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RESEARCH ARTICLE

$g^{\#}b$ -CLOSED SETS IN TOPOLOGICAL SPACES.

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Abstract

In topological spaces closed sets and open sets are highly used in many practical and engineering problems.In this paper a new class of sets, namely $g^{\#}b$ -closed sets is introduced in topological spaces. Moreover we analyze the relations between $g^{\#}b$ -closed sets and already existing various closed sets. Also we find some basic properties and applications of $g^{\#}b$-closed, $g^{\#}b$-Neighbourhoods and $g^{\#}b$ -Limit points

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Introduction:-

Generalized closed sets play a very important role in general topology and they are now the research topics of many topologists worldwide. Generalized closed sets have been studied extensively in recent years by many topologists. The investigation of generalized closed sets has led to several new and interesting results,In1963, N.Levine [5] introduced semi-open sets in topology and studied their properties. N.Levine [6] introduced the concept of generalized closed sets and studied their properties in 1970. Mashhour [9] [1982] introduced pre-open sets in topological spaces. Andrijevic [1] introduced one such new version called b-open sets in 1996.Maki, et al.[1993] [8] introduced generalized α- closed and α-generalized closed sets (briefly, gα-closed, αg-closed). M. K. R. S. VeeraKumar [18](2003), $g^{\#}$-closed sets in topological spaces. In this paper, we introduce a new class of generalized closed sets called $g^{\#}b$-closed sets in topological spaces and its various properties are discussed

Preliminaries:-

Throughout this paper (X, τ) (or simply X) represent topological spaces ,For a subset A of X, cl(A), int(A) and A^c denote the closure of A, the interior of A and the complement of A respectively.
Let us recall the following definition, which are useful in the sequel.
Definition 2.1
A subset A of a space (X, τ) is called a
(i). Pre open set[9] if $A \subseteq int(cl(A))$.
(ii). semi-open set[5] if $A \subseteq cl(int(A))$.
(iii). α-open set [8] if $A \subseteq int(cl(int(A)))$.
(iv). b-open [1] if $A \subseteq cl(int(A)) \cup int(cl(A))$,
(v). *b-open [10] if $A \subseteq cl (int (A)) \cap int (cl (A))$.

(v). $b^{\#}$-open [15] if $A = cl(int(A)) \cup int(cl(A))$,

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The complements of the above mentioned open sets are their respective closed sets.

Definition 2.2
1. A subset A of X is called a generalized closed (briefly g-closed)set[6] if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is open in (X, τ).
2. A subset A of X is called an α-generalized closed (briefly αg-closed) set[8] if $\alpha cl(A) \subseteq H$ whenever $A \subseteq$ Hand H open in (X, τ).
3. A subset A of X is called a generalized α-closed (briefly gα-closed) set[8] if $\alpha cl(A) \subseteq H$ whenever $A \subseteq H$ and H is α-open in (X, τ).
4. A subset A of X is called $g^{\#}$-closed set [19] if $cl(A) \subseteq H$ whenever $A \subseteq H$ and H is αg-open in (X,τ);
5. A subset A of X is called $^{\#}$gp-closed set[2] if $pcl(A) \subseteq H$ whenever $A \subseteq H$ and H is αg-open in (X,τ);
6. A subset A of X is called a $g^{\#}$s closed set[18] (written as $g^{\#}$s-closed) if $scl(A) \subseteq H$ whenever $A \subseteq H$ and H is αg-open set of (X, τ) .
7. A subset A of X is called a $g^{\#}\alpha$-closed [11] if $\alpha cl(A) \subseteq H$ whenever $A \subseteq H$ and H is g-open in (X, τ) .
8. A subset A of X is called generalized $b^{\#}$-closed (briefly $gb^{\#}$-closed)[17] if $b^{\#}cl(A) \subseteq H$ whenever $A \subseteq H$ and H is open in (X, τ).
9. A subset A of X is called generalized b-closed (briefly, gb-closed) [4] if $bcl(A) \subseteq H$ whenever $A \subseteq H$ and H is open in (X, τ).
10. A subset A of X is called generalized b*-closed set (briefly, gb*-closed) if $int(cl(A)) \subseteq H$ Whenever $A \subseteq H$ and H is gb-open set in (X, τ) .
11. A subset A of X is called generalized star b-closed set (briefly, g*b-closed) set[16] if $bcl(A) \subseteq H$ whenever $A \subseteq H$ where H is g-open in (X, τ) .

The complements of the above mentioned closed sets are their respective open sets.

Properties Of $g^{\#}$b -Closed Sets:-

In this section we introduce the following definition and study further some of their properties.

A subset A of a space (X, τ) is called a $g^{\#}$b closed set if $bcl(A) \subseteq H$ whenever $A \subseteq H$ and H is a αg-open set in (X, τ).The complement of a $g^{\#}$b-closed set is called $g^{\#}$b-open set of (X, τ)

Proposition:-

(i) Every closed set is pre closed set.
(ii) Every α closed set is pre closed set and semi closed set.
(iii) Every semi closed set is b closed set.
(iv) Every $b^{\#}$ closed and *b closed set is b closed set

The converse of the above Proposition need not be true.

Proposition:-

(i) Every pre closed set is $^{\#}$gp closed set.
(ii) Every semi closed set is $g^{\#}$s closed set.
(iii) Every α closed set is $g^{\#}\alpha$ closed set.
(iv) $g^{\#}$ closed and $^{\#}$gp closed set is gb* closed set.
(v) Every $g^{\#}$s closed set is gb closed set

The converse of the above Proposition need not be true.

Theorem 3.3:-

Every b-closed set is $g^{\#}$b-closed.

Proof:-

Let A be any b-closed set in X. Let H be any αg-open set containing A. Since A is a b-closed set ,we have bcl(A) = A. Therefore $bcl(A) \subseteq H$ .Hence A is $g^{\#}$b -closed in X.The converse of the above theorem need not be true.

Example 3.4:-

Let X ={a, b, c, d}, τ ={X, ϕ, {a}, {b},{a,b}{b.c},{a,b,c}}.{a,c} is $g^{\#}$b-closed set but not b-closed set

Theorem 3.5:-
Every closed(resp. α-closed, pre closed, semi closed) set is $g^{\#}b$ -closed.

Proof:-
The proof follows from the definitions and the fact that every closed (resp. α-closed, semi closed) set is $g^{\#}b$-closed.

Example 3.6:-
Let X ={a, b, c, d, e}, τ ={X, ϕ, {a}, {b},{a,b}}, Let A={a,c,d } .Here A is $g^{\#}b$-closed set but not pre-closed set in(X, τ)

Example 3.7:-
Let X ={a, b, c} with the topology τ ={X, ϕ,{a,b}}. Let A= {a, c} is $g^{\#}b$ -closed set but not semi-closed set in (X, τ)
Let A be any $^{\#}gp$ -closed set and H be any αg-open set containing A. from the definitions and the fact that every pre open set is b open.bcl(A) ⊆pcl(A) ⊆H.Hence A is $g^{\#}b$–closed.The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.8:-
Let X ={a, b, c, d} with the topology τ ={X, ϕ,{a},{b}{b,c}} {a,b,c}, Let A= {a}is $g^{\#}b$ -closed set but not α-closed set in(X, τ)

Theorem 3.9:-
Every $g^{\#}s$-closed set is $g^{\#}b$-closed set but not conversely.

Proof:-
Let A be any $g^{\#}s$ -closed set and H be any αg-open set containing A. from the definitions and the fact that every semi open set is b open.bcl(A) ⊆scl(A) ⊆H . Hence A is $g^{\#}b$-closed.The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.10:-
Let X ={a, b, c} with the topology τ ={X, ϕ,{a,b}}. Let A= {b} is $g^{\#}b$ -closed set but not $b^{\#}g$-closed set in (X, τ)

Theorem 3.11:-
Every $^{\#}gp$-closed set is $g^{\#}b$-closed set but not conversely.

Proof:-
Example3.11:-
Let X ={a, b, c, d, e} with the topology τ ={X, ϕ,{a},{b},{a.b}}, Let A= {a,e}.A is $g^{\#}b$ -closed set but not $^{\#}gp$ -closed set in(X, τ)

Theorem 3.12 :-
Every $g^{\#}\alpha$-closed set is $g^{\#}b$-closed set but not conversely.

Proof:-
Let A be any $g^{\#}\alpha$ -closed set and H be any g-open set containing A. from the definitions and the fact that every α open set is b open.bcl(A) ⊆αcl(A) ⊆H. Hence A is $g^{\#}b$–closed.The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.13:-
Let X ={a, b, c, d} with the topology τ ={X, ϕ,{a},{b},{a.b},{b,c},{a,b,c}}, Let A= {a,c}.Ais$g^{\#}b$ -closed set but not $g^{\#}\alpha$ -closed set in(X, τ)

Theorem 3.14:-
Every $g^{\#}$ closed set is $^{\#}$gp-closed.

Proof:-
The proof follows from the definitions
The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.15:-
Let X ={a, b, c, d} with the topology τ ={X, ϕ,{a},{b}{a,b},{b,c},{a.b.c}}, Let A= {c} is $^{\#}$gp -closed set but not $g^{\#}$ -closed set in(X,τ)

Theorem 3.16:-
Every $g^{\#}$ closed set is gb*-closed.

Proof:-
The proof follows from the definitions
The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.17:-
Let X ={a, b,c}with the topology τ ={X, ϕ,{a}},Let A={c} is gb*-closed set but not $g^{\#}$-closed set in (X,τ)

Theorem 3.18
Every gb*-closed set is $g^{\#}$b -closed.

Proof:-
Let A be any gb*-closed set in X . Let H be gb -open set containing A. Since every αg-open set is gb-open, we have bcl(A)⊆H. Hence A is $g^{\#}$b closed
The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.19:-
Let X ={a, b, c, d} with the topology τ ={X, ϕ,{a},{b}{b,c}} {a,b,c}, Let A= {a}is $g^{\#}$b -closed set but not gb*-closed set in(X, τ)

Theorem 3.20:-
Every g $b^{\#}$ -closed set is $g^{\#}$b -closed.

Proof:-
Let A be $b^{\#}$g-closed set in X. Let A⊆H where H is open. Thus H is b-open. Since A is $b^{\#}$g-closed, $b^{\#}$cl(A) ⊆H .But bcl(A)⊆$b^{\#}$cl(A).Thus we have bcl(A) ⊆H whenever A ⊆H and H is b-open. Therefore A is $g^{\#}$b-closed set.
The converse of the above theorem need not be true in general, as shown in the following example.

Example:3.21
Let X ={a, b, c} with the topology τ ={X, ϕ,{a,b}}. Let A= {b}.A is $g^{\#}$b -closed set but not $b^{\#}$g-closed set in (X, τ)

Theorem 3.22:-
Every $g^{\#}$b closed set is g*b-closed.

Proof:-
Let A be $g^{\#}$b -closed set in X. Let A⊆H where H is open. Thus H is b-open. Since A is $g^{\#}$b -closed, bcl(A)⊆H whenever A ⊆H and H is b-open. Therefore A is g*b -closed set
The converse of the above theorem need not be true in general, as shown in the following example.

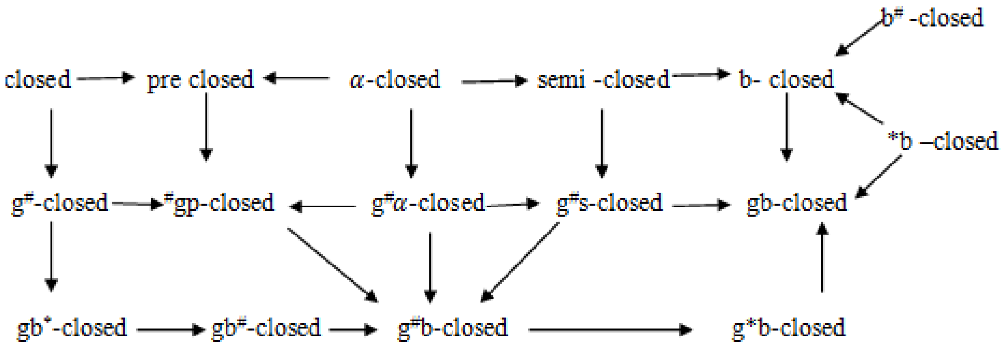
Example 3.23:-
Let X ={a, b, c, d} with the topology τ ={X, ϕ,{a},{b}{b,c},{a,b,c}, Let A= {a}is g^{*}b -closed set but not $g^{\#}$b -closed set in(X, τ)

Theorem 3.24:-
Every gb*-closed set is $gb^{\#}$-closed

Proof:-
The proof follows from the definitions
The converse of the above theorem need not be true in general, as shown in the following example.

Example 3.24:-
Let X ={a, b, c} with the topology τ ={X, ϕ,{b,c}}, Let A= {a,b} is gb* -closed set but not$gb^{\#}$ -closed set in(X ,τ)

Diagram-I:-



Characterization of $g^{\#}$b–Closed Sets:-
Theorem 4.1
The intersection of any two subsets of $g^{\#}$b-closed sets in X is $g^{\#}$b-closed sets.

Proof:-
Let A and B be the subsets of $g^{\#}$b-closed sets, A⊆U and bcl(A) ⊆U, B⊆U and bcl(B) ⊆U, U is an αg-open. Therefore, A∩B⊆A and bcl(A∩B) ⊆bcl(A), A∩B⊆B and bcl(A∩B) ⊆bcl(B). Hence, bcl(A∩B) ⊆U and U is an αg-open. Thus, A∩B is $g^{\#}$b-closed set.
Remark 4.2
If the subsets A and B are $g^{\#}$b-closed sets, their union need not be $g^{\#}$b-closed set.

Example 4.3:-
Let X = {a, b, c, d} with τ={X, ϕ,{a},{b},{c},{a,b},{b,c}{a, c},{a,b,c}}. In this topological space(X, τ), the subsets {c} and {a,b} are $g^{\#}$b-closed, but their union {a,b,c} is not $g^{\#}$b-closed.

Theorem 4.4:-
If A is a $g^{\#}$b-closed set of (X, τ), then bcl(A)-A does not contain any non emptyαg-closed set.

Proof:-
Let F be a αg-closed set contained in bcl(A)-A. Then A ⊆ X-F and X-F is a αg-open set of (X, τ). Since A is a $g^{\#}$b-closed set of (X, τ), then bcl(A) ⊆ X-F. Now F ⊆ X-bcl(A). Then F ⊆ (X-bcl(A)) ∩ (bcl(A)-A)) ⊆ (X-bcl(A)) ∩bcl(A) = ϕ. Hence F = ϕ.

Theorem 4.5:-
If A is a $g^{\#}$b-closed set of (X, τ) and A ⊆ B ⊆bcl(A), then B is also a $g^{\#}$b-closed set.

Proof:-
Let H be an αg-open set of (X, τ) such that B ⊆ H. Then A ⊆H.Since A is $g^{\#}$b-closed, then bcl(A) ⊆ H. Since B ⊆bcl(A), then scl(B) ⊆bcl(bcl(A)). Thus bcl(B) ⊆ H. Therefore B is a $g^{\#}$b-closed set of (X, τ).

Theorem 4.6:-
A subset A is a $g^{\#}$b-closed set in (X, τ) if and only if bcl(A) - A contains no non-empty αg -closed set in (X, τ).

Proof:-
Let F be a αg -closed set contained in bcl(A) - A. Then $A^C \subseteq F^C$ and F^C is a αg -open set of (X, τ). Since A is $g^{\#}$b-closed set, bcl(A)$\subseteq F^C$. This implies F ⊆ X - bcl(A). Then F ⊆ (X -bcl(A)) ∩ (bcl(A)- A).F⊆ (X-bcl(A))∩bcl(A)= ϕ. Therefore F = ϕConversely, suppose that bcl(A) - A contain no non-empty αg -closed set in (X, τ).Let H be a αg -open such that A ⊆H. If bcl(A) ⊄H, then bcl(A) ∩ G^C is a non-empty αg -closed set of bcl(A) - A, which is a contradiction. Therefore bcl(A) ⊆ H and hence A is an $g^{\#}$b-closed set in (X, τ).

Theorem 4.7:-
If A is both αg-open and $g^{\#}$b-closed in X then A is b-closed.

Proof:-
Suppose A is αg-open and $g^{\#}$b-closed in X.Since A ⊆ A, bcl(A) ⊆ A. But Always A ⊆ bcl (A).
Therefore A = bcl (A). Hence A is b-closed.

Corollary 4.8:-
Let A be a αg-open set and $g^{\#}$bclosed set in X. Suppose that F is b-closed in X. Then A∩ F is $g^{\#}$b-closed in X.

Proof:-
By theorem 4.7, A is b-closed. So A∩ F is b-closed and hence A ∩ F is an $g^{\#}$b-closed in X.

Theorem 4.9:-
If A is both open and αg-closed in X, then A is $g^{\#}$b-closed in X.

Proof:-
Let A⊆ H and H be αg-open in X. Now A⊆ A. By hypothesis αcl(A) ⊆ A. Since every α-closed set is b-closed ,bcl(A) ⊆ αcl(A). Thus bcl(A) ⊆ A ⊆ H. Hence A is $g^{\#}$b-closed in X.

Theorem 4.10:-
Let A be a $g^{\#}$b-closed set in (X ,τ).Then A is b-closed iffbcl (A) - A is αg-closed.

Proof:-
Suppose A is preclosed in X. Then bcl(A) = A and so bcl(A) - A = ϕ which is αg-closed in X.Conversely , Suppose bcl(A) -A is αg-closed in X.Since A is $g^{\#}$b-closed , bcl(A) - A does not contain any non-empty αg-closed set in X. That is bcl(A) - A = ϕ**.** Hence A is b-closed.

Theorem 4.1:-
Let A ⊆Y ⊆X and suppose that A is $g^{\#}$b-closed set in X,A is then $g^{\#}$b-closed set relative to Y.

Proof:-
Given that A ⊆Y ⊆X and A is $g^{\#}$b- closed set in X, to show that Ais$g^{\#}$b-closed set relative to Y. Let A ⊆Y∩U, where U is an αg-open in X,thenbcl(A) ⊆U and bcl(A) ∩⊆Y⊆Y∩U. Therefore, bcl(A) ∩Y is the b-closure of A in Y. Thus, A is $g^{\#}$b-closed set relative to Y.

5.$g^{\#}$b-Neighbourhoods and $g^{\#}$b -Limit points:-
In this section, we define and study about $g^{\#}$b -neighbourhood, $g^{\#}$b -limit point and $g^{\#}$b -derived set of a set and show that some of their properties are analogous to those for open sets.

Definition 5.1:-
Let (X, τ) be a topological space and let x ∈ X. A subset N of X is said to beg$^{\#}$b-neighbourhood of a point x∈X if there exists a $g^{\#}$bopen set H such that x∈ H ⊂N.

Definition 5.2:

Let (X, τ) be a topological space and A be a subset of X.A subset N of X is said to be $g^{\#}$b-neighbourhood of A if there exists a $g^{\#}$bopen set H such that A∈ H ⊂N.The collection of all $g^{\#}$b-neighbourhood of x ∈ X is called the $g^{\#}$bneighbourhood system at 'x' and shall be denoted by $g^{\#}$bN(x).It is evident from the above definition that a $g^{\#}$b-open set is a $g^{\#}$bneighbourhood of each of its points. But a $g^{\#}$b-neighbourhood of a point need not be a $g^{\#}$b-open set. Also every $g^{\#}$b-open set containing x is a $g^{\#}$bneighbourhood of x.

Theorem 5.3:-
A subset of a topological space is $g^{\#}$b-open if it is a $g^{\#}$b-neighbourhood of each of its points.

Proof:-
Let a subset H of a topological space be $g^{\#}$b-open. Then for every x ∈ H, x∈ H ⊂H and therefore H is a $g^{\#}$b-neighbourhood of each of its points. Analogous to those for open sets. The converse of the above Theorem need not be true as seen from the following example.

Example 5.4:-
Let X={a,b,c,d} with topology τ ={X, ϕ , {a},{b}{a, b},{b,c},{a,b,c}}.In this topological spac(X, τ),$g^{\#}$b-cl(X) = {X, ϕ ,{a},{c},{d},{a, c},{a, d}, {b, c},{b, d},{c, d},{a, c, d},{b, c, d}} the set{a, b, d} is the neighbourhood of {a, d}, since a, d∈{a, d} ⊂{a, b, d} ,and {a, b, d} is the $g^{\#}$b-neighbourhood of each of its points. However,{a, b, d} is not $g^{\#}$b-closed in X.

Theorem 5.5:-
Let (X, τ) be a topological space. If A is a $g^{\#}$b-closed subset of X and x ∈ X - A, then there exists a $g^{\#}$b-neighbourhood N of x such that N ∩ A =ϕ

Proof:-
Since A is $g^{\#}$b-closed, then X - A is $g^{\#}$b-open set in (X, τ). By the above Theorem 5.3, X - A contains a $g^{\#}$b-neighbourhood of each of its points. Hence there exists a $g^{\#}$b-neighbourhood N of x, such that N ⊂ X - A.That is, no point of N belongs to A and hence N ∩ A =ϕ.

Theorem 5.6:-
Let (X, τ) be a topological space and A ⊆ X. Then x ∈$g^{\#}$bcl(A) if and only if for any $g^{\#}$b-neighbourhood N of x in (X, τ) ,A ∩ N = ϕ.

Proof:-
Suppose x ∈$g^{\#}$bcl(A). Let us assume that there is a $g^{\#}$b-neighbourhood N of the point x in (X, τ) such that N ∩ A = ϕ. Since N is a $g^{\#}$b-neighbourhood of x in (X, τ) by definition of $g^{\#}$b-neighbourhood there exists an $g^{\#}$b-open set H of x such that x∈ H ⊂N. Therefore we have H ∩ A= ϕ and so A ⊆ H^C. Since X - H is an $g^{\#}$b-closed set containing A.We have by definition of $g^{\#}$b-closure, $g^{\#}$bcl(A) ⊆ X-H and therefore x ∉$g^{\#}$bcl(A), which is a contradiction to hypothesis x ∈$g^{\#}$bcl(A). Therefore A∩ N = ϕ.

Conversely,Suppose for each $g^{\#}$b-neighbourhood N of x in (X, τ).A ∩ N = ϕ. Suppose that x ∈$g^{\#}$bcl(A). Then by definition of $g^{\#}$bcl(A), there exists a $g^{\#}$b-closed set H of (X, τ) such that A⊆H and x ∉ H. Thus x∈X - H and X - H is $g^{\#}$b-open in (X, τ) and hence X - H is a $g^{\#}$b-neighbourhood of x in (X, τ) .But A∩(X-H)= ϕ which a contradiction.Hence x ∈$g^{\#}$bcl(A).

Theorem 5.7:-
Let (X,τ) be a topological space and p ∈X. Let $g^{\#}$bN(p) be the collection of all $g^{\#}$b- neighbourhoods of p. Then

1. $g^{\#}$bN(p) ≠ ϕ and p∈ each member of $g^{\#}$bN(p).
2. The intersection of any two members of $g^{\#}$bN(p) is again a member of $g^{\#}$bN(p).
3. If N ∈ $g^{\#}$b N(p) and M⊆N, then M ∈$g^{\#}$b N(p).
4. Each member N∈$g^{\#}$b N(p) is a superset of a member H∈$g^{\#}$b N(p) where H is a $g^{\#}$b-open set.

Proof:-

1. Since X is a $g^{\#}$b-open set containing p, it is a $g^{\#}$b-neighbourhood of every p∈X. Hence there exists at least one $g^{\#}$b-neighbourhood namely X for each p∈X. Here $g^{\#}$bN(p)≠ ϕ. Let N ∈$g^{\#}$bN(p),N is a $g^{\#}$b-neighbourhood of p.Then there exists a $g^{\#}$b-open set H such that p∈ H ⊂N.So p∈N.Therefore p ∈every member N of $g^{\#}$b-N(p).
2. Let N ∈$g^{\#}$bN(p) and M ∈$g^{\#}$b N(p). Then by definition of $g^{\#}$b-neighbourhood, there exists $g^{\#}$b-open sets H and F such that p∈ H ⊆N and p ∈F ⊆ M. Hence p∈ H ∩F⊆M∩N, Note that H ∩F is a $g^{\#}$b-open set since intersection of $g^{\#}$b-open sets is $g^{\#}$b-open. Therefore it follows that N∩M is a $g^{\#}$b-neighbourhood of p. Hence N∩M ∈$g^{\#}$b N(p).
3. If N ∈$g^{\#}$bN(p) then there is an $g^{\#}$b -open set H such that p∈ H ⊆N.Since M ⊆ N, M is a $g^{\#}$b- neighbourhood of p. Hence M ∈$g^{\#}$b N(p).Let N ∈$g^{\#}$bN(p) .Then there exist an $g^{\#}$b-open set H such that p∈ H ⊆N. Since H is$g^{\#}$b-open and p ∈ H, H is $g^{\#}$b-neighbourhood of p. Therefore H ∈$g^{\#}$bN(p) and also H⊆N.

Definition 5.8:-

Let (X,τ) be a topological space and A be a subset of X.Then a point x∈X is called a $g^{\#}$b-limit point of A if and only if every $g^{\#}$b-neighbourhood of x contains a point of A distinct from x. That is [N{x}]∩A ≠ ϕ for each $g^{\#}$b-neighbourhood N of x. Also equivalently if and only if every $g^{\#}$b-open set H containing x contains a point of A other than x.

In a topological space (X,τ) the set of all $g^{\#}$b-limit points of a given subset A of X is called a $g^{\#}$b-derived set of A and it is denoted by $g^{\#}$bd(A).

Theorem 5.9:-

Let A and B be subsets of a topological space (X,τ). Then

1. $g^{\#}$bd(ϕ)= ϕ
2. If A ⊆ B, then $g^{\#}$bd(A) ⊆$g^{\#}$bd(B),
3. If x ∈$g^{\#}$bd(A), then x ∈$g^{\#}$bd[A-{x}],
4. $g^{\#}$bd(A) ∪$g^{\#}$bd(B) ⊆$g^{\#}$bd(A∪B),
5. $g^{\#}$bd(A∩B) ⊆ $g^{\#}$b d(A) ∩$g^{\#}$bd(B).

Proof:-

1. Let x be any point of X and x ∈$g^{\#}$bd(ϕ)). That is x is a $g^{\#}$b-limit point of ϕ. Then for every$g^{\#}$b-open set H containing x, we should have [H-{x}]∩ ϕ ≠ ϕ.which is impossible. Hence $g^{\#}$bd(ϕ)= ϕ
2. If X∈$g^{\#}$b d(A), that is if x is $g^{\#}$b-limit point of A, then by Definition 5.8 [H - {x}] ∩ A ≠ ϕ. for every $g^{\#}$b-open set H containing x. Since A⊆B implies [H - {x}] ∩ A ⊆ [H - {x}] ∩ B. Thus if x is a $g^{\#}$b-limit point of A it is also a$g^{\#}$b-limit point of B, that is x ∈$g^{\#}$bd(B). Hence $g^{\#}$bd(A) ⊆$g^{\#}$bd(B).
3. (iii)If x ∈$g^{\#}$bd(A), that is x is a $g^{\#}$b-limit point of A. Then by Definition 5.8 every $g^{\#}$b-open set H containing x contains at least one point other than x of A -{x}. That is H ∩ (A -{x}) ≠ ϕ Hence x is a $g^{\#}$b-limit point of A -{x} and as such it belongs to $g^{\#}$bd[A -{x}]. Therefore x ∈$g^{\#}$bd(A) ⟹x∈ $g^{\#}$b d[A-{x}].
4. Since A ⊆ A∪B and B ⊆A∪B, it follows from (ii) $g^{\#}$bd(A)⊆$g^{\#}$bd(A∪B) and $g^{\#}$bd(B) ⊆$g^{\#}$bd(A∪B) and hence $g^{\#}$bd(A)∪$g^{\#}$bd(B) ⊆$g^{\#}$b(A∪B).

5. Since A∩B⊆A and ∩ B ⊆ A, by (ii) $g^{\#}$bd(A∩B) ⊆$g^{\#}$bd(A) and $g^{\#}$bd(A∩B) ⊆$g^{\#}$bd(B).Consequently $g^{\#}$bd(A∩B) ⊆ $g^{\#}$bd(A) ∩$g^{\#}$bd(B).

Theorem 5.10:-

Let (X,τ) be a topological space and A be subset of X. If A is $g^{\#}$b-closed, then $g^{\#}$bd(A) ⊆ A.

Proof:-

Let A be to $g^{\#}$b-closed, Now we will show that $g^{\#}$bd(A)⊆A. Since A is $g^{\#}$b-closed, X-A is $g^{\#}$b-open.To each x∈X- A there exists $g^{\#}$bneighbourhood H of x such that H⊆X-A. Since A∩(X - A) = ϕ, the $g^{\#}$bneighbourhood H contains no point of A and so x is not a $g^{\#}$b-limit point of A. Thus no point of X - A can be $g^{\#}$b-limit point of A that is, A contains all its $g^{\#}$b-limit points. That is $g^{\#}$bd(A)⊆A.

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