tilde{g}_\alpha$ -closed, ψg- closed and $\psi\hat{g}$-closed but not $\psi^*\alpha$ - closed in (X, τ).

Example 3.7 Let X = Y = {a, b, c} with the topologies τ= {ϕ,{a},{b},{a, b},X} and σ= {ϕ, {a, b}, Y}. Let f : (X, τ) $\rightarrow$ (Y, σ) be a map defined by f(a) = c, f(b) = b, f(c) = a. Then f is ψ- continuous but not $\psi^*\alpha$-continuous, since for the closed set {c} in (Y, σ), f^{-1}({c}) = {a} is ψ- closed but not $\psi^*\alpha$ - closed in (X, τ).

Theorem 3.8 Every strongly continuous (resp. totally continuous) map f : (X, τ) $\rightarrow$ (Y, σ) is $\psi^*\alpha$-continuous but not conversely.

Proof: Let V be a closed set in (Y, σ). Since f : (X, τ) $\rightarrow$ (Y, σ) is strongly continuous (resp. totally continuous), f^{-1}(V) is clopen in (X, τ). Since every closed set is $\psi^*\alpha$ - closed, f^{-1}(V) is $\psi^*\alpha$ - closed in (X, τ).

Example 3.9 Let X = Y = {a, b, c} with the topologies τ = {ϕ, {a}, {b}, {a, b}, {a, c},X} and σ = {ϕ,{a}, {a, b}, {a, c},Y}. Let f : (X, τ) $\rightarrow$ (Y, σ) be a map defined by f(a) = a, f(b) = c, f(c) = b. Then f is $\psi^*\alpha$ - continuous but not strongly continuous and not totally continuous, since {b} is closed in (Y, σ) but f^{-1}({b}) = {c} is not open in (X, τ).

Remark 3.10 The following examples show that semi-continuous maps and $\psi^*\alpha$-continuous maps are independent.

Example 3.11 Let X = Y = {a, b, c} with the topologies τ={ϕ,{a},{b},{a, b},X} and σ= {ϕ, {a, b}, Y}. Let f :(X, τ) $\rightarrow$ (Y, σ) be a map defined by f(a) = c, f(b) = a, f(c) = b. Then f is semi-continuous but not $\psi^*\alpha$ - continuous, since for the closed set {c} in (Y, σ), f^{-1}({ c}) = {a} is semi- closed but not $\psi^*\alpha$ - closed in (X, τ).

Example 3.12 Let X = Y = {a, b, c} with the topologies τ ={ϕ, {a, b}, X} and σ ={ϕ, {a}, Y}.Let f : (X, τ) $\rightarrow$ (Y, σ) be the identity map. Then f is $\psi^*\alpha$ - continuous but not semi-continuous, since for the closed set {b, c} in (Y, σ) f^{-1}({b, c}) = {b, c} is $\psi^*\alpha$-closed but not semi-closed in (X, τ).

Remark 3.13 The following examples show that the notion of g-(resp. g^*, $\hat{g}$, $g^\#$, $^*g$, $\tilde{g}$, $g^\#p^\#$)continuity and $\psi^*\alpha$-continuity are independent.

Example 3.14 Let X = Y = {a, b, c, d} with the topologies τ={ϕ,{d},{a, b},{a, b, d},X} and σ= {ϕ, {a}, Y}. Let f : (X, τ) $\rightarrow$ (Y, σ) be a map defined by f(a) = b, f(b) = a, f(c) = c, f(d)=d. Then f is g-(resp. g^*, $\hat{g}$, $g^\#$,$^*g$, $\tilde{g}$, $g^\#p^\#$)continuous but not $\psi^*\alpha$ - continuous, since for the closed set {b, c, d} in (Y, σ), f^{-1}({b, c, d})={a, c, d} is g-(resp. g^*, $\hat{g}$, $g^\#$,$^*g$, $\tilde{g}$, $g^\#$ $p^\#$)closed but not $\psi^*\alpha$ - closed in (X, τ).

Example 3.15 Let X = Y = {a, b, c} with the topologies τ = {ϕ, {a}, {a, b}, X} and σ = {ϕ, {a, b}, Y}. Let f : (X, τ) $\rightarrow$ (Y, σ) be a map defined by f(a) = b, f(b) = c, f(c) = a. Then f is $\psi^*\alpha$ - continuous but not g-(resp. not g^*,not $\hat{g}$, not $g^\#$, not $^*g$, not $\tilde{g}$, not $g^\#$ $p^\#$-) continuous, since for the closed set {c} in (Y, σ) f^{-1}({c}) = {b} is $\psi^*\alpha$-closed but not g-(resp. not g^*,not $\hat{g}$, not $g^\#$, not $^*g$, not $\tilde{g}$, not $g^\#$ $p^\#$) closed in (X, τ).

Theorem 3.16 A map f : (X, τ) $\rightarrow$ (Y, σ) is $\psi^*\alpha$-continuous if and only if the inverse image of every open set in (Y, σ) is $\psi^*\alpha$-open in (X, τ)

Proof: (Necessity) Let U be an open set in (Y, σ). Then Y-U is closed in (Y, σ). Since f is $\psi^*\alpha$-continuous, f^{-1}(Y-U)= X- f^{-1}(U) is $\psi^*\alpha$-closed in (X, τ). Hence f^{-1} (U) is $\psi^*\alpha$ -open in (X, τ).

(Sufficiency) Assume that f^{-1}(V) is $\psi^*\alpha$-open in (X, τ)for each open set V in (Y, σ).Let V be any closed set in (Y,σ).Then Y-V is open in (Y, σ). By assumption, f^{-1}(Y-V)= X- f^{-1}(V) is $\psi^*\alpha$ -open in (X, τ) which implies that f^{-1}(V) is $\psi^*\alpha$ -closed in (X, τ). Hence f is $\psi^*\alpha$ -continuous.

Theorem 3.17 If f : (X, τ) $\rightarrow$ (Y, σ) is $\psi^*\alpha$ - continuous then f($\psi^*\alpha$cl(V)) $\subseteq$ cl(f(V)) for every subset V of (X, τ).

Proof: Let V be any subset of (X, τ).Then cl(f(V)) is closed in (Y, σ).Since f is $\psi^*\alpha$- continuous,f^{-1}(cl(f(V))) is $\psi^*\alpha$-closed in (X, τ).Since f(V) $\subseteq$ cl(f(V)), V $\subseteq$ f^{-1}(f(V))$\subseteq$$f^{-1}$(cl(f(V))) and hence f^{-1}(cl(f(V))) is a $\psi^*\alpha$ - closed set containing V. By definition of $\psi^*\alpha$-closure, we have $\psi^*\alpha$cl(V) $\subseteq$ f^{-1}(cl(f(V))) which implies that f($\psi^*\alpha$cl(V)) $\subseteq$ cl(f(V)).

Remark 3.18 Let f : (X, τ) $\rightarrow$ (Y, σ) be a continuous map. Then for every subset V of (X, τ), f($\psi^*\alpha$cl(V)) $\subseteq$ cl(f(V)).

Proof: Since every continuous map is $\psi^*\alpha$-continuous and by Theorem 3.17, the result follows.

Theorem 3.19 Let f : (X, τ) $\rightarrow$ (Y, σ) be a map from a topological space (X, τ) into a topological space (Y, σ). Then the following statements are equivalent:

(a) For each point x in (X, τ) and each open set V in (Y, σ) containing f(x), there exists a $\psi^*\alpha$ - open set U in (X, τ) containing x such that f(U) $\subseteq$ V.

(b) For every subset A of (X, τ), f($\psi^*\alpha$cl(A)) $\subseteq$ cl(f(A)).

(c) For every subset B of (Y, σ), $\psi^*\alpha$cl(f^{-1}(B)) $\subseteq$ f^{-1}(cl(B)).

Proof: (a) $\Leftrightarrow$ (b) Let y $\in$ f($\psi^*\alpha$cl(A)). Then y = f(x) for some x $\in$ $\psi^*\alpha$cl(A) $\subseteq$ X. Let V be any open set in (Y, σ) containing f(x).Then by hypothesis, there exists a $\psi^*\alpha$- open set U in (X, τ) containing x such that f(U) $\subseteq$ V. By Theorem 5.4 [2] we get U $\cap$ A $\neq$ ϕ.Then f(U $\cap$ A) $\neq$ ϕ. which implies that V $\cap$ f(A) $\neq$ ϕ. Hence y = f(x)$\in$cl(f(A)). Therefore f($\psi^*\alpha$cl(A)) $\subseteq$ cl(f(A)).

Conversely, let x $\in$ X and let V be any open set in (Y, σ) containing f(x). Let A = f^{-1}(V^c) then x$\notin$A. By (b), f($\psi^*\alpha$cl(A)) $\subseteq$ cl(f(A)) $\subseteq$ cl(f(f^{-1}(V^c))) $\subseteq$ cl(V^c) = V^c. Therefore f^{-1}(f($\psi^*\alpha$cl(A))) $\subseteq$ f^{-1}(V^c) which implies

$\psi^*\alpha cl(A) \subseteq f^{-1}(V^c) = A$. Hence $A = \psi^*\alpha cl(A)$. Since $x \notin A$, $x \notin \psi^*\alpha cl(A)$. Then there exists a $\psi^*\alpha$- open set U containing x such that $U \cap A = \phi$ and hence $f(U) \subseteq f(A^c) \subseteq V$.

(b) $\Leftrightarrow$ (c) Suppose that (b) holds and let B be any subset of Y. Replacing A by $f^{-1}(B)$ from (b), $f(\psi^*\alpha cl\ (f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Hence $\psi^*\alpha cl\ (f^{-1}(B)) \subseteq f^{-1}(cl(B))$.

Conversely, suppose that (c) holds and let $B = f(A)$ where A is a subset of X. Then $\psi^*\alpha cl\ (A) \subseteq \psi^*\alpha cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$. Therefore $f(\psi^*\alpha cl(A)) \subseteq cl(B) = cl(f(A))$.

Theorem 3.20 If $f : (X, \tau) \to (Y, \sigma)$ is $\psi^*\alpha$ - continuous and $g : (Y, \sigma) \to (Z,\eta)$ is continuous (resp. strongly continuous) then $g\circ f : (X, \tau) \to (Z,\eta)$ is $\psi^*\alpha$- continuous.

Proof: Let V be any closed set in (Z,η). Since g is continuous (resp. strongly continuous), $g^{-1}(V)$ is closed (resp. clopen) in (Y, σ). Since f is $\psi^*\alpha$- continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^*\alpha$ - closed in (X, τ). Therefore $g\circ f$ is $\psi^*\alpha$-continuous.

Theorem 3.21 If $f : (X, \tau) \to (Y, \sigma)$ is continuous (resp. α-continuous) and $g : (Y, \sigma) \to (Z,\eta)$ is continuous then $g\circ f : (X, \tau) \to (Z, \eta)$ is $\psi^*\alpha$ - continuous.

Proof: Let V be any closed set in (Z,η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). Since f is continuous (resp. α-continuous), $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed (resp. α-closed) in (X, τ). Since every closed (resp. α-closed) set is $\psi^*\alpha$ - closed, $(g\circ f)^{-1}(V)$ is $\psi^*\alpha$ - closed. Therefore $g\circ f$ is $\psi^*\alpha$ - continuous.

Theorem 3.22 If $f : (X, \tau) \to (Y, \sigma)$ is α-irresolute and $g : (Y, \sigma) \to (Z,\eta)$ is continuous then $g\circ f : (X, \tau) \to (Z, \eta)$ is $\psi^*\alpha$ - continuous.

Proof: Let V be any closed set in (Z,η). Since g is continuous, $g^{-1}(V)$ is closed in (Y,σ). Since every closed set is α - closed, $g^{-1}(V)$ is α -closed. Since f is α- irresolute, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is α - closed in (X, τ). Since every α - closed set is $\psi^*\alpha$- closed, $(g\circ f)^{-1}(V)$ is $\psi^*\alpha$-closed. Hence $g\circ f$ is $\psi^*\alpha$ - continuous.

Theorem 3.23 Let $f : (X, \tau) \to (Y, \sigma)$ be α - irresolute and $g : (Y, \sigma) \to (Z,\eta)$ be $\psi^*\alpha$ - continuous. If (Y, σ) is a ${}_{\psi*\alpha}T_{\alpha}$-space then $g\circ f : (X, \tau) \to (Z, \eta)$ is α - continuous.

Proof: Let U be any closed set in (Z,η). Since g is $\psi^*\alpha$ - continuous, $g^{-1}(U)$ is $\psi^*\alpha$ - closed in (Y, σ). Since (Y, σ) is a ${}_{\psi*\alpha}T_{\alpha}$ - space, $g^{-1}(U)$ is α - closed in (Y, σ). Since f is α - irresolute, $(g\circ f)^{-1}(U) = f^{-1}(g^{-1}(U))$ is α - closed in (X, τ). Hence $g\circ f$ is α - continuous.

Remark 3.24 The composition of two $\psi^*\alpha$ - continuous maps need not be a $\psi^*\alpha$ - continuous map as seen from the following example:

Example 3.25 Let $X = Y = Z = \{a, b, c\}$. Consider $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a, b\}, Y\}$ and $\eta = \{\phi, \{a\}, Z\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = b$, $f(c) = c$ and $g : (Y, \sigma) \to (Z,\eta)$ be a map defined by $g(a) = b$, $g(b) = a$, $g(c) = c$. Then the maps f and g are $\psi^*\alpha$ - continuous but their composition $g\circ f : (X, \tau) \to (Z, \eta)$ is not a $\psi^*\alpha$ - continuous map, since $\{b, c\}$ is closed in (Z,η) but $(g\circ f)^{-1}(\{b, c\}) = \{a, c\}$ is not $\psi^*\alpha$- closed in (X, τ).

Theorem 3.26 Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z,\eta)$ be $\psi^*\alpha$ - continuous maps. Then $g\circ f : (X, \tau) \to (Z, \eta)$ is also a $\psi^*\alpha$ - continuous map, if (Y, σ) is a ${}_{\psi*\alpha}T_c$-space.

Proof: Let V be any closed set in (Z,η). Since g is $\psi^*\alpha$ - continuous, $g^{-1}(V)$ is $\psi^*\alpha$ - closed in (Y, σ). Since (Y, σ) is a ${}_{\psi*\alpha}T_c$ -space, $g^{-1}(V)$ is closed in (Y, σ). Since f is $\psi^*\alpha$ - continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi^*\alpha$ - closed in (X, τ). Hence $g\circ f$ is a $\psi^*\alpha$ - continuous map.

Theorem 3.27 Let $f : (X, \tau) \to (Y, \sigma)$ be $g\alpha$ - continuous and $g : (Y, \sigma) \to (Z,\eta)$ be continuous. Then $g\circ f : (X, \tau) \to (Z, \eta)$ is a $\psi^*\alpha$ - continuous map, if (X, τ) is a ${}_{g\alpha}T_{\psi*\alpha}$ - space.

Proof: Let V be a closed set in (Z,η). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is $g\alpha$ -continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g\alpha$ - closed in (X, τ). Since (X, τ) is a ${}_{g\alpha}T_{\psi*\alpha}$ -space, $(g\circ f)^{-1}(V)$ is $\psi^*\alpha$ - closed in (X, τ). Hence $g\circ f$ is a $\psi^*\alpha$ - continuous map.

Theorem 3.28 Let $f : (X, \tau) \to (Y, \sigma)$ be αg - continuous and $g : (Y, \sigma) \to (Z,\eta)$ be continuous. Then $g\circ f : (X, \tau) \to (Z, \eta)$ is a $\psi^*\alpha$ - continuous map, if (X, τ) is a ${}_{\alpha g}T_{\psi*\alpha}$-space.

Proof: Let V be a closed set in (Z,η). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is αg-continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is αg- closed in (X, τ). Since (X, τ) is a ${}_{\alpha g}T_{\psi*\alpha}$ -space, $(g\circ f)^{-1}(V)$ is $\psi^*\alpha$ - closed in (X, τ). Hence $g\circ f$ is a $\psi^*\alpha$ - continuous map.

Theorem 3.29 Let $f : (X, \tau) \to (Y, \sigma)$ be ψg- continuous and $g : (Y, \sigma) \to (Z,\eta)$ be continuous. Then $g\circ f : (X, \tau) \to (Z, \eta)$ is a $\psi^*\alpha$ - continuous map, if (X, τ) is a ${}_{\psi g}T_{\psi*\alpha}$ -space

Proof: Let V be a closed set in (Z,η). Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ). Since f is ψg -continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is ψg - closed in (X, τ). Since (X, τ) is a ${}_{\psi g}T_{\psi*\alpha}$ -space, $(g\circ f)^{-1}(V)$ is $\psi^*\alpha$ - closed in (X, τ). Hence $g\circ f$ is a $\psi^*\alpha$ - continuous map.

References:-

1. Balachandran, K., Sundaram, P. and Maki, H. (1991): On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. Math., 12: 5 - 13.
2. Balamani,N.and Parvathi,A.(2016): Between α -closed sets and $\tilde{g}_{\alpha}$ -closed sets, International Journal of Mathematical Archive, 7(6): 1-10.
3. Balamani,N.and Parvathi,A.(2016): Separation axioms by $\psi^{*}\alpha$-closed sets, International Journal of Engineering Sciences and Research Technology, 5(10): 183-186.
4. Bhattacharyya,P. and Lahiri, B.K.(1987):Semi-generalized closed sets in topology, Indian J. Math., 29: 376-382.
5. Devi, R. Balachandran, K. and Maki, H. (1997): On generalized α - continuous maps and α - generalized continuous maps, Far East J. Math. Sci., Special Volume (I): 1-15.
6. Jain, R.C. (1980): Role of regular open sets in general topology, Ph.D., thesis, Meerut University, Meerut, India.
7. Levine, N. (1960): Strong continuity in topological spaces, Amer. Math. Monthly, 67: 269 - 275
8. Levine, N. (1963):Semi open sets and semi continuity in topological spaces, Amer. Math. Monthly, 70: 36 - 41.
9. Levine, N. (1970): Generalized closed sets in topology, Rend. Circ. Math. Palermo, 19: 89 - 96.
10. Maheswari, S.N. and Prasad, R. (1980): On α - irresolute maps, Tamkang J. Math., 11: 209 - 214.
11. Mashhour, A.S., Hasanein, I.A. and EI-Deeb, S.N. (1983): α - continuous and α - open mappings, Acta. Math. Hungar, 41: 213 - 218.
12. Pious Missier.S., Subramanian. and Alli.K(2013): On $g^{\#}p^{\#}$ continuous maps in topological spaces Journal of Global Research in Mathematical Archives 1(5): 26-30.
13. Rajesh. N.and Ekici.,E,(2008): On $\tilde{g}$-continuous functions, God .Zb. Inst.Mat.,75-86
14. Ramya,N.,and Parvathi,A.(2011): $\psi\hat{g}$-closed sets in topological spaces, International Journal of Mathematical Archive- 2(10):1992 – 1996.
15. Ramya,N.,and Parvathi,A.(2012):Strong forms $\psi\hat{g}$-continuous function in topological spaces, Journal of Mathematical and Computational Science- 2(1): 101-109.
16. Rose Mary and Lellis Thivagar,M. (2010): Stronger form of $\alpha\hat{g}$-continuous functions ,Inter.J.of Eng. Comp.Sci.and Math,1(1):71-79
17. Thivagar M.L and Nirmala Rebacca,P.(2011): A New type of topological mapping International J. of Math.Sci and Engg. Appls, IV:43-61
18. Veera kumar, M.K.R.S.(2000): Between semi-closed sets and semi-pre closed sets, Rend. Istit.Mat. Univ. Trieste, (ITALY) XXXXII :25-41.
19. Veerakumar, M.K.R.S. (2000): Between closed sets and g - closed sets, Mem. Fac. Sci. Koch. Univ. Ser. A, Math., 21: 1 - 19.
20. Veerakumar,M.K.R.S.(2003): On $\hat{g}$-closed sets in topological spaces, Bull.Allah. Math. Soc, 18: 99-112
21. Veerakumar, M.K.R.S. (2003): $g^{\#}$-closed sets in topological spaces, Mem. Fac. Sci. Koch. Univ. Ser. A, Math., 24: 1 - 13.
22. Veera kumar, M.K.R.S.(2006): Between g^{*}-closed sets and g- closed sets ,Antarctica. J.Math.,3(1): 43-65.