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RESEARCH ARTICLE

REVIEW FOR QUANTUM EFFECTS ON GRAVITATIONALLY BOUND NEUTRON STAR BINARIES.

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Abstract

Since the quantum field theory was invented at the end of 1920s, attempts have been made to apply it to gravitational field. After more than 20 years, the formal quantum theory of gravity, which describes any systems as an action function in a canonical Hamiltonian method, achieved for the first time a state of completion. Contrary to the situation held for the canonical theory, a covariant treatment also was developed to deal with the physical conditions such that effects of vacuum processes must be taken into account. In this review, we start from a brief introduction to the history of construction of quantum theory of gravity and consider the applications of two quantization methods to inspiraling neutron star binary systems. We mainly focus on the gravitational quantum effects on dynamics of the gravitational quanta radiated from different systems by using canonical method and on the gravitational potential of the binaries by employing the covariant treatment. The possible detections and constraints to the quantum effects are also discussed.

Introduction:-

Almost as soon as the electromagnetic fields gave birth to the quantum field theory (QFT) at the end of last 20s, attempts were made to apply it to other fields. In 1930, Rosenfeld was first to apply QFT to gravitational field and developed general methods for handling the technical difficulties involved in quantizing gravitation [1, 2]. It was used to compute the gravitational self-energy of a photon in the lowest order of perturbation theory, and the results with quadratic divergence were obtained, which confirmed the divergence malady of the field theory. During the last 30s, which was a physics great booming era, it was recognized that the quanta of gravitational field produce no observable effects until the Planck energy is reached. That is to say the energy of gravitational quanta should be Planck scale of the order of 10^{-19} GeV, corresponding to the Planck length $\sqrt{\frac{\hbar G}{c^3}} \approx 10^{-30}$ m. After these initial studies, there had been no essentially new developments for quantum gravidynamics about 20 years.

In 1950s, stimulated by the renormalization method that makes sense of the infinities in perturbation theory by altering the values of quantities to compensate for effects of their self-interactions, DeWitt reperformed Rosenfeld's self- energy calculation in a Lorentz-covariant and gauge-invariant manner [3], which was just the lowest-order calculations for perturbation theory. However, the renormalization methods were critically attacked because of its explicitly manipulation of divergent quantities when involving the interaction of point masses with gravity. At about the same time, Bergman tried to quantize the gravitational field by using the commutation relations for particle

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position and momenta [4], which set out upon the classical canonical road in search of the Hamiltonian. Although immediately running into the problem of constraints, Bergman performed much valuable ground work in formulating and resolving the difficulties. In the meantime, Dirac published a general Hamiltonian theory [5], which is in principle applicable to any system described by an action functional. Pirani and Schild applied Diracs primary theory soon to gravitational field [6]. However, the theory had been remaining an incomplete state for several years, until the first and second international relativity conferences held in 1955 and 1957. It was shown that, by using Pirani-Schild formalism, four primary constraints, which meant that the state functional must be independent of the components of metric tensor, could be transformed into pure momenta by a phase transformation [7]. While the three of secondary constraints, implying that the state functional must be independent of the coordinates chosen in the space-like cross sections and hence cannot be taken to be arbitrary functional of the metric components, are actually the generators of infinitesimal transformations of three spatial coordinates [7]. Dirac then began to apply his method and the constraints to gravitational field [8–10], and the formal quantum theory of gravity achieved for the first time a state of completion, which is the canonical Hamiltonian quantization method [11] and focus on some bizarre features, arising in the case of closed finite words, of possible cosmological and even metaphysical significance.

When dealing with the questions such as particle scattering, pair-production, pair-annihilation, and decay of individual quanta, the canonical theory is left untouched. A manifestly covariant treatment for quantum theory of gravity [12–14] was constructed by analogy with the conventional scattering-matrix theory, which lends itself to study the questions when physical conditions such that the effects of vacuum process are must be taken into account. However, the manifest covariance in conventional scattering-matrix theory denotes covariant Lorentz invariance, which is an expression of a geometrical symmetry processed by a system. While the gravity theory bases on the manifest general covariant propagators, which is accomplished by introducing a variable background metric, instead of a flat background. Consequently, a formalism, in which general covariance penetrates the theory, by introducing a c-number background metric [13], instead of the flat background, was developed, which allows us to introduce generally covariant particles propagators rather than just Lorentz covariant.

The physical interpretation to the quantum theory of gravity, on the classical side, was to provide a characterization of gravitational radiation and energy [15]. While the interpretation in the quantum domain requires analysis for the technical structure of canonical theory [16], which we introduce in section 2.1. In the remaining part of section II, we apply the canonical Hamiltonian quantization methods to inspiraling neutron star (NS) binaries and review the quantum effects on gravitational radiations, in which we mainly discuss the Higgs-like mechanism and mass generation processes of gravitational quanta radiated from double neutron star (DNS) binaries and neutron star-white dwarf (NS- WD) systems. We review the manifest covariant theory of gravity and its application to gravitational interactions of two-body bound systems in section III, where the quantum corrections to the static gravitational potential of the NS binaries systems are reviewed. Finally we give a summary for the quantum effects in NS binaries and discuss the possible detections and constraints.

In this review, we just consider the wide NS binaries, with separation of about 10^9 m, which move closer in a spiral way and expect to coalesce and merge in the Hubble time. For our calculations, we use the notations: Latin indices range over the values “1, 2, 3” and Greek indices over the values “0, 1, 2, 3”. The comma “,” denotes differentiation, while the semicolon“ ; ”represents the covariant differentiation. The so-called “absolute units” $\hbar = c = 1$ are used. We choose the space-time metric as -+++. The Riemann and Ricci tensors, and the curvature scalar are, respectively, taken as

$$\mathcal{R}_{\alpha\beta\gamma}{}^{\delta} = \Gamma_{\beta\gamma}{}^{\delta}{}_{,\alpha} - \Gamma_{\alpha\gamma}{}^{\delta}{}_{,\beta} + \Gamma_{\beta\gamma}{}^{\sigma}\Gamma_{\alpha\sigma}{}^{\delta} - \Gamma_{\alpha\gamma}{}^{\sigma}\Gamma_{\beta\sigma}{}^{\delta}, (1.1)$$

$$\mathcal{R}_{\alpha\beta} \equiv \mathcal{R}_{\gamma\alpha\beta}{}^{\gamma}, (1.2)$$

$${}^{(4)}\mathcal{R} \equiv \mathcal{R}_{\mu}{}^{\mu} \equiv g^{\mu\nu}\mathcal{R}_{\mu\nu}, (1.3)$$

$$\Gamma_{\mu\nu}{}^{\sigma} \equiv \frac{1}{2} g^{\sigma\rho}\left(g_{\mu\rho,\nu} + g_{\nu\rho,\mu} - g_{\mu\nu,\rho}\right), \qquad g_{\mu\sigma}g^{\sigma\nu} = \delta^{\nu}_{\mu}. (1.4)$$

In these conventions, ${}^{(4)}\mathcal{R}$ is non-negative in a space-time containing normal matter and satisfies the Einstein’s equations. The corresponding tensor ${}^{(3)}\mathcal{R}$ is positive in the 3-space-like cross sections of positive curvatures.

Canonical Quantization And Massive Gravitational Quanta In NS Binaries:-

I. Canonical Hamiltonian Quantization Method

The beginning of the canonical theory follows the decomposition of the metric tensor in terms of new variables "α, β, γ" [11],

$$(g_{\mu\nu}) = \begin{pmatrix} -\alpha^2 + \beta_k\beta^k & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix},\ (g^{\mu\nu}) = \begin{pmatrix} -\alpha^{-2} & \alpha^{-2}\beta^j \\ \alpha^{-2}\beta^i & \gamma_{ij} - \alpha^{-2}\beta^i\beta^j \end{pmatrix}, (2.1)$$

where $\gamma^{ik}\gamma_{kj} = \delta^i_j, \beta^i = \gamma^{ij}\beta_j$. Then the conventional Einstein Lagrangian density can, in terms of the new variables, be taken as

$$\mathcal{L} \equiv \sqrt{g}^{(4)}\mathcal{R} = \alpha\gamma^{\frac{1}{2}}\left(K_{ij}K^{ij} - K^2 + {}^{(3)}\mathcal{R}\right) - 2\left(\gamma^{\frac{1}{2}}K\right)_{,0} + 2\left(\gamma^{\frac{1}{2}}K\beta^i - \gamma^{\frac{1}{2}}\gamma^{ij}\alpha_{,j}\right)_{,i}, (2.2)$$

where $g \equiv -\det(g_{\mu\nu})$ and $\gamma \equiv \det(\gamma_{ij})$ are the determinant of 4-metric $g_{\mu\nu}$ and 3-metric γ_{ij}, respectively, and $g = \alpha^2\gamma$ here. The quantity K_{ij} transforms as a symmetric tensor under spatial coordinate transformations, which is called the extrinsic curvature tensor and describes the curvature of 4-dimensional space-time that it is embedded,

$$K_{ij} \equiv \frac{1}{2}\alpha^{-1}\left(\beta_{i;j} + \beta_{j;i} - \gamma_{ij,0}\right), K^{ij} \equiv \gamma^{ik}\gamma^{jl}K_{kl}, K \equiv \gamma^{ij}K_{ij}. (2.3)$$

The contracted forms ${}^{(3)}\mathcal{R}$ and $K_{ij}K^{ij} - K^2$ in Eq. (2.2) are referred to as the intrinsic and extrinsic curvatures, which play the roles of potential energy and kinetic energy, respectively. The integrand of Eq. (2.2) gives the Lagrangian,

$$L \equiv \int \alpha\gamma^{\frac{1}{2}}\left(K_{ij}K^{ij} - K^2 + {}^{(3)}\mathcal{R}\right)d^3x, (2.4)$$

which has the classical form, i.e. "kinetic energy minus potential energy". The Lagrangian (2.4) is general 3-dimensional coordinate invariant and obeys the primary and secondary constraints. The primary constraints have the explicit forms

$$\frac{\partial L}{\partial \alpha_{,0}} = 0, \frac{\partial L}{\partial \beta_{i,0}} = 0, (2.5)$$

which express the fact that the Lagrangian (2.4) is independent of the arbitrary velocities $\alpha_{,0}$ and $\beta_{i,0}$. While the secondary or dynamical constraints show the coordinates-independent properties of Lagrangian (2.4) and read

$$\mathcal{H} = 0, (2.6)$$

$$x^i = 0. (2.7)$$

Here, $\mathcal{H} \equiv \gamma^{\frac{1}{2}}\left(K_{ij}K^{ij} - K^2 + {}^{(3)}\mathcal{R}\right)$, with Hamiltonian $H = \int \mathcal{H}d^3x$ appears as the difference of the extrinsic curvature and intrinsic curvature. $x^i \equiv -2\pi^{ij}_{,j} - \gamma^{il}(2\gamma_{jl,k} - \gamma_{jk,l})\pi^{jk}$, where $\pi_{ij} = \frac{\partial L}{\partial \gamma_{ij,0}} = -\gamma^{\frac{1}{2}}(K^{ij} - \gamma^{ij}K)$ relates to Hamiltonian via $H = \int\left(\pi\alpha_{,0} + \pi^i\beta_{i,0} + \pi^{ij}\gamma_{ij,0}\right)d^3x - L$, with $\pi = \frac{\partial L}{\partial \alpha_{,0}}$ and $\pi^i = \frac{\partial L}{\partial \beta_{i,0}}$. The condition (2.6) is the Hamiltonian constraint, which states that the intrinsic curvature equals the extrinsic one in a flat space-time and essentially describes the intrinsic dynamics of the gravitational field.

Then we can quantize the gravitational field. In the quantum theory, Poisson brackets become commutators. That is to say the constraints Eqs. (2.5), (2.6), and (2.7) become conditions on state vector Ψ that describe the field [5, 9],

$$\frac{\partial L}{\partial \alpha_{,0}}\Psi = 0, \frac{\partial L}{\partial \beta_{i,0}}\Psi = 0, \mathcal{H}\Psi = 0, \mathcal{X}^i\Psi = 0, (2.8)$$

instead of operator equations. While the basic commutation relations of the canonical variables read

$$[\alpha, \pi'] = i\delta(\vec{r} - \vec{r}'), \left[\beta_i, \pi^{j'}\right] = i\delta^{j'}_i, [\gamma_{ij}, \pi^{k'l'}] = i\delta^{k'l'}_{ij}. (2.9)$$

II. Graviton — Massless or Massive?

For almost a century, the theory of general relativity [17] has been widely accepted as the correct theory in describing gravitational force [18–20] by experiments of gravitationally bound astronomical systems, e.g. solar system, binary pulsars, coalescing binary black holes, and so on. In quantum field theory [21], the fundamental forces are mediated by the exchanges of corresponding particles. Graviton is the hypothetical elementary particle that mediates the force of gravitation in the framework of quantum theory of gravity. According to Einstein general relativity, the gravitational force propagates at the speed of light. Therefore, the graviton must be a spin-2 tensor boson and is expected to be massless. Massless particles are characterized by how they transform under rotations transverse to their directions of motion. The transformation rule for bosons is characterized by a non-negative integer helicity. Any sort of interaction terms of helicity-0 massless particles carried by scalar field is consistent with Lorentz invariance. For positive helicities, the field must carry gauge symmetry if we write interactions with manifest Lorentz symmetry and locality. The consistent self-interactions of helicity-1 massless particles are the non-

Abelian gauge theories. The required gauge symmetry of helicity-2 particle carried by tensor field is the linearized general coordinate invariance. Moreover, general relativity, as a quantum theory, must be treated as an effective field theory valid at energies up to a cutoff at the Planck energy, beyond which some unknown high-energy effects will correct the Einstein-Hilbert action.

It is well known that the supernova data [22, 23] indicate that the universe is accelerating in its expansion. From general relativity, there must be dark energy density of 10^{-29} g/cm^3, which is simplest interpreted as a constant term in the Einstein-Hilbert action and gives a small vacuum energy of 10^{-65}. Whereas arguments from QFT suggest a much larger value, up to the order of unity [24]. It is therefore tempting to modify general relativity in the infrared, by adding additional scalar degrees of freedom (d.o.f.), which producing the accelerating universe from nothing [25], instead of a dark energy component. That is the massive gravity [26], which is an extension of general relativity by simply adding a mass term to the Einstein-Hilbert action [27]. As a spin-2 theory, a healthy theory of massive gravity should have 5 d.o.f.. However, many theories of massive gravity tend to suffer from an additional ghost d.o.f.. Recent years, more efforts were made to realize a ghost-free massive gravity, by adding some more general polynomial terms onto the massive term in the action, which have achieved acceptable results [25, 26].

Many experiments and observations have been dedicated to the constraints for graviton mass [28], such as experiments in solar system, cosmological observations, and direct and indirect gravitational-wave detections from inspiralling and coalescing compact binaries. For the graviton with a mass m_g, the Newtonian gravitational potential $\sim\frac{1}{r}$ of a static point-like source is changed to a Yukawa one, with an exponential suppression $e^{-m_g r}$, from which we would probe length scales associated with the Compton wavelength of the mass to find large deviations from Newtonian potential, look for very small deviations to the gravitational force at distances less than the Compton wavelength, and thus constrain the graviton mass. By neglecting the relativistic corrections, the best and most rigorous model-independent bound on the graviton mass in solar systems comes from the planet Mars yields $m_g < 7.2 \times 10^{-23}$ eV at 2σ level [29]. With the recent direct detections of gravitational waves (GWs), e.g. GW150914 [30] and GW151226 [31], a more careful calculation for GW150914 waveform by aLIGO gives $m_g < 1.2 \times 10^{-22}$ eV at 90% confidence [32], which is largely model-independent and mainly relies on the dispersion relation the helicity-2 modes of the massive graviton $E^2 = \vec{k}^2 + m_g{}^2$. Some indirect GW detection with pulsar timing data of the periodic pulses of binary pulsars from PSR B1913+16 and PSR B1534+12 yield the bound $m_g < 7.6 \times 10^{-20}$ eV [33] at 90% confidence, by considering the induced sizable changes for the period due to their GW emissions. While an array of 300 spaced millisecond pulsars with 100 ns timing accuracy a 10 yr observation would bound the graviton mass $m_g < 3 \times 10^{-23}$ eV [34], by analyzing the angular correlation between the timing residual of pulsar pairs modified by a graviton mass. Smaller bounds on graviton mass were given from the cosmological observations [28].

III. Scalarizations in NS Binaries

From recent detected GW events in black hole (BH) [30, 31] and NS binaries [35], its clear that the Einsteins general relativity has been so far a sound theory in describing gravitational radiation during the phase of merger and ringdown in coalescing compact binaries, whose dynamics locates in the strong-field regime. However, several observations indicated an excess orbital decay in the Hulse-Taylor system, PSR 1913+16, predicted by general relativistic quadrupole formula [36, 37]. The long baseline of precise timing observations for PSR J1738+0333 [38] have indicated an excess orbital decay of $+2.0^{+3.7}_{-3.6}$ fs/s. Both systems are wide inspirialing binary pulsars, in which the field equations are always solved in weak field limits. The Lunar Laser Ranging experiments give the limit on dipole radiation of $\dot{P}^{\rm dipole}=1.9^{+3.8}_{-3.7}$ fs/s, which directly translates to a dipole radiation constraint on the deviations from the quadruple formula.

It was proposed that a nontrivial scalar configuration comes about in strong-field regime [39]. By making an analogy with the spontaneous magnetization of ferromagnets below the Curie temperature, a NS, with a compactness of $\frac{Gm_{NS}}{R_{NS}}$ (m_{NS} and R_{NS} are the mass and radius of NS, respectively. G is the Newtonian gravitational constant.) above a given critical value, will exhibit a nontrivial configuration, and a scalar field settles in the interior [40], i.e. a spontaneous scalarization occurs for the NS. The NS-WD binaries usually contain a massive recycled NS [41-43], owing to the recycling process [44], which thus more tends to undergo a spontaneous scalarization. It was indicated that NS in binary pulsar, with a mass of 1.4 $M_\odot$, would develop a strong scalar charge even in absence of external scalar solicitation for strong couplings and with vanishing asymptotic value [40]. The spontaneously scalarized component modifies the exterior space-time and produces external scalar field φ_{ss} in its vicinity, which contributes to a scalar asymptotic solution. In the meantime, a scalarization of NS suffers from a change of compactness [45], which enhances the gravitational interaction with its companion. As a result, the companion star is also scalarized, which is assigned to be an induced scalarization [46], and the other external scalar field φ_{is} subsequently appears around the secondly scalarized component.

The two external scalar fields, φ_{ss} and φ_{is} around the spontaneously scalarized NS "ss" and the induced scalarized companion star"is", dynamically interplay with each other, governed by the following relations [46]

$$^{(n+1)}\varphi_{ss} = {}^{(0)}\varphi_{ss} + \frac{^{(n)}\varphi_{is}}{r}, (2.10)$$

$$^{(n+1)}\varphi_{is} = {}^{(0)}\varphi_{is} + \frac{^{(n)}\varphi_{ss}}{r}. (2.11)$$

Here, $^{(n)}\varphi_{ss}$ and $^{(n)}\varphi_{is}$ represent the nth $(n \geq 0)$ induced external scalar fields around the scalarized components "ss" and "is", respectively. "r" denotes the distance from the center of the binary. The feedback mechanism described by Eq. (2.10) and Eq. (2.11) results in an iteratively induced scalarization of two components, which enhances the strength of external scalar fields, as well as the gravitational interaction between two scalarized stars. Accordingly, the Newtonian gravitational interaction of the binary is modified according to [40]

$$V_{int} = -\frac{Gm_{ss}m_{is}}{R_{ss-is}} - \frac{G\omega_{ss}\omega_{is}}{R_{ss-is}}, (2.12)$$

where m_{ss} and m_{is} represent the masses of the spontaneously scalarized NS and the induced scalarized companion, ω_{ss} and ω_{is} denote the scalar charges of corresponding components with the definition of $\omega_{ss,is} = -\partial \ln m_{ss,is}(\varphi_{ss,is}) / \partial\varphi_{ss,is}$ [47], and R_{ss-is} is the orbital separation of the binary. The local Newtonian gravitational constant is accordingly modified as an effective one,

$$G_{eff} = G\,(1 + \omega_{ss}\omega_{is} + \cdots), (2.13)$$

The second term in the bracket, $\omega_{ss}\omega_{is}$ describes the 1- and 2-order post-Newtonian corrections to the dynamics of the binary systems, and the "$\cdots$" denotes the terms of dissipative corrections to the Newtonian dynamics that accounts of the backreaction of GW emission. Either the continuous enhancement of external scalar fields or the different scalar charges carried by two components that sources an emission of dipolar gravitational scalar radiation will contribute to a gravitational scalar counterpart field ϕ in an inspiraling NS binary. We assign the inspiraling NS binaries with both quadruple tensor gravitational wave radiations and dipolar gravitational scalar counterparts to be the dynamically scalarized systems. The associated process is referred to the dynamical scalarization of NS binaries.

It was shown that "spontaneous scalarization" leads to very significant deviations from Einstein's general relativity in conditions involving binary-pulsar systems [45], which do not necessarily vanish when the weak-field scalar coupling tends to zero. The non-perturbative strong-field deviations away from general relativity due to the appearance of scalar fields, measured by a dimensional scalar coupling factor [45], could have a significant impact on the emission of GWs in NS systems [39]. The equations of motion for scalarized NSs binary systems have been modified, which produce dipolar gravitational scalar counterparts of gravitational tensor waves, depending on the coupling strength between scalar fields and the star matter [46]. As a consequence, the dynamics of scalarized inspiraling NS binary is encoded not only by the gravitational tensor metric $g_{\mu\nu}$, but also by a gravitational scalar counterpart field, which naturally renders the scalar-tensor theory [47] of gravity to be the alternative theory to Einstein's general relativity describing the scalarized binary systems.

Please note that in our discussion, we just consider the systems with relatively wide separation of 10^9 m, which will evolve about 10^9 years before coalescence and merger. The reason why we don't consider the coalescing systems is that the scalarization cannot occur during the merger phase because of the very short duration of less than one second for BH binaries and of less than one hundred seconds for NS systems.

IV. Massive Gravitational Quanta in DNS Binaries

1. Massive gravitational scalar background field in DNS systems

The external scalar fields, φ_{ss} and φ_{is}, around two components in a scalarized NS binaries, dynamically interact with each other following Eqs. (3.1.1) and (3.1.2). The feedback effects contribute to a continual enhancement of scalar configurations inside two components, as well as the external scalar fields. As a consequence, a convergence of $^{(n)}\varphi_{ss}$ and $^{(n)}\varphi_{is}$ occurs, which produces a gravitational scalar background field ϕ_B. Therefore, the binary system immerses in the gravitational scalar background field ϕ_B, and the dynamics of the DNS binary deviates from Einstein's general relativity, which has an influence on its orbital evolution [47-49].

Because of a very approximate compactness of two NSs [41] in a DNS binary, we can neglect the effects of differences in couplings between scalar field and the NS matter. Therefore, the scalar-tensor action that describes the scalarized DNS binary can be written as,

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{pl}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\,\partial_\mu\phi_B\,\partial_\nu\phi_B - V_{DNS}(\phi_B)\right). (2.14)$$

Here, $M_{pl} = \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck constant. $\mathcal{R}$ and g are the Ricci scalar and the determinant of the gravitational tensor metric $g_{\mu\nu}$, respectively. $V_{DNS}(\phi_B)$ is the gravitational scalar potential of DNS, which consists of the dynamical coupling of ϕ_B to $\varphi_{ss,is}$ and a self-coupling term of ϕ_B,

$$V_{DNS}(\phi_B) = \frac{\alpha}{2}\varphi_{ss}\varphi_{is}\phi_B^2 + \frac{\lambda}{4}\phi_B^4, (2.15)$$

Here, $\alpha \equiv -M_{pl}\frac{d\log m_{ss,is}}{d\phi_B}$ is a dimensionless coupling constant and characterizes the coupling strength between ϕ_B and matters in the scalarized stars, whose value depends on the compactness of stars consisting of the binary [39, 40, 47]. λ is the self-coupling constant of gravitational scalar counterpart field, which is roughly of the order of unity.

The iterative interplay and convergence of ${}^{(n)}\varphi_{ss,is}$ perturb ϕ_B and cause small gravitational scalar background fluctuations σ ($\sigma \ll \phi_B$). The background fluctuating field also has effects on both the gravitational tensor metric and the gravitational background scalar field, via an exponential transformation $e^{\lambda\sigma}$, which follows the couplings [39, 45, 46],

$$g_{\mu\nu}^* = e^{-2\lambda\sigma}g_{\mu\nu}, \sqrt{-g^*} = e^{4\lambda\sigma}\sqrt{-g}, (2.16)$$

$$\phi^* = e^{-\lambda\sigma}\phi. (2.17)$$

where $g_{\mu\nu}^*$ and g^* are transformed gravitational tensor metric and its determinant. ϕ_B^* is the transformed gravitational background scalar field. By expanding the transformed metric $g_{\mu\nu}^*$ about a Minkowski background in terms of Eq. (2.16), we express them as

$$g_{\mu\nu}^* = \eta_{\mu\nu} + h_{\mu\nu}^*, h_{\mu\nu}^* = h_{\mu\nu} + 2\eta_{\mu\nu}\lambda\sigma, (2.18)$$

where $|h_{\mu\nu}|, |h_{\mu\nu}^*| \ll 1$. The Eq. (2.16) remains unchanged. Under the transformation of σ, we, using Eq. (2.16) and Eq. (2.17), find that the kinetic term in action (2.14) is transformed into a canonical kinetic term,

$$\frac{1}{2}\sqrt{-g}g^{\mu\nu}\,\partial_\mu\phi_B\,\partial_\nu\phi_B = \frac{1}{2}\sqrt{-g^*}g^{\mu\nu *}\mathcal{D}_\mu\phi_B^*\mathcal{D}_\nu\phi_B^*, (2.19)$$

$$\mathcal{D}_\mu \equiv \partial_\mu + \lambda\,\partial_\mu\sigma, (2.20)$$

which is scale-invariant. The transformed action then reads

$$S^* = \int d^4x\sqrt{-g^*}\left(\frac{M_{pl}^2}{2}\mathcal{R}^* - \frac{1}{2}g^{\mu\nu *}\mathcal{D}_\mu\phi_B^*\mathcal{D}_\nu\phi_B^* - V_{DNS}(\phi_B^*)\right). (2.21)$$

In the process of conformal transformation, the solutions of external scalar fields $\varphi_{ss,is}$ with mass dimensions [40] involves a dimensional constant μ with the Planck mass scale, which appears in the transformed gravitational scalar potential $V_{DNS}(\phi_B^*)$,

$$V_{DNS}(\phi_B^*) = \frac{\alpha}{2}\mu^2\phi_B^{*2} + \frac{\lambda}{4}\phi_B^{*4}. (2.22)$$

The Planck-scale constant $\mu = \sqrt{\frac{1}{8\pi G_{eff}}}$ here appears to be related to the scalar charges of the scalarized NSs via the effective gravitational constant G_{eff} according to Eq. (2.13) [47]. It is the appearance of the mass-dimensional constant μ that is responsible for a process of spontaneous breaking of symmetry, which allows us to apply the similar recipe to the Higgs mechanism in the standard model. Thus the gravitational scalar background field becomes a massive one.

Actual NSs observed in DNS binaries, with important deviations from general relativity in strong-field regime, would develop strong scalar charges in the absence of an external scalar field for enough negative values of α, i.e. $\alpha < 0$ [39, 40, 46]. The self-coupling constant λ is of the order of unity, i.e. $\lambda > 0$. By considering that the interplay between φ_{ss} and φ_{is} is long-range force, the behavior of transformed gravitational background scalar field ϕ_B^* near spatial infinity endows it with a vacuum expectation value (VEV) $v_{\phi_B^*}$,

$$v_{\phi_B^*}^2 = -\frac{\alpha\mu^2}{2\lambda}, (2.23)$$

which is obtained from the condition $\frac{dV_{DNS}(\phi_B^*)}{d\phi_B^*}|_{(\phi_B^*)_{min}} = 0$. Therefore, the gravitational scalar background field ϕ_B^* is a combination of its VEV $v_{\phi_B^*}$ and the fluctuating field of the spatial infinity approximate value. Substituting the VEV (2.23) into the Lagrangian of ϕ_B^* extracted from Eq. (2.21), we get the mass of ϕ_B^* [50],

$$(m_s^{DNS})^2 = -\alpha\mu^2. (2.24)$$

It was proven that non-perturbative strong-gravitational-field effects developed in NSs for a dimensionless coupling constant $\alpha \lesssim -4$, which causes order-of-unity deviations from general relativity [39]. The general properties of binary systems consisting of scalarized NSs can be described by $\alpha \gtrsim -4.5$, because of binary-pulsar measurements [38, 43, 51]. For $\alpha \lesssim -5$, NSs in binary pulsar, with mass of 1.4 $M_{\odot}$, would develop strong scalar charges even in absence of external scalar solicitation, and a more negative value of α corresponds to a less compact NS [40]. Most of the measured more massive NS in detected DNS systems have masses of ~1.3-1.44 $M_{\odot}$ [41]. Consequently, the coupling constant locates in a range of $\alpha = -5 \sim -6$ within a quadratic coupling model described in Eq. (2.22) [40]. The scalar charges mildly vary with the compactness of NSs [46] and will be ~1 only in the last stages of the evolution of NS binaries or close transient encounters. For NSs in 9 so-far detected DNS systems, the scalar charges are around 0.2 within solar-system bound [52] in the Fierz-Jordan-Brans-Dicke (FJBD) theory, by considering its dependence on the ``sensitivities'' s~0.2 [53, 54]. Accordingly, the mass of gravitational scalar background field is of the order of Planck mass scale [45], which plays the role of the gravitational scalar counterpart of gravitational waves in scalarized inspiraling DNS binary. During this process, the gravitational background scalar fluctuation field σ is the only massless field, which plays the role of Higgs-like field.

2. Mass of gravitons from DNS

Variation of the transformed action (2.21), with respect to the transformed metric (2.16) and the transformed scalar background field (2.17), yields the following e.o.m. in vacuum,

$$(\mathcal{R}_{\mu\nu}^* - \frac{1}{2}g_{\mu\nu}^*\mathcal{R}^*)M_{pl}^2 = \partial_\mu\phi_B^*\,\partial_\nu\phi_B^* - \frac{1}{2}g^{\mu\nu*}(\partial_\alpha\phi_B^*)^2 + \frac{\alpha}{2}\mu^2 g^{\mu\nu*}\phi_B^{*2} + \frac{\lambda}{4}g^{\mu\nu*}\phi_B^{*4}, (2.25)$$

$$\Box_{g^*}g^{\mu\nu*}\phi_B^* = \alpha\mu^2 g^{\mu\nu*}\phi_B^* + \lambda g^{\mu\nu*}\phi_B^{*3}, (2.26)$$

where $\Box_{g^*}$ is the curved space d'Alembertian that is defined by $\Box_{g^*}=\sqrt{-g^*}\,\partial_\upsilon\left(\sqrt{-g^*}g_{\mu\nu}^*\,\partial_\mu\right)$.

Let us write the gravitational scalar background field ϕ_B^* as the combination of its VEV $v_{\phi_B^*}$ and a fluctuating field $\widetilde{\phi}_B^*$ of the spatial infinity approximate value of ϕ_B^*, according to

$$\phi_B^* = v_{\phi_B^*} + \widetilde{\phi}_B^*. (2.27)$$

Then we expand the transformed scalar potential $V_{DNS}(\phi_B^*)$ of Eq. (2.22) in a Taylor series about the VEV of ϕ_B^* (2.23),

$$V_{DNS}(\phi_B^*) = V(v_{\phi_B^*}) + V(v_{\phi_B^*})'\ \widetilde{\phi}_B^* + \frac{1}{2}\ V(v_{\phi_B^*})''\widetilde{\phi}_B^{*2} + \cdots. (2.28)$$

Considering the weak field scalar perturbation of ϕ_B^* in Eq. (2.27), we expand the field equations in weak field limit,

$$(\mathcal{R}_{\mu\nu}^* - \frac{1}{2}g_{\mu\nu}^*\mathcal{R}^*)M_{pl}^2 = \partial_\mu\phi_B^*\,\partial_\nu\phi_B^* - \frac{1}{2}g^{\mu\nu*}(\partial_\alpha\phi_B^*)^2 + \frac{1}{2}(m_s^{DNS})^2(\phi_B^* - v_{\phi_B^*})^2 g^{\mu\nu*}, (2.29)$$

$$(\Box_{g^*} - (m_s^{DNS})^2)\phi_B^* = (m_s^{DNS})^2 v_{\phi_B^*}. (2.30)$$

Here, we use the expansion Eq. (2.28) and just consider the leading-order terms. The expanded field equations that are consistent at all orders in $(\frac{v}{c})^n$ is required. We solve the e.o.m. of massive scalar field Eq. (2.30) in a static spherically symmetric configuration, which yields an exterior solution of ϕ_B^* far from the DNS system,

$$\phi_B^* = v_{\phi_B^*} + v_{\phi_B^*}G_{eff}\frac{M_r}{r}e^{-m_s^{DNS}r}, (2.31)$$

where $M_r = \frac{m_{ss}m_{is}}{m_{ss}+m_{is}}$ is the reduced mass of DNS. Therefore, the mass m_s^{DNS} is the ordinary mass parameter in the Klein-Gordon equation (2.30) of ϕ_B^*, arising from the Higgs-like potential (2.22). In the meanwhile, the mass of ϕ_B^* plays precisely a role in the Yukawa-like correction $\sim e^{-m_s^{DNS}r}$ to the standard Newtonian form of gravitational potential $\sim\frac{GM_r}{r}$ in the scalarized DNS binary system.

We expand the left-hand side of Eq. (2.29) in weak field limit, by using the weak field perturbations of tensor metric Eq. (2.18) and the small perturbative coupling $\theta^{\mu\nu} = h^{\mu\nu*} - \frac{1}{2}h^*\eta^{\mu\nu} - \frac{\tilde{\phi}^*_B}{v_{\phi^*_B}}\eta^{\mu\nu}$, with $h^* = \eta^{\mu\nu}h^*_{\mu\nu}$. Imposing the harmonic gauge $\partial^\nu\left(h^*_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^*\right) = 0$ and $\partial^\nu\theta_{\mu\nu} = 0$ and neglecting the higher-order terms, we rewrite the e.o.m. of gravitons as

$$-\frac{M^2_{pl}}{2}\Box_\eta\bar{h}^*_{\mu\nu} - \frac{M^2_{pl}}{2}\Box_\eta\theta_{\mu\nu} - M^2_{pl}\eta^{\mu\nu}\Box_\eta(\frac{\tilde{\phi}^*_B}{v_{\phi^*_B}}) = \partial_\mu\phi^*_B\,\partial_\nu\phi^*_B - \frac{1}{2}\eta^{\mu\nu}(\partial_\alpha\phi^*_B)^2 + \frac{1}{2}\eta^{\mu\nu}(m^{DNS}_s)^2\left(\phi^*_B - v_{\phi^*_B}\right)^2, (2.32)$$

where $\bar{h}^*_{\mu\nu} = h^*_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^*$ and the flat-space d'Alembertian $\Box_\eta = \eta^{\mu\nu}\,\partial_\mu\,\partial_\upsilon$. Let us study the scalar-mediated propagations of gravitons outside the scalarized DNS binary. Substituting the solution of ϕ^*_B (2.31) into the e.o.m. of gravitons (3.2.19), we then write the wave solution of gravitons (2.32) as [55]

$$\bar{h}^*_{\mu\nu} = \int d\omega \int \frac{d^3\vec{k}}{(2\pi)^3}Ae^{i(\vec{k}\cdot\vec{r}-\omega t)} \cdot \sum_{n=-\infty}^{\infty}\phi^{*n}_B(\vec{r})e^{in\sigma}. (2.33)$$

Here, "A" denotes the amplitude of tensor gravitational waves radiated from the orbital decaying DNS. $\sum_{n=-\infty}^{\infty}\phi^{*n}_B(\vec{r})e^{in\sigma}$ is the Fourier expansion of the gravitational scalar field ϕ^*_B, with the gravitational background scalar fluctuation fields σ, which is converged by the nth induced scalar background $^{(n)}\varphi_{ss,is}$ of two NSs. ϕ^{*n}_B is in the form of exterior solutions (2.31).

The Klein-Gordon equation of gravitons (2.33) therefore reads

$$\left(\Box - \frac{(m^{DNS}_s)^2 v_{\phi^*_B}G_{eff}}{\tilde{R}}\right)\bar{h}^*_{\mu\nu} = 0, (2.34)$$

where $\tilde{R}$ is the semi-major axis of the elliptical DNS binary system. By defining [56]

$$(m^{DNS}_g)^2 = \frac{(m^{DNS}_s)^2 v_{\phi^*_B}G_{eff}}{\tilde{R}}, (2.35)$$

we find that the gravitons acquire a mass of m_g. In the scenario, the propagations of massive gravitational background scalar field ϕ^*_B modifies the Newtonian potential of DNS and contributes to a Yukawa-like one $\sim\frac{e^{-m^{DNS}_s r}}{r}$. The Yukawa-corrected potential has influence on the propagations of tensor gravitational waves, via the entrance of massive scalar component and the scalar charges into the e.o.m. of gravitational waves. As a consequence, the massive gravitational background scalar field and two external scalar fields, manifested as scalar charges in the effective gravitational constant, have been eaten by the massless gravitons, remaining healthy massive gravitons with five d.o.f..

3. Applications to detected DNS binaries in our galaxy

According to the expressions of (2.23) and (2.24), we rewrite the mass of gravitons, Eq. (2.35), as

$$(m^{DNS}_g)^2 = (-\alpha)^{\frac{2}{3}}\mu^3(2\lambda)^{-1/2}G_{eff}\tilde{R}^{-1}. (2.36)$$

It is found that the mass of graviton depends on three quantities, i.e. the separation of DNS system represented by $\tilde{R}$, the coupling strength between gravitational scalar field and the NSs that is characterized by the dimensionless coupling constant α, and the scalar charges that is related to both the Planck scale constant μ and the effective gravitational constant G_{eff}. Consequently, the mass of gravitons rests with the intrinsic properties of DNS, i.e. the separation of the binaries and the compactness of two NSs, which is not a certain value and mildly variable.

Accordingly, the gravitons radiated in a DNS binary with a semi-separation of 10^9 m have masses of the order of $\sim 10^{-23}$ eV, which mildly vary with the compactness of NSs and the separation between them (TABLE I). The gravitons radiate from closer DNS binaries possess a higher mass, which corresponds with current simulations that higher-frequency gravitational waves come from the closer binaries.

Table I:-

Source	m_{ss} ($M_\odot$)	m_{is} ($M_\odot$)	P_{orb} (Days)	Ecc	m^{DNS}_g 10^{-23}eV

PSR J1811-1736	$1.5^{+0.12}_{-0.4}$	$1.06^{+0.45}_{-0.1}$	18.8	0.828	0.106
PSR J1829+2456	$1.35^{+0.46}_{-0.15}$	$1.15^{+0.1}_{-0.25}$	1.176	0.139	0.711
PSR J1913+16	1.44 ± 0.0006	1.39 ± 0.0006	0.323	0.617	0.972
PSR B1534+12	1.35 ± 0.0020	1.33 ± 0.0020	0.421	0.274	1.292
PSR B2127+11C	1.36 ± 0.080	1.35 ± 0.080	0.335	0.681	1.408
PSR J1756-2251	$1.40^{+0.04}_{-0.06}$	$1.18^{+0.06}_{-0.04}$	0.320	0.181	1.574
PSR J1906+0746	1.37	1.25	0.166	0.085	2.427
PSR J0737-3039	1.34 ± 0.010	1.25 ± 0.010	0.102	0.088	3.356

Notes. m_{ss} and m_{is} are the masses of spontaneous scalarized NS and induced scalarized NS in units of solar mass, respectively. P_{orb} denotes the orbital period in units of days. “Ecc” is the eccentricity for each binary system.

V. Massive Gravitational Quanta in NS-WD Binaries

1. Massive gravitational scalar radiation field in NS-WD

It is well known that NS is a more compact object than WD. Consequently, the strength of couplings between the scalar configurations inside stars and the NS/WD matter are different. A distinct dependence of masses on the scalar fields for NS and WD actually sources an emission of dipolar gravitational scalar radiation in a post-Newtonian inspiraling scalarized binary [46], in addition to the quadruple tensor gravitational waves. Accordingly, the dynamics of a scalarized inspiraling NS-WD system is governed by a gravitational scalar radiated field ϕ_r, together with the gravitational tensor metric $g_{\mu\nu}$. The scalar charge of a scalarized NS-WD binary can be extracted from the behavior of the gravitational scalar radiated field near spatial infinity [47], i.e.

$$\phi_r = \phi_r^0 + \frac{\phi_r^1}{r} + \mathcal{O}\left(\frac{1}{r^2}\right), (2.37)$$

where the iterative interplay and convergence of the external scalar fields φ_{ss} and φ_{is} around NS and WD are considered, and ϕ_r^0 is the asymptotic value of the gravitational scalar radiated field in spatial infinity. Accordingly, the dynamics of an inspiraling scalarized NS-WD binary system, suffering from the post-Newtonian corrections, is described by the following scalar-tensor action,

$$S = \int d^4x\,\sqrt{-g}\left(\frac{\mathcal{R}}{16\pi G} - \frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi_r\,\partial_\nu\phi_r - V_{NS-WD}(\phi_r)\right) + \sum\nolimits_n \int_{\gamma_n} ds\; m_{(ss,is)_n}(\phi_r)\,. (2.38)$$

The gravitational scalar potential of NS-WD binary $V_{NS-WD}(\phi_r)$ results from two interactions, i.e. the self-interactions of ϕ_r and the interactions between ϕ_r and matter fields of NS and WD. The gravitational scalar radiated field is associated with the non-perturbative strong-field effects [39], which contributes to a potential of the runaway form [57] that satisfies $\lim_{\phi_r\to\infty} V_{NS-WD}(\phi_r) \to 0$, $\lim\limits_{\phi_r\to\infty} \frac{V_{NS-WD}(\phi_r)'}{V_{NS-WD}(\phi_r)} \to 0$, $\lim\limits_{\phi_r\to\infty} \frac{V_{NS-WD}(\phi_r)''}{V_{NS-WD}(\phi_r)'} \to 0$, ..., as well as $\lim_{\phi_r\to 0} V_{NS-WD}(\phi_r) \to \infty$, $\lim\limits_{\phi_r\to 0} \frac{V_{NS-WD}(\phi_r)'}{V_{NS-WD}(\phi_r)} \to \infty$, $\lim\limits_{\phi_r\to 0} \frac{V_{NS-WD}(\phi_r)''}{V_{NS-WD}(\phi_r)'} \to \infty$, ... ($V_{NS-WD}(\phi_r)' \equiv dV_{NS-WD}(\phi_r)/d\phi_r$, and $V_{NS-WD}(\phi_r)'' \equiv d^2V_{NS-WD}(\phi_r)/d{\phi_r}^2$, etc.). Thus, the self-interactions of gravitational scalar radiated field, whose behavior is described by Eq. (2.37), lead to a monotonically decreasing potential,

$$V_{\phi_r} = \frac{\xi^5}{\phi_r}, (2.39)$$

where ξ has the unit of mass. The NS/WD matter interacts directly with the gravitational scalar radiated field ϕ_r through a conformal coupling of the form $e^{-\alpha_{ss,is}\phi_r/\mu}$. The values of $\alpha_{ss,is}$ are also usually negative for WDs [58]. So the exponential coupling function is an increasing function of ϕ_r. The combined effects of self-interactions of ϕ_r described by Eq. (2.39) and the conformal coupling give us the form of the scalar potential $V_{NS-WD}(\phi_r)$ in Eq. (2.38),

$$V_{NS-WD}(\phi_r) = \frac{\xi^5}{\phi_r} + \varepsilon_{\varphi_{ss,is}} e^{-\frac{\alpha_{ss,is}\phi_r}{\mu}}. (2.40)$$

It can be found that $V_{NS-WD}(\phi_r)$ is an explicit function of energy density $\varepsilon_{\varphi_{ss,is}}$ of the external scalar fields $\varphi_{ss,is}$, which depends on the masses of the stars (a function of the density for each star $\rho_{ss,is}$) and the coupling strength between interior scalar configuration and matter components of NS/WD [40].

The summation part of Eq. (2.38) describes the action of matter components making up the NS and the WD. In the sum over "n" we give the world line action for the number of any species of matter and particles consisting in the NS and the WD and we use γ_n to represent the integral of the matter action along world line. The couplings of matter components inside the stars to the scalar field arise from the dependence of the masses $m_{ss,is}$ on ϕ_r. The

NS/WD matter couples to the gravitational tensor metric $g_{\mu\nu}$ via the conformal transformation $e^{-\alpha_{ss,is}\phi_r/\mu}$, according to the rescaling relation,

$$g^*_{\mu\nu} = e^{-2\alpha_{ss,is}\phi_r/\mu} g_{\mu\nu}. (2.41)$$

The combined gravitational scalar potential $V_{NS-WD}(\phi_r)$ Eq. (2.40) in NS-WD system, consisting of a monotonically decreasing potential (2.39) and a monotonically increasing interaction $e^{-\alpha_{ss,is}\phi_r/\mu}$, actually displays a minimum. By minimizing the differentiation of the gravitational scalar potential with respect to ϕ_r, i.e.

$$V_{NS-WD}(\phi_r)' - \sum_{ss,is} \frac{\alpha_{ss,is}}{\mu} \varepsilon_{\varphi_{ss,is}} e^{-\frac{\alpha_{ss,is}\phi_r}{\mu}} = 0, (2.42)$$

we can get the minimum value of ϕ_r at the minimum potential ϕ_r^{min}. Around this minimum, the gravitational scalar radiated field acquires an effective mass, which is obtained by evaluating the second derivative of the potential at ϕ_r^{min},

$$(m_s^{NS-WD})^2 = V_{NS-WD}(\phi_r)''|_{\phi_r^{min}} + \sum_{ss,is} \frac{{\alpha_{ss,is}}^2}{\mu^2} \varepsilon_{\varphi_{ss,is}} e^{-\frac{\alpha_{ss,is}\phi_r}{\mu}}|_{\phi_r^{min}}. (2.43)$$

Equations (2.42) and (2.43) imply that both the local value of the gravitational scalar radiated field ϕ_r^{min} and the mass of scalar counterpart depend on the local energy density of external scalar fields produced by two scalarized components. It can be found, from Eq. (2.43), that the gravitational scalar radiated field become more massive in a higher $\varepsilon_{\varphi_{ss,is}}$ environment.

The gravitational scalar interaction between NS and WD, mediated by a massive gravitational scalar radiated field, typically acquires an exponential Yukawa suppression, which results in a finite range of Yukawa type of potential energy,

$$U(r) = -2\alpha_{ss}\alpha_{is} \frac{Gm_{ss}m_{is}}{r} e^{-m_s^{NS-WD} r}. (2.44)$$

Here the product $2\alpha_{ss}\alpha_{is}$ is referred to as the interaction strength. The mass of gravitational radiated scalar field is characterized by the inverse of the range of a Yukawa potential (2.44). Most of the NS-WD binaries have very small orbital eccentricity of $\sim 10^{-5} - 10^{-6}$ [41], i.e. approximate circular orbits. Accordingly, the scalarized NS and WD orbit with each other and form a ring-configuration-orbit on the binary plane. The distance "a" from the center of the NS-WD binary plane to the outer boundary of the ring configuration corresponds to the semi-separation of an NS-WD binary, which is of the order of $\sim 10^9$ m [41], and the central thickness of the ring approximately equals the diameter of the WD, i.e. $\Delta a \sim 10^6$ m. By comparing the radius of NS and WD with the separation between them, we can get $\frac{\Delta a}{a} \sim 10^{-3} \ll 1$. Accordingly, the orbit of NS-WD binary can be assigned to be a thin-ring orbit. The gravitational scalar interaction between NS and WD are therefore screened in the thin ring configuration, with an interaction range of the same order of the orbital width $\Delta a \sim 10^6$ m. Consequently, the corresponding mass of gravitational radiated scalar field in NS-WD binary is estimated as $m_s^{NS-WD} \equiv \Delta a^{-1} \sim 10^{-21}$ eV.

2. Solutions of gravitational scalar radiation field

The e.o.m. of the gravitational scalar radiation field can be derived by varying the action (2.38) with respect to ϕ_r, which reads

$$\Box\phi_r = V_{NS-WD}(\phi_r)' - \sum_{ss,is} \frac{\alpha_{ss,is}}{\mu} \varepsilon_{\varphi_{ss,is}} e^{-\frac{\alpha_{ss,is}\phi_r}{\mu}}. (2.45)$$

In the framework of thin-orbit-ring configuration for scalarized NS-WD binary system, with spherically symmetric semi-sphere outer boundary, we consider an infinitesimal volume element within the orbital ring. The gravitational scalar field can be solved in a static, spherically symmetric regime. Accordingly, the e.o.m. (2.45) is reduced to

$$d^2\phi_r/dr'^2 = (1/r')(d\phi_r/dr') = V_{NS-WD}(\phi_r)' + \sum_{ss,is} \frac{\alpha_{ss,is}}{\mu} \varepsilon_{\varphi_{ss,is}} e^{-\frac{\alpha_{ss,is}\phi_r}{\mu}}, (2.46)$$

where the coordinates r' denotes the distance from the center of orbital ring for the binary. This differential equation (3.3.10) is subject to the following boundary conditions,

$$\phi_r = \phi_{r_{out}}, \quad \text{as } r' = \Delta a/2;$$
$$\phi_r = \phi_{r_{in}}, \quad \text{as } 0 \le r' \le \Delta a/2;$$
$$\phi_r = \phi_{r_0}, \quad \text{as } r' \to \infty. (2.47)$$

Here, the thickness of the orbital ring Δa is related to $\phi_{r_{out}}$, $\phi_{r_{in}}$, and the Newtonian potential of the binary system $\Phi_c = M_c/8\pi\mu^2 a$, which is given by [57]

$$\Delta a/a = (\phi_{r_{out}} - \phi_{r_{in}})/6\sum_{ss,is}\alpha_{ss,is}\mu\Phi_c\,.\,(2.48)$$

In analogy with the electrostatic shield of an electronic conducting shell, the deposited scalar energy is screened and dominates in the ring orbit, and the scalar charges are distributed on the surface of outer boundary with the radius of $r' = \Delta a/2$. Therefore, the gravitational scalar field inside the ring $\phi_{r_{in}}$ can be considered as perturbations, i.e. $\phi_{r_{in}} \ll \phi_{r_{out}}$.

Aiming to investigate the influence of the massive gravitational scalar field on the propagations of gravitons, we are just interested in the solutions outside the system, i.e. solutions in the region of $r' > \Delta a/2$. By solving Eq. (2.46) and using the boundary conditions (2.47), we get the exact exterior solutions,

$$\phi_r(r' > \Delta a/2) = \phi_{r_{out}}\left(1 - \frac{\phi_{r_{out}} - \phi_{r_{in}}}{6\sum_{ss,is}\alpha_{ss,is}\mu\Phi_c}\right)\frac{a e^{-m_s^{NS-WD}\left(r' - \frac{\Delta a}{2}\right)}}{r'} + \phi_{r_{out}}.\,(2.49)$$

By considering that the field density contrast $\phi_{r_{in}} \ll \phi_{r_{out}}$ and in the limit of thin-ring orbit $\Delta a \ll a$, the combination of Eq. (2.48) and Eq. (2.49) gives the approximative solutions

$$\phi_r(r' > \Delta a/2) \approx \frac{\sum_{ss,is}\alpha_{ss,is}}{4\pi\mu}\frac{3\Delta a}{a}\frac{M_c e^{-m_s^{NS-WD}r'}}{r'} + \phi_{r_{out}}.\,(2.50)$$

3. Scalar-mediated mass of gravitons

Variation of the action (2.38) with respect to the metric gives us the following e.o.m.,

$$(\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R})M_{pl}^2 = \partial_\mu\phi_r\,\partial_\nu\phi_r - \frac{1}{2}g^{\mu\nu}(\partial_\alpha\phi_r)^2 + g^{\mu\nu}(\xi^5/\phi_r + \varepsilon_{\varphi_{ss,is}}e^{-\frac{\alpha_{ss,is}\phi_r}{\mu}}).\,(2.51)$$

We consider the weak-field scalar and tensor perturbations, i.e. $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\phi_r = \phi_{r_{out}} + \delta\phi_r$, as well as the small perturbative coupling $\sigma^{\mu\nu} = h^{\mu\nu} - \frac{1}{2}h\eta^{\mu\nu} - \frac{\delta\phi_r}{\phi_{r_{out}}}\eta^{\mu\nu}$. Expanding the left-hand side of Eq. (2.51) in the weak-field limits, we rewrite the e.o.m. of gravitons as

$$(\Box_\eta\bar{h}_{\mu\nu} + \frac{1}{2}\Box_\eta\sigma_{\mu\nu} + \Box_\eta\,(\delta\phi_r/\phi_{r_{out}}))M_{pl}^2 = \partial_\mu\delta\phi_r\,\partial_\nu\delta\phi_r - \frac{1}{2}\eta_{\mu\nu}(\partial_\alpha\delta\phi_r)^2 + \frac{1}{2}\eta_{\mu\nu}(m_s^{NS-WD})^2\left(\phi_r - \phi_{r_{out}}\right)^2,$$
(2.52)

where we impose the harmonic gauge conditions of $\partial^\nu\left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h\right) = 0$ and $\partial^\nu\sigma_{\mu\nu} = 0$, and the expansion of $V_{NS-WD}(\phi_r)$ in Taylor series about $\phi_{r_{out}}$ is also used.

We then substitute the exterior approximative solutions (2.50) of ϕ_r into Eq. (2.52) and follow the gauge selection described in [55]. It is evident that the motion of gravitons has the wave solutions, with the modifications resulting from the gravitational scalar radiation [59, 60],

$$\bar{h}_{\mu\nu} = \int d\omega \int \frac{d^3\vec{k}}{(2\pi)^3} A e^{i(\vec{k}\cdot\vec{r} - \omega t)} \int d\omega' e^{i(k_r' r - \omega' t)}\phi_{r_{out}}(\omega')\frac{M_r}{r}\left(1 + \sum_{ss,is}\alpha_{ss,is}e^{-m_s^{NS-WD}r}\right).\,(2.53)$$

Here, we consider the plane-wave solutions of the gravitational scalar radiation $\phi_r = \int \phi_{r_{out}}(\omega')e^{i(k_r' r - \omega' t)}d\omega'$ [61]. The quantities "ω", "$\vec{k}$", and "A" denote the frequency, the wave vector and the amplitude of tensor gravitational waves radiated from the orbital decaying NS-WD system, while those with "′" are the corresponding quantities for the gravitational scalar radiation.

Then the Klein-Gordon equation of gravitons reads

$$[\Box - (m_s^{NS-WD})^2(\phi_{r_0}/\mu\Delta a)]\bar{h}_{\mu\nu} = 0.\,(2.54)$$

Consequently, we can find that the gravitons acquire a mass, which is expressed as [62]

$$(m_g^{NS-WD})^2 = (m_s^{NS-WD})^2(\phi_{r_0}/\mu\Delta a).\,(2.55)$$

The asymptotic value of the gravitational scalar radiation field near spatial infinity ϕ_{r_0} is constrained to $\phi_{r_{max}}^0 \lesssim 10^{-3}$ in weak-field tests, for a coupling strength of about -6 [40]. In the binary-pulsar measurements [38, 43, 51], ϕ_{r_0} is constrained to very close to zero, and it is usually to be taken as $\phi_{r_0}G^{1/2} = 10^{-3} - 10^{-5}$. Because of a less compactness and relatively lower surface gravity of WD, we consider $\phi_{r_0}G^{1/2} = 10^{-3}$ in NS-WD binary. Accordingly, we estimate the gravitons radiated from NS-WD binaries can acquire a mass of the order of $\sim 10^{-23}$eV. From Eq. (2.43), the mass of gravitational scalar radiation field is a function of energy density of stars' scalar configurations and also depends on the strength of the scalar field and the scalar coupling strength. Therefore, the value of graviton mass m_g^{NS-WD} in NS-WD systems mildly varies according to the microphysics of the interior of NS/WD and intrinsic properties of the binaries.

VI. Discussions

The mass generation mechanism of gravitational quanta in both DNS and NS-WD binaries requires the scalarization of the system, which necessitates firstly the scalarization of each component. Noting that the spontaneous scalarization takes place in the interior of a NS, locating in a binary system, when its compactness $\frac{Gm}{c^2R_{NS}}$ is above a given critical threshold even in the absence of scalar sources. The subsequently induced scalarizations also occur in each companion star of the spontaneously scalarized NSs. That is to say both spontaneous scalarization and induced scalarization occur in the interior of a single star. So the components in DNS binaries and NS-WD systems undergo scalarizations via same mechanisms. However, the binding energy of NS and WD are different, which contributes to differences in the dependence of the masses of NS/WD on the scalar configurations (i.e. scalar charges). An obvious difference in the dependence of the masses on the scalar field of two components in one binary system actually sources an emission of dipolar gravitational scalar radiation [46]. As a result, the causes of gravitational scalar counterpart field are distinct in DNS and NS-WD systems.

In inspiraling DNS binaries, the two NS components possess very similar binding energy [41], and the scalar charges are very close to each other. Consequently, the dipolar gravitational radiation is negligible. With the iteratively interplay, the strengths of external scalar field around each component is enhanced, and a convergence finally occurs. As a consequence, a gravitational scalar background field appears, which plays the role of the gravitational scalar counterpart of quadruple gravitational tensor waves. In inspiraling NS-WD systems, owing to a different binding energy of NS and WD, the dependence of masses of NS/WD on scalar configurations is different. Therefore, the two components in NS-WD binary carry different scalar charges, which is responsible for a dipolar gravitational scalar radiation. Accordingly, the gravitational scalar radiated field plays the role of the gravitational scalar counterpart for quadruple gravitational tensor waves in a post-Newtonian corrected inspiraling scalarized NS-WD system. Consequently, the scalarized inspiraling DNS and NS-WD systems, suffering from gravitational scalar counterparts, dip in gravitational scalar potentials, resulting from different mechanism, which contribute to distinct physical processes. In the inspiraling scalarized DNS binaries, because of the iterative interplay of two external scalar fields, the gravitational scalar background field suffers from fluctuations. The scalar fluctuations couple to both tensor metrics and gravitational scalar background field, which transfer the couplings of scalar fields in to a Higgs-like gravitational scalar potential Eq. (2.22), with an appearance of a mass-dimensional constant. It is the appearance of Planck-scale mass-dimensional constant that is responsible for a process of spontaneous breaking of symmetry. Thus the gravitational scalar background field becomes a massive one, in which the gravitational scalar fluctuation field is the massless field and plays the role of Higgs-like field. Therefore, the mass of gravitational scalar counterpart in inspiraling scalarized DNS, expressed by Eq. (2.24), is of the order of Planck mass scale, which depends on the coupling strength between the gravitational background scalar field and NS matter. In inspiraling scalarized NS-WD binaries, the gravitational scalar potential is then consisting of a monotonically decreasing self-interaction of the gravitational scalar radiated field and a scalar-energy-density-dependent exponential increasing coupling to NS/WD matters. The non-monotonic potential displays a minimum, which contributes to a massive gravitational scalar counterpart. The reason why the gravitational scalar counterpart in NS-WD system becomes massive is that the gravitational scalar radiated field oscillates around a local minimum of the gravitational scalar potential, with high scalar energy density. By considering the Yukawa-suppression effects on an environment of high scalar energy density, we estimate the mass of dipolar gravitational scalar counterpart of quadruple tensor gravitational waves in NS-WD binaries, expressed by Eq. (2.43), as of the order of $\sim 10^{-21}$ eV, which depends on the orbital scale of the NS-WD system.

As far as the gravitons, the mass generation mechanism is that two external scalar fields and the gravitational scalar counterpart, carrying three d.o.f. in all, are eaten by the massless gravitons, with two d.o.f.. Finally, the gravitons possess five d.o.f. and become massive ones. The graviton masses have the same order of $\sim 10^{-23}$eV, which depend on the intrinsic properties of the systems.

According to Higgs mechanism in standard model, the mass generation associates with spontaneous symmetry breaking. The mass generation of gravitational quanta in NS binaries basically due to the appearance of the gravitational scalar counterpart field, which spontaneously breaks the Lorentz invariance constructed in the framework of general relativity, during the process of orbital decay. As the magnitude of deviations from general relativity depends non-linearly on the binding energy, the more massive NSs, e.g. PSRs J0348+0432 with mass of $1.97 \pm 0.04 M_{\odot}$ and J1614-2230 with mass of $2.01 \pm 0.04 M_{\odot}$, can be more promising systems used to probe the non-perturbative strong-field deviations away from general relativity, which is qualitatively very different compared to other binary-pulsar experiments. The effect is true even for DNS binaries that have small differences on their binding energies.

Covariant Treatment And Quantum Corrections To The Gravitational Potentail In Scalarized NS Binaries:-

I. Covariant Quantum Theory of Gravity

Contrary to the situation in the canonical quantum theory of gravity discussed in section 2.1, a manifestly covariant treatment for quantum theory of gravity [13, 14] was constructed by analogy with the conventional scattering-matrix theory, which lends itself to study the questions such as the scattering, pair-production, pair-annihilation, and decay of individual quanta. However, the manifest covariance in conventional scattering-matrix theory denotes covariant Lorentz invariance, which is an expression of a geometrical symmetry processed by a system. The gravity theory bases on the manifest general covariant propagators, which is accomplished by introducing a variable background metric, instead of a flat background.

The theory begins with the selection of an action functional

$$S = \int \mathcal{L} d^4 x, (3.1)$$

The Lagrangian is a function of dynamical variables and a finite number of their space-time derivatives at a single point, whose choice basically irrelevant to the development of the theory of a given field and should be determined only by the convenience, which are in practice limit drastically by various criteria, such as covariance, self-consistency of the field equations, the existence of the vacuum as a state of lowest energy, and the positive definiteness of the quantum mechanical Hilbert space.

The covariant quantum theory of gravity begins with a treatment of the propagations of small disturbances on a classical background, which plays the fundamental role as a technical instrument for probing vacuum process and as an arbitrary fiducial point for the quantum fluctuations. The transition from classical to quantum regime is made via the Poisson bracket of Peierls [63]. We use the commutation relations for the asymptotic fields to define the incoming and outgoing states. With the aid of a canonical form for the commutator function [13], we can define two distinct Feynman propagators relative to an arbitrary background. One of these is manifestly covariant and propagates both nonphysical and physical quanta. While the other propagates only physical quanta and lacks manifest covariance, which is used to define the external line wave function. By the construction of full S-matrix theory [13, 14], we can calculate the gravitational scattering of two scalar particles, scattering of gravitons by scalar particles, and graviton-graviton scattering.

II. Quantum Corrections to Gravitational Potential

In scalarized NS binaries, the gravitational interactions between two scalarized components realize via the exchanges of corresponding gravitational quanta, i.e. gravitons and gravitational scalar particles. In classical gravitational theory, the modified Newtonian potential expressed by Eq. (2.12) is approximately valid for the gravitational interactions in a gravitationally bounding scalarized NS binary system, due to the exchanges of both gravitons and scalar particles. With a definition of bound state potential in the framework of general relativity, the relativistic corrections to the Newtonian law, arising from higher order effects $(\frac{v^2}{c^2})^n$ and nonlinear terms in the field equations of order $\frac{Gm}{r}$ $(m = m_{ss}, m_{is})$, in a Hamiltonian treatment were completed [64], which was obtained in the form of

$$V_{cl} = a_{cl} \frac{G m_{ss} m_{is}}{R_{ss-is}} \frac{G(m_{ss}+m_{is})}{R_{ss-is}}, (3.2)$$

where a_{cl} is a numerical constant which would depend on the precise definition of the potential. The classical relativistic corrections to the Newtonian potential of two bodies also were discussed [65, 66], with general agreement with the above result, although in unavoidably ambiguously defining the potential.

The theory of general relativity has been widely accepted in describing the gravitationally bound systems consisting of two compact extended objects. The post-Newtonian approximation to general relativity [67, 68], i.e. systematically solving the Einstein equations with nonrelativistic sources, is employed as the conventional approach to calculating the initial inspiral of the system as it slowly loses energy to gravitational radiation [55]. In order to investigate the gravitational radiation power spectra emitted by nonrelativistic bound systems, the dynamics of two components, which are treated as point particles, in a binary system coupled to gravity was described in an effective field theory framework [69], in which the observables appearing in long-wavelength physics are consistent with general coordinate invariance of general relativity.

General relativity, with low energy degrees of freedom and gravitational interactions, is a consistent effective field theory [70], which allows, in principle, its quantization to be carried out without knowledge of microphysics details. By using the effective field theory approach with background field quantization methods [13, 14], the leading long-distance quantum corrections to the one-particle-irreducible Newtonian potential were calculated [70, 71], which result in a finite correction,

$$V_{qu} = a_{qu} \frac{G m_{ss} m_{is}}{R_{ss-is}} \frac{G}{{R_{ss-is}}^2}, (3.3)$$

where a_{qu} is a numerical constant. However, many works dedicated to the choices between various definitions of the potential depending on the physical situation and the way of defining the total energy. By using the Arnowitt-Deser-Misner formula for the total energy of the gravitational system, the Wilson loop description for the gravitational potential has been done [72, 73]. The quantum corrections to Newtonian potential for an arbitrary gravitational field that includes the back-reaction produced by a quantum scalar field of mass was considered by deriving an in-in effective equations [74]. For the simplicity and intuitiveness, a number of authors employed the scattering amplitude itself to define the potential [65, 75-78]. The obtained particular effects, from summing one-loop Feynman diagrams with off-shell gravitons, applies to point particle masses [78]. The quantum gravitational effects of a pair of localized polarizable objects, associated with two-graviton exchange from the induced gravitational quadrupole moments due to quantum fluctuations in the metric, was computed [79].

In this section, we shall treat the components in the scalarized system as massive scalar point sources and employ the background field method [13, 14], in order to investigate the quantum corrections to the modified Newtonian potential (2.12) in scalarized NS binaries.

1. Effective Field Theory Description

Neglecting the extended scales, we treat the stars in scalarized NS binary systems as massive scalar point particles. The action that describes the scalarized binary reads,

$$S = \int d^4x (\mathcal{L}_g + \mathcal{L}_{sbinary}) = \int d^4x \sqrt{-g} \left(\frac{2}{\kappa^2} \mathcal{R} - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} m^2 \varphi^2 - \frac{1}{2} \omega^2 \phi_g^2 \varphi^2 \right), \quad (3.4)$$

Here, $\varphi = \varphi_{ss}, \varphi_{is}, m = m_{ss}, m_{is}, \omega = \omega_{ss}, \omega_{is}$. ϕ_g is the gravitational scalar counterpart developed during the dynamical scalarization [50]. $\mathcal{L}_g$ and $\mathcal{L}_{sbinary}$ represent the effective Lagrangian of the gravitational tensor terms and the terms of scalar particles, respectively. $\kappa = \sqrt{32\pi G}$ is the gravitational coupling.

In order to treat action (3.4) as an effective field theory, one must include all possible higher derivative couplings of the fields in the gravitational Lagrangian [70]. We consequently write an effective Lagrangian for gravitational tensor terms and scalar particles in describing the gravitationally scalarized binary as

$$\mathcal{L}_g^{eff} = \frac{2\mathcal{R}}{\kappa^2} + c_1 \mathcal{R}^2 + c_2 \mathcal{R}^{\mu\nu} \mathcal{R}_{\mu\nu} + \cdots, (3.5)$$

$$\mathcal{L}_{sbinary}^{eff} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} \left(m^2 + \omega^2 \phi_g^2 \right) \varphi^2 + \bar{c}_1 \mathcal{R}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \bar{c}_2 \mathcal{R} \partial_\mu \varphi \partial^\mu \varphi + \bar{c}_3 \mathcal{R} \left(m^2 + \omega^2 \phi_g^2 \right) \varphi^2 + \cdots. (3.6)$$

Here, the coefficients $c_1, c_2, \cdots$ are dimensionless constants that determine the scale of the energy expansion of pure gravity [80], and $\bar{c}_1, \bar{c}_2, \bar{c}_3, \cdots$ are energy-scale dependent coupling constants determined currently by binary observational measurements.

We expand the metric as a background part $\bar{g}_{\mu\nu}$ and a quantum contribution $\kappa h_{\mu\nu}$,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}, \qquad g^{\mu\nu} = \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h_\lambda^\mu h^{\lambda\nu} + \cdots, \qquad \sqrt{-g} = \sqrt{-\bar{g}} \left(1 + \frac{1}{2} \kappa h + \cdots \right), (3.7)$$

where $h^{\mu\nu} = \bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}h_{\alpha\beta}$ and $h = \bar{g}^{\mu\nu}h_{\mu\nu}$. For the simplicity of graviton propagator, a gauge fixing term [81, 82], with the form of [83]

$$\frac{1}{\kappa^2}(\partial_\mu\sqrt{-\bar{g}}g^{\mu\nu})^2 \quad (3.8)$$

should be introduced, which then gives the bare graviton propagator

$$\mathcal{D}_{\mu\nu\alpha\beta}(q) = \frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{q^2} = \frac{i(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta})}{2q^2}, \quad (3.9)$$

where q is the momentum. In calculating quantum corrections at one loop, we need to consider the Lagrangian to quadratic order. Consequently, the expansion of Lagrangian for gravitons and scalar fields, as well as the mixed ones, can be written as follows,

$$\mathcal{L}(\varphi^2) = -\frac{1}{2}\left(\partial_\mu\varphi\,\partial^\mu\varphi + m^2\varphi^2 + \omega^2\phi_g^2\varphi^2\right), \quad (3.10)$$

$$\mathcal{L}(h^2) = -\frac{1}{2}(h_{\alpha\beta,\gamma}h_{\alpha\beta,\gamma} - 2h_{\gamma\beta,\alpha}h_{\gamma\alpha,\beta} + 2h_{\beta\beta,\alpha}h_{\gamma\alpha,\gamma} - h_{\beta\beta,\alpha}h_{\gamma\gamma,\alpha}), \quad (3.11)$$

$$\mathcal{L}(h\varphi^2) = \frac{\kappa}{2}\left[h^{\mu\nu}\,\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}h^{\alpha\alpha}\left(\partial_\beta\varphi\,\partial_\beta\varphi + m^2\varphi^2 + \omega^2\phi_g^2\varphi^2\right)\right], \quad (3.12)$$

$$\mathcal{L}(h^3) = \frac{\kappa}{2}\Big[h_\lambda\left(h_{\gamma\beta,\alpha}h_{\gamma\alpha,\beta} - \frac{1}{2}h_{\alpha\beta,\gamma}h_{\alpha\beta,\gamma} - h_{\beta\beta,\alpha}h_{\gamma\alpha,\gamma} + \frac{1}{2}h_{\beta\beta,\alpha}h_{\gamma\gamma,\alpha}\right) + h_{\mu\nu}(h_{\alpha\beta,\mu}h_{\alpha\beta,\nu} - 2h_{\mu\beta,\alpha}h_{\nu\alpha,\beta} + 2h_{\mu\beta,\alpha}h_{\nu\beta,\alpha} + 2h_{\beta\beta,\alpha}h_{\mu\alpha,\nu} - 2h_{\beta\beta,\alpha}h_{\mu\nu,\alpha} + 2h_{\beta\mu,\beta}h_{\alpha\alpha,\nu} - 2h_{\beta\beta,\mu}h_{\alpha\alpha,\nu} + 2h_{\beta\alpha,\beta}h_{\mu\nu,\alpha} - 4h_{\beta\mu,\alpha}h_{\beta\alpha,\nu})\Big], \quad (3.13)$$

$$\mathcal{L}(h^2\varphi^2) = -\frac{\kappa^2}{2}\Big[h^{\mu\alpha}h^{\nu\alpha}\,\partial_\mu\varphi\,\partial_\nu\varphi - \frac{1}{2}h^{\alpha\alpha}h^{\mu\nu}\,\partial_\mu\varphi\,\partial_\nu\varphi + \left(\frac{1}{2}h^{\alpha\alpha}h^{\beta\beta} - \frac{1}{2}h^{\alpha\beta}h^{\alpha\beta}\right)\left(\partial_\alpha\varphi\,\partial_\alpha\varphi + m^2\varphi^2 + \omega^2\phi_g^2\varphi^2\right)\Big]. \quad (3.14)$$

In some given NS binaries, the gravitational scalar counterparts can become massive [50], with a mass of m_s. Accordingly, the gravitational scalar interactions in these systems between two scalarized components realize by the exchange of massive scalar fields with the massive scalar propagator $\frac{i}{q^2-m_s^2+i\epsilon}$, which results in analytic contributions to the gravitational potential. Owing to an exponential Yukawa suppression, the propagations of massive mode, with obvious representation as $\frac{1}{q^2-m_s^2} = -\frac{1}{m_s^2}(1 + \frac{q^2}{m_s^2} + \cdots)$ in momentum q, are screened in the range of binary orbit [50]. So the analytical contributions are local effects. On the scale lager than the binary orbit, non-analytic effects, arising from the propagations of massless gravitons and scalar fields, dominate in magnitude over the analytic corrections in the low energy limit of the effective field theory.

2. Calculations and Results

According to dimensional analysis, we can figure out the modifications to the potential (2.12) of the form

$$V(r) = -\frac{Gm_{ss}m_{is}}{r} - \frac{G\omega_{ss}\omega_{is}}{r} + a_{cl}\frac{Gm_t}{c^2r} + a_{qu}\frac{G\hbar}{c^3r^2} + \cdots, \quad (3.15)$$

where m_t contains both gravitational mass (m_{ss}, m_{is}) and the scalar contributions ($\omega_{ss}\phi_g, \omega_{is}\phi_g$). What we shall do in this section is to calculate the numerical coefficients a_{cl} and a_{qu} for an appropriate definition of potential.

In our calculations, we only consider the non-analytic contributions from the one-loop diagrams, which contain two or more massless propagating particles. The general form for diagrams contributing to the scattering matrix in the momentum (q) space representation is

$$\mathcal{M}(q) \sim A + Bq^2 + \cdots + \alpha\frac{\kappa^4}{q^4} + \beta_1\kappa^4\ln(-q^2) + \beta_2\kappa^4\frac{m}{\sqrt{-q^2}} + \beta_3\kappa^4\frac{\omega}{\sqrt{-q^2}} + \cdots. \quad (3.16)$$

Here, A, B, ⋯, in the terms with power series of momentum q, correspond to analytic pieces, which only dominate in high-energy regime of the effective field theory and are of no interest to our calculations. The coefficients$\alpha, \beta_1, \beta_2, \beta_3, \cdots$associate with the long range, non-analytic interactions, where $\beta_1, \beta_2, \beta_3$ involve terms yield the leading post-Newtonian and quantum corrections to the gravitational potential. The resulting amplitudes are transformed to produce the scattering potential, by performing Fourier transformation and using the following integrals,

$$\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{1}{|\vec{q}|^2} = \frac{1}{4\pi r},$$
$$\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \frac{1}{|\vec{q}|} = \frac{1}{2\pi^2 r^2},$$
$$\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \ln(|\vec{q}|^2) = -\frac{1}{2\pi r^3}. \quad (3.17)$$

Definition of potential

Because the NS binaries are gravitational bound systems, we consider the expectation value for the matrix iT and use the scattering amplitude itself to define the gravitational potential. The full scattering amplitude are calculated in order to represent the non-relativistic potential generated by the non-analytic pieces [75, 78],

$$\langle f|iT|i\rangle = (2\pi)^4\delta^4(q_1 - q_1' + q_2 - q_2')[i\mathcal{M}(\vec{q})] = -iV(\vec{q})2\pi\delta(E - E'). \quad (3.18)$$

Here, q_1, q_2 and q_1', q_2' are the incoming and outgoing momentum, respectively. $E - E'$ is the energy difference between the incoming and outgoing states. $\mathcal{M}(\vec{q})$ is the non-analytical part of the amplitude in momentum space representation. $V(\vec{q}) = -\frac{1}{2m_{ss}}\frac{1}{2m_{is}}\mathcal{M}(\vec{q})$. Taking the non-relativistic limit and Fourier transformation, we can get the corresponding coordinate space representation,

$$V(\vec{r}) = \frac{1}{2m_{ss}}\frac{1}{2m_{is}} \int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \mathcal{M}(\vec{q}). \quad (3.19)$$

Vertex rules

From the effective Lagrangian (3.10)-(3.14), our calculations for the Feynman diagrams involve in two scalar-one graviton vertex ($h\varphi^2$), two scalar-two graviton vertex ($h^2\varphi^2$), and three-graviton vertex (h^3). The two scalar-one graviton vertex is given by

$$\tau_1^{\mu\nu}(k_1, k_2, m) = \frac{i\kappa}{2}\left[k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - \eta^{\mu\nu}(k_1 \cdot k_2 - m^2)\right], \quad (3.20)$$

where k_1, k_2 denote the four-momentum of the incoming and outgoing scalar particles, respectively. The two scalar-two graviton vertex can be written as

$$\tau_2^{\mu\nu\alpha\beta}(k_1, k_2, m) = i\kappa^2\left\{\left[I^{\mu\nu\rho\lambda}I_\lambda^{\alpha\beta\sigma} - \frac{1}{4}\left(\eta_{\mu\nu}I^{\alpha\beta\rho\sigma} + \eta_{\alpha\beta}I^{\mu\nu\rho\sigma}\right)\right] \times \left(k_{1\rho}k_{2\sigma} + k_{2\rho}k_{1\sigma}\right) - \frac{1}{2}\left(I^{\mu\nu\alpha\beta} - \frac{1}{2}\eta^{\mu\nu}\eta^{\alpha\beta}\right) \times [(k_1 \cdot k_2) - m^2]\right\}, \quad (3.21)$$

where $I_{\mu\nu\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha})$ and the pairs of indices ($\mu\nu$) and ($\alpha\beta$) are associated with two graviton lines. The three-graviton vertex is derived via the background field method, which has the form [70]

$$\begin{aligned}
\tau_{3\alpha\beta\gamma\delta}^{\mu\nu}(k, q) = \frac{i\kappa}{2} \times \Big(&\mathcal{P}_{\alpha\beta\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\right] \\
&+ 2q_\lambda q_\sigma\left(I_{\alpha\beta}^{\sigma\lambda}I_{\gamma\delta}^{\mu\nu} + I_{\gamma\delta}^{\sigma\lambda}I_{\alpha\beta}^{\mu\nu} - I_{\alpha\beta}^{\mu\sigma}I_{\gamma\delta}^{\nu\lambda} - I_{\gamma\delta}^{\mu\sigma}I_{\alpha\beta}^{\nu\lambda}\right) \\
&+ \left[q_\lambda q^\mu\left(\eta_{\alpha\beta}I_{\gamma\delta}^{\nu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}^{\nu\lambda}\right) + q_\lambda q^\nu\left(\eta_{\alpha\beta}I_{\gamma\delta}^{\mu\nu} - \eta_{\gamma\delta}I_{\alpha\beta}^{\nu\lambda}\right) - q^2\left(\eta_{\alpha\beta}I_{\gamma\delta}^{\mu\nu} - \eta_{\gamma\delta}I_{\alpha\beta}^{\mu\nu}\right)\right. \\
&\left.- \eta^{\mu\nu}q_\sigma q_\lambda\left(\eta_{\alpha\beta}I_{\gamma\delta}^{\sigma\lambda} - \eta_{\gamma\delta}I_{\alpha\beta}^{\sigma\lambda}\right)\right] \\
&+ \Big\{2q_\lambda\left[I_{\alpha\beta}^{\lambda\sigma}I_{\gamma\delta\sigma}^{\nu}(k-q)^\mu + I_{\alpha\beta}^{\lambda\sigma}I_{\gamma\delta\sigma}^{\mu}(k-q)^\nu - I_{\gamma\delta}^{\lambda\sigma}I_{\alpha\beta\sigma}^{\nu}k^\mu - I_{\gamma\delta}^{\lambda\sigma}I_{\alpha\beta\sigma}^{\mu}k^\nu\right] + q^2\left(I_{\alpha\beta\sigma}^{\mu}I_{\gamma\delta}^{\lambda\sigma} + I_{\alpha\beta}^{\nu\sigma}I_{\gamma\delta\sigma}^{\mu}\right) \\
&+ \eta^{\mu\nu}q_\sigma q_\lambda\left(I_{\alpha\beta}^{\lambda\rho} + I_{\gamma\delta}^{\lambda\rho}I_{\alpha\beta\rho}^{\sigma}\right)\Big\} \\
&+ \Big\{[k^2 + (k-q)^2]\left[I_{\alpha\beta}^{\mu\sigma}I_{\gamma\delta\sigma}^{\nu} + I_{\gamma\delta}^{\mu\sigma}I_{\alpha\beta\sigma}^{\nu} - \frac{1}{2}\eta^{\mu\nu}\mathcal{P}_{\alpha\beta\gamma\delta}\right] - \left[I_{\gamma\delta}^{\mu\nu}\eta_{\alpha\beta}k^2 + I_{\alpha\beta}^{\mu\nu}\eta_{\gamma\delta}(k-q)^2\right]\Big\}\Big). \quad (3.22)
\end{aligned}$$

Tree diagram

The set of tree diagrams, coming from the exchanges of both gravitons and scalar particles in Fig. 1, are the well-known lowest order potential in the non-relativistic limit. Because the scalar configuration couples to the star matters inside each component, the exchanges of gravitons actually blend with that of scalar particles. We put them into one single Feynman diagram in Fig. 1. However, the gravitational interactions between star matters realize via the exchanges of gravitons, while the scalar configurations settled in the star components gravitationally interact with each other by the exchanges of scalar particles [46], we separate the exchanges of gravitons from that of scalar particles when calculating the contributions to the potential. By using the Feynman rules and choosing a parameterization of the momentum, the piece of graviton exchanges, with a momentum q, can be defined as

$$i\mathcal{M}_1^g = \tau_1^{\mu\nu}(k_1, k_2, m_{ss})\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{q^2}\tau_1^{\alpha\beta}(k_3, k_4, m_{is}), (3.23)$$

where $q = k_1 - k_2 = k_3 - k_4$. The component with mass m_{ss} and scalar charges ω_{ss} has incoming momentum k_1 and outgoing momentum k_2, and the other component with mass m_{is} and scalar charges ω_{is} has incoming momentum k_3 and outgoing momentum k_4, respectively.

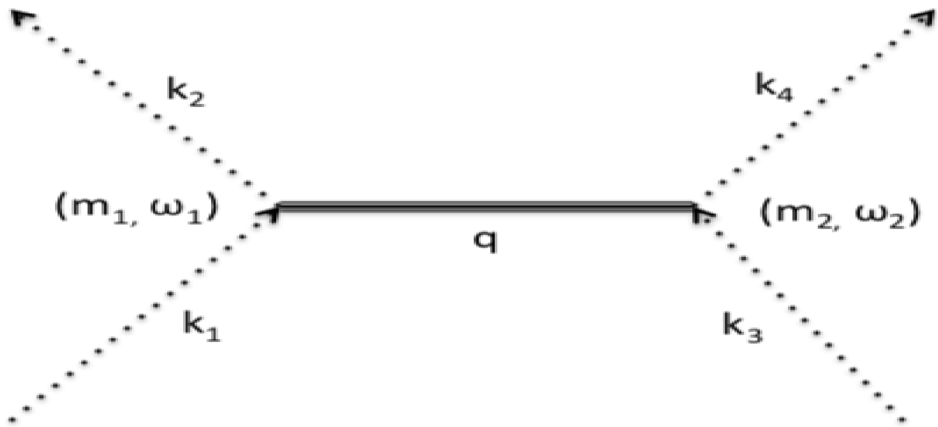

Fig 1:-The tree diagram contributes to the scalar-modified Newtonian potential. The graviton-graviton scattering yields the Newtonian potential, while the gravitational scalar interaction results from the exchanges of scalar particles, which accompanies the graviton exchanges. The triple solid lines represent the exchanges of gravitons and accompanied exchanges of scalar particles in a scalarized binary system, in which the central thick line denotes the gravitons, and the thin lines on two sides are the scalar particles. The dash lines are the scalar fields.

By contracting all indices for the tree level and performing Fourier transforms, we obtain the scattering potential

$$V_1^g(r) = -\frac{Gm_{ss}m_{is}}{r}, (3.24)$$

which gives the Newtonian law.

The settled scalar configuration inside the star affects star's masses [45] and thus enhances the gravitational attraction between the binary's components, which make the scalarized system act as a source of the gravitational scalar field [50]. Therefore, the vertex associated with the propagations of scalar fields between two stars involves scalar mass dimensional quantities, i.e. $\omega_1\phi_g$ and $\omega_2\phi_g$. Accordingly, we define the diagram for the exchanges of scalar particles with a propagating momentum l as

$$i\mathcal{M}_1^s = \tau_1^{\mu\nu}(k_1, l, \omega_{ss})\frac{i}{l^2}\tau_1^{\alpha\beta}(l, k_3, \omega_{is}), (3.25)$$

where k_1 is the incoming momentum for components with scalar charge ω_{ss} and k_3 is the incoming momentum with external line for components with scalar charge ω_{is} . In non-relativistic limit, we perform Fourier transformation

$$V(\vec{r}) = \frac{1}{2\omega_{ss}}\frac{1}{2\omega_{is}}\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\mathcal{M}(\vec{q}), (3.26)$$

and find the result

$$V_1^s(r) = -\frac{G\omega_{ss}\omega_{is}}{r}, (3.27)$$

which arises from the classically gravitational scalar interaction between two scalarized stars. Therefore, the combined result of FIG.1 yields the modified Newtonian potential (2.12).

Box and crossed-box diagrams

The box (FIG. 2(a)) and crossed-box (FIG. 2(b)) diagrams just involve 1-graviton-two scalar particle ($h\varphi^2$).

The contributions from Box diagram FIG. 2(a) can be written as

$$i\mathcal{M}^{g}_{2(a)} = \int \frac{d^4l}{(2\pi)^4}\frac{i}{(k_1+l)^2-m_{ss}^2}\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{l^2}\frac{i\mathcal{P}_{\rho\sigma\gamma\delta}}{(l+q)^2}\frac{i}{(k_3-l)^2-m_{is}^2} \times \tau_1^{\mu\nu}(k_1,k_1+l,m_{ss})\tau_1^{\rho\sigma}(k_1+l,k_2,m_{ss})\tau_1^{\alpha\beta}(k_3,k_3-l,m_{is})\tau_1^{\gamma\delta}(k_3-l,k_4,m_{is}), (3.28)$$

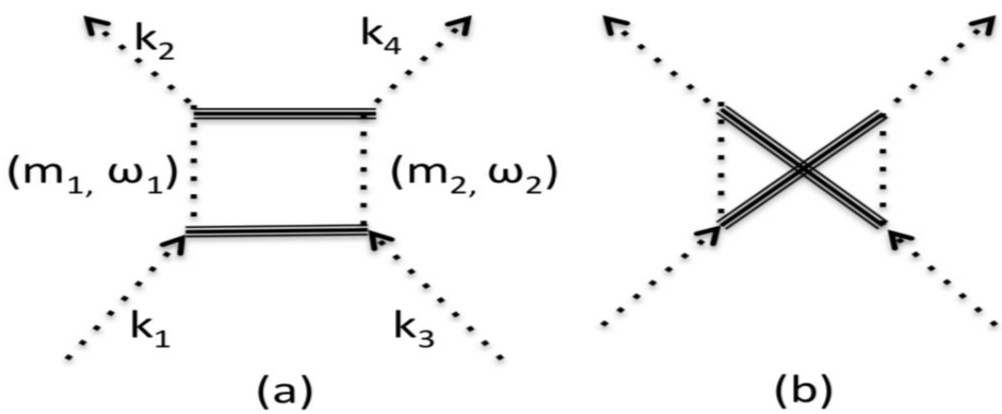

Fig 2:-The set of box (a) and crossed-box (b) diagrams contribute to the non-analytic components of the potential.

for the gravitational tensor interactions, and as

$$i\mathcal{M}^{s}_{2(a)} = \int \frac{d^4l}{(2\pi)^4}\frac{i}{(l-k_1)^2-m_{ss}^2}\frac{i}{(k_3+l)^2-m_{is}^2}\frac{i}{l^2}\frac{i}{(l+q)^2} \times \tau_1^{\mu\nu}(k_1,l,\omega_{ss})\tau_1^{\rho\sigma}(k_1,l-q,\omega_{ss})\tau_1^{\alpha\beta}(l,k_3,\omega_{is})\tau_1^{\gamma\delta}(k_3,l+q,\omega_{is}), (3.29)$$

for gravitational scalar interactions in FIG. 2(a). The star with mass and scalar charge (m_{ss},ω_{ss}) has incoming momentum k_1 and outgoing momentum k_2, and the other component with (m_{is},ω_{ss}) has incoming momentum k_3 and outgoing momentum k_4, respectively.

The non-analytic contributions to the potential from the crossed-box diagram FIG. 2(b) are defined as

$$i\mathcal{M}^{g}_{2(b)} = \int \frac{d^4l}{(2\pi)^4}\frac{i}{(k_1+l)^2-m_{ss}^2}\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{l^2}\frac{i\mathcal{P}_{\rho\sigma\gamma\delta}}{(l+q)^2}\frac{i}{(k_4+l)^2-m_{is}^2} \times \tau_1^{\mu\nu}(k_1,k_1+l,m_{ss})\tau_1^{\rho\sigma}(k_1+l,k_2,m_{ss})\tau_1^{\alpha\beta}(k_3,k_4+l,m_{is})\tau_1^{\gamma\delta}(k_4+l,k_4,m_{is}), (3.30)$$

for pieces of graviton exchanges and as

$$i\mathcal{M}^{s}_{2(b)} = \int \frac{d^4l}{(2\pi)^4}\frac{i}{(l-k_1)^2-m_{ss}^2}\frac{i}{(k_3+l)^2-m_{is}^2}\frac{i}{(l-q)^2}\frac{i}{l^2} \times \tau_1^{\mu\nu}(k_1,l,\omega_{ss})\tau_1^{\rho\sigma}(k_1,l-q,\omega_{ss})\tau_1^{\alpha\beta}(l-q,k_3,\omega_{is})\tau_1^{\gamma\delta}(k_3,l,\omega_{is}), (3.31)$$

for that of exchanges of scalar particles.

For the calculations of diagrams, we employ the algebraic program and the contraction rules, which are discussed in references [78, 84] in order to reduce the integrals, and we also use the integrals listed in the appendix in these two references. The results from graviton exchanges are in agreement with that of [78], i.e.

$$V_{2(a)+2(b)}(r) = -\frac{47}{3\pi}\frac{Gm_{ss}m_{is}}{r}\frac{G}{r^2}. (3.32)$$

The contributions from scalar particles exchanges give

$$V_{2(c)+2(d)}(r) = -\frac{G\omega_{ss}\omega_{is}}{r}\left[\frac{2G(m_{ss}+m_{is})}{3r}+\frac{11}{3\pi}\frac{G}{r^2}\right]. (3.33)$$

Triangle diagrams

The triangle diagrams contributing to the non-analytic pieces arise from graviton exchanges involving in the effective Lagrangian $\mathcal{L}(h\varphi^2)$ and $\mathcal{L}(h^2\varphi^2)$ (see FIG. 3). The expressions are same as [78],

$$i\mathcal{M}^{g}_{3(a)} = \int \frac{d^4l}{(2\pi)^4}\frac{i}{(k_1+l)^2-m_{ss}^2}\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{l^2}\frac{i\mathcal{P}_{\rho\sigma\gamma\delta}}{(l+q)^2} \times \tau_1^{\mu\nu}(k_1,k_1+l,m_{ss})\tau_1^{\rho\sigma}(k_1+l,k_2,m_{ss})\tau_2^{\alpha\beta\gamma\delta}(k_3,k_4,m_{is}), (3.34)$$

$$i\mathcal{M}_{3(b)}^{g} = \int \frac{d^4l}{(2\pi)^4} \frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{l^2} \frac{i\mathcal{P}_{\rho\sigma\gamma\delta}}{(l+q)^2} \frac{i}{(l-k_3)^2 - m_{is}^2} \times \tau_1^{\alpha\beta}(k_3, k_3 - l, m_{is}) \tau_1^{\gamma\delta}(k_3 - l, k_4, m_{is}) \tau_2^{\mu\nu\rho\sigma}(k_1, k_2, m_{ss}). \quad (3.35)$$

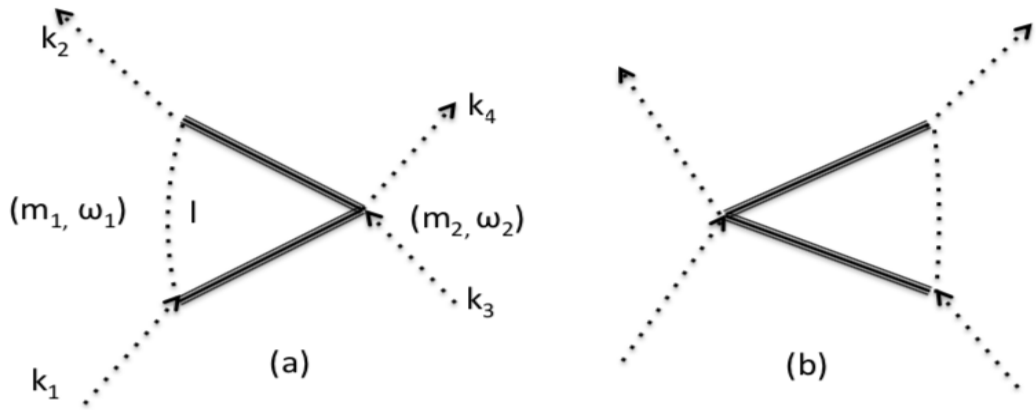

Fig 3:-The set of triangle diagrams contribute to the non-analytic pieces of the potential.

Taking the non-relativistic limit, we produce the results

$$V_{3(a)+3(b)}^{g}(r) = -\frac{Gm_{ss}m_{is}}{r}\left[4\frac{G(m_{ss}+m_{is})}{r} + \frac{28}{\pi}\frac{G}{r^2}\right]. \quad (3.36)$$

The pieces from the exchange of scalar particles have the following expressions,

$$i\mathcal{M}_{3(a)}^{s} = \int \frac{d^4l}{(2\pi)^4} \frac{i}{(k_1+l)^2 - m_{ss}^2} \frac{i}{l^2} \frac{i}{(l+q)^2} \times \tau_1^{\mu\nu}(k_1, l, \omega_{ss}) \tau_1^{\rho\sigma}(l, k_2, \omega_{ss}) \tau_2^{\alpha\beta\gamma\delta}(l, l+q, \omega_{is}), \quad (3.37)$$

$$i\mathcal{M}_{3(b)}^{s} = \int \frac{d^4l}{(2\pi)^4} \frac{i}{l^2} \frac{i}{(l+q)^2} \frac{i}{(k_3+l)^2 - m_{is}^2} \times \tau_2^{\mu\nu\rho\sigma}(l, l+q, \omega_{ss}) \tau_1^{\alpha\beta}(l, k_3, \omega_{is}) \tau_1^{\gamma\delta}(l+q, k_3, \omega_{is}), \quad (3.38)$$

The non-analytic contributions to the potential arising from gravitational scalar interactions are obtained as

$$V_{3(a)+3(b)}^{s}(r) = -\frac{G\omega_{ss}\omega_{is}}{r}\left[4\frac{G(\omega_{ss}+\omega_{is})}{r} + \frac{6}{\pi}\frac{G}{r^2}\right]. \quad (3.39)$$

Circular diagram

By taking the symmetry into account, we write down the expressions of circular diagram (FUG. 4) involving in both gravitons exchanges and the exchanges of scalar particles as follows, respectively,

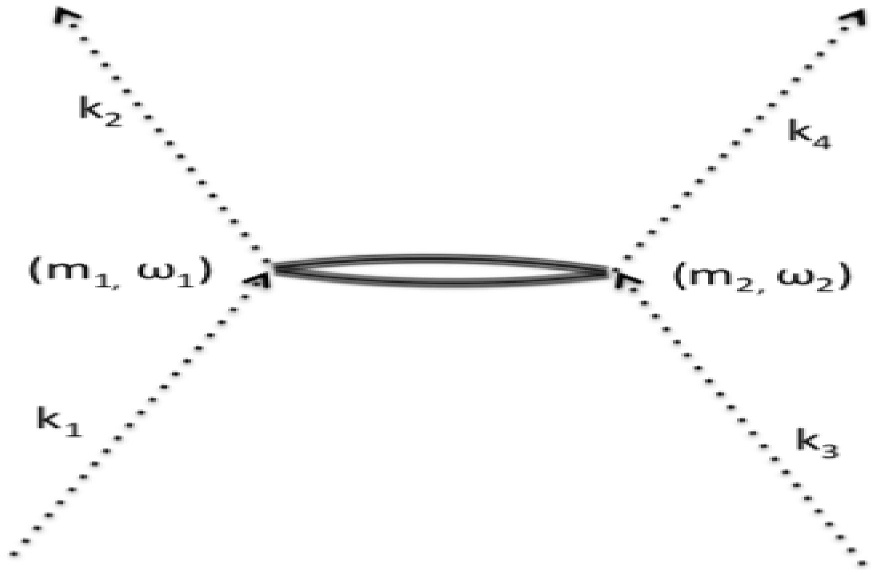

Fig 4:-The set of circular diagram contribute to the non-analytic components of the potential.

$$i\mathcal{M}_4^g = \frac{1}{2!}\int \frac{d^4l}{(2\pi)^4}\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{(l+q)^2}\frac{i\mathcal{P}_{\rho\sigma\gamma\delta}}{l^2} \times \tau_2^{\mu\nu\rho\sigma}(k_1, k_2, m_{ss})\tau_2^{\alpha\beta\gamma\delta}(k_3, k_4, m_{is}), (3.40)$$

$$i\mathcal{M}_4^s = \frac{1}{2!}\int \frac{d^4l}{(2\pi)^4}\frac{i}{(l+q)^2}\frac{i}{l^2} \times \tau_2^{\mu\nu\rho\sigma}(k_1, k_2, \omega_{ss})\tau_2^{\alpha\beta\gamma\delta}(k_3, k_4, \omega_{is}). (3.41)$$

Performing the same contractions and integrals as reference [78], we obtain the corrections to the potential,

$$V_4^g(r) = -\frac{22}{\pi}\frac{Gm_{ss}m_{is}}{r}\frac{G}{r^2}, (3.42)$$

$$V_4^s(r) = \frac{2}{\pi}\frac{G\omega_{ss}\omega_{is}}{r}\frac{G}{r^2}. (3.43)$$

One-particle reducible diagrams

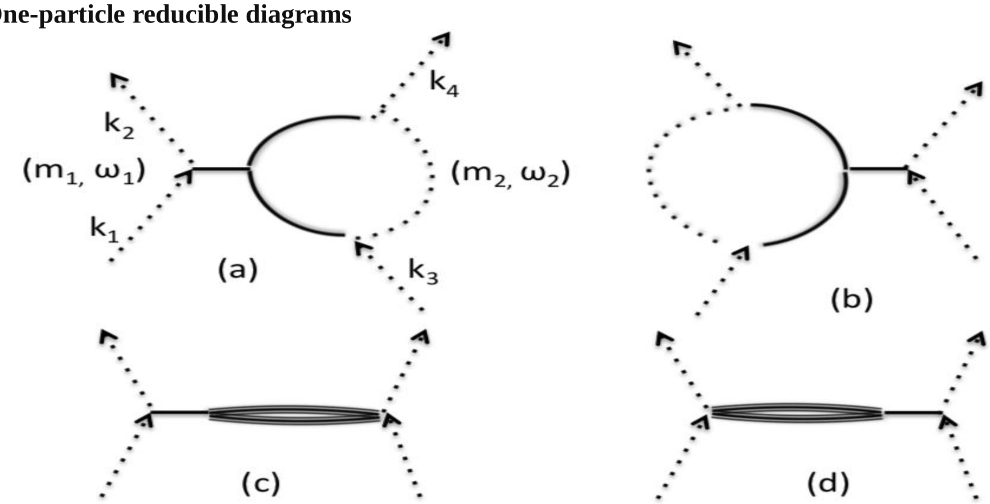

Fig 5:-Two classes of the set of one-particle reducible diagrams yield non-analytic corrections to the potential. The solid lines denote the gravitons.

There are two classes of set of one-particle reducible (1PR) diagrams (see FIG. 5), i.e. the massive loop diagrams (FIG. 5(a) and FIG. 5(b)) and the mixed scalar-graviton diagrams (FIG. 5(c) and FIG. 5(d)). For the massive loop diagrams contributing to non-analytic corrections, we have the expressions,

$$i\mathcal{M}^{g}_{5(a)} = \int \frac{d^4l}{(2\pi)^4}\frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{q^2}\frac{i\mathcal{P}_{\lambda\kappa\alpha\beta}}{l^2}\frac{i\mathcal{P}_{\phi\epsilon\gamma\delta}}{(l+q)^2}\frac{i}{(l-k_3)^2 - m_{is}^2}$$
$$\times \tau_1^{\mu\nu}(k_1,k_2,m_{ss})\tau_3^{\rho\sigma\lambda\kappa(\phi\epsilon)}(-l,q)\tau_1^{\alpha\beta}(k_3,k_3-l,m_{is})\tau_1^{\gamma\delta}(k_3-l,k_4,m_{is}), (3.44)$$
$$i\mathcal{M}^{g}_{5(b)} = \int \frac{d^4l}{(2\pi)^4}\frac{i}{(l+k_1)^2 - m_{ss}^2}\frac{i\mathcal{P}_{\mu\nu\phi\epsilon}}{l^2}\frac{i\mathcal{P}_{\rho\sigma\lambda\kappa}}{(l+q)^2}\frac{i\mathcal{P}_{\gamma\delta\alpha\beta}}{q^2}$$
$$\times \tau_1^{\mu\nu}(k_1,k_1+l,m_{ss})\tau_1^{\rho\sigma}(k_1+l,k_2,m_{ss})\tau_3^{\phi\epsilon\lambda\kappa(\gamma\delta)}(-l,q)\tau_1^{\alpha\beta}(k_3,k_4,m_{is}), (3.45)$$
which yield the contributions to the potential,
$$V^{g}_{5(a)+5(b)}(r) = -\frac{Gm_{ss}m_{is}}{r}\left[-\frac{G(m_{ss}+m_{is})}{r} + \frac{5}{3\pi}\frac{G}{r^2}\right]. (3.46)$$
By noting that a symmetry factor of $\frac{1}{2!}$, the pure graviton exchanges contributing to the mixed diagrams (FIG. 5(c) and FIG. 5(d)) can be defined as follows,
$$i\mathcal{M}^{g}_{5(c)} = \frac{1}{2!}\int \frac{d^4l}{(2\pi)^4}\frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{q^2}\frac{i\mathcal{P}_{\lambda\kappa\alpha\beta}}{l^2}\frac{i\mathcal{P}_{\phi\epsilon\gamma\delta}}{(l+q)^2}\times\tau_1^{\mu\nu}(k_1,k_1+l,m_{ss})\tau_3^{\rho\sigma\lambda\kappa(\phi\epsilon)}(l,-q)\tau_2^{\alpha\beta\gamma\delta}(k_3,k_4,m_{is}), (3.47)$$
$$i\mathcal{M}^{g}_{5(d)} = \frac{1}{2!}\int \frac{d^4l}{(2\pi)^4}\frac{i\mathcal{P}_{\mu\nu\phi\epsilon}}{l^2}\frac{i\mathcal{P}_{\rho\sigma\lambda\kappa}}{(l+q)^2}\frac{i\mathcal{P}_{\gamma\delta\alpha\beta}}{q^2}\times\tau_2^{\mu\nu\gamma\delta}(k_1,k_2,m_{ss})\tau_3^{\gamma\delta\lambda\kappa(\phi\epsilon)}(-l,q)\tau_1^{\alpha\beta}(k_3,k_4,m_{is}), (3.48)$$
and the contributions from the mixed scalar-graviton diagrams are defined as
$$i\mathcal{M}^{m}_{5(c)} = \frac{1}{2!}\int \frac{d^4l}{(2\pi)^4}\frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{q^2}\frac{i}{l^2}\frac{i}{(l+q)^2}\times\tau_2^{\mu\nu\rho\sigma}(l,l+q,\omega_{ss})\tau_1^{\gamma\delta}(-l,-l-q)\tau_1^{\alpha\beta}(k_3,k_4,m_{is}), (3.49)$$
$$i\mathcal{M}^{m}_{5(d)} = \frac{1}{2!}\int \frac{d^4l}{(2\pi)^4}\frac{i}{l^2}\frac{i}{(l+q)^2}\frac{i\mathcal{P}_{\gamma\delta\alpha\beta}}{q^2}\times\tau_1^{\mu\nu}(k_1,k_2,m_{ss})\tau_1^{\rho\sigma}(l,l+q)\tau_2^{\gamma\delta\alpha\beta}(-l,-l-q,\omega_{is}). (3.50)$$

By performing the algebra analysis, we obtain the following corrections, respectively,
$$V^{g}_{5(c)+5(d)}(r) = \frac{26}{3\pi}\frac{Gm_{ss}m_{is}}{r}\frac{G}{r^2}, (3.51)$$
$$V^{m}_{5(c)+5(d)}(r) = -\frac{1}{6\pi^2}\frac{G(\omega_{ss}{}^2m_{is}{}^2 + \omega_{is}{}^2 + m_{ss}{}^2)}{m_{ss}m_{is}}\frac{G}{r^3}. (3.52)$$

Vacuum polarization diagrams

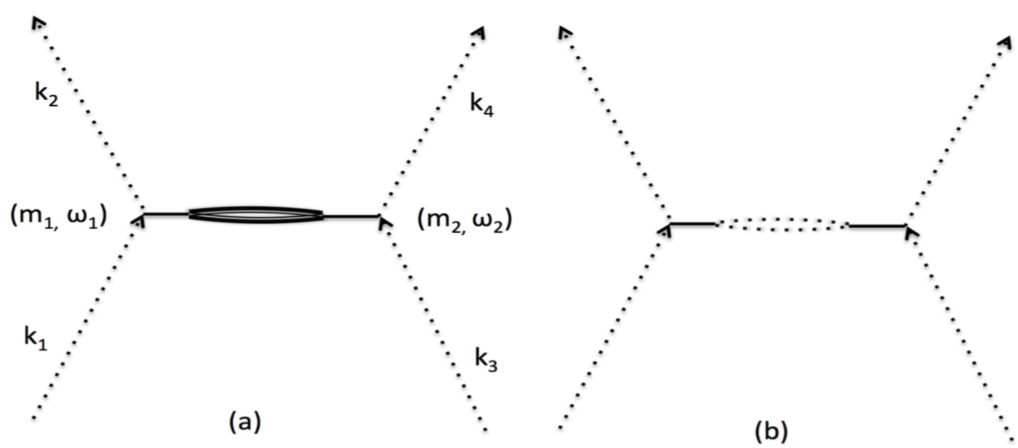

Fig 6:-The set of vacuum polarization diagrams contribute to the non-analytic corrections to the potential. The graviton loop diagram contains a ghost one, which is marked by the double line in (a).

By considering the gauge choice Eq. (3.8), which yields a Faddeev-Popov ghost along with the graviton loop, a ghost loop exists in FIG. 6(a). Accordingly, the vacuum polarization FIG. 6(a) for graviton and ghost loop diagram has the expression

$$i\mathcal{M}_{6(a)} = \tau_{\mu\nu}(k_1, k_2, m_{ss}) \frac{i\mathcal{P}^{\mu\nu\rho\sigma}}{q^2} \Pi_{\rho\sigma\gamma\delta} \frac{i\mathcal{P}^{\gamma\delta\alpha\beta}}{q^2} \tau_{\alpha\beta}(k_3, k_4, m_{is}), (3.53)$$

where the vacuum polarization tensor $\Pi_{\rho\sigma\gamma\delta}$ [85, 86] satisfies the Slavnov-Taylor identity $q_\mu q_\nu \mathcal{D}_{\mu\nu\rho\sigma}(q)\Pi_{\rho\sigma\gamma\delta}(q)\mathcal{D}_{\gamma\delta\alpha\beta}(q) = 0$.

It gives the contributions to the potential

$$V_{6(a)}(r) = -\frac{43}{30\pi}\frac{Gm_{ss}m_{is}}{r}\frac{G}{r^2}. (3.54)$$

While the corrections from the scalar loop diagram FIG. 6(b), which involves in a symmetry factor of 1/2!, can be written as

$$i\mathcal{M}_{6(b)} = \frac{1}{2!}\int \frac{d^4l}{(2\pi)^4}\frac{i\mathcal{P}_{\mu\nu\rho\sigma}}{q^2}\frac{iI_{\rho\sigma\gamma\delta}}{l^2}\frac{iI_{\rho\sigma\gamma\delta}}{(l+q)^2}\frac{i\mathcal{P}_{\gamma\delta\alpha\beta}}{q^2} \times \tau_1^{\mu\nu}(k_1, k_2, m_{ss})\tau_1^{\rho\sigma}(l, l+q)\tau_1^{\gamma\delta}(-l, -l-q)\tau_1^{\alpha\beta}(k_3, k_4, m_{is}). (3.55)$$

We find the results from scalar loop vacuum polarization diagram,

$$V_{6(b)}(r) = -\frac{1}{20\pi}\frac{Gm_{ss}m_{is}}{r}\frac{G}{r^2}. (3.56)$$

III. Results for the Gravitational Potential

Adding up all the non-analytical contributions, we get the total gravitational potential, containing both classical relativistic corrections and quantum corrections [87],

$$V_T(r) = -\frac{Gm_{ss}m_{is}}{r} - \frac{G\omega_{ss}\omega_{is}}{r} - \frac{3G^2 m_{ss}m_{is}(m_{ss}+m_{is})}{c^2r^2} - \frac{2G^2\omega_{ss}\omega_{is}(m_{ss}+m_{is})}{3c^2r^2} - \frac{4G^2\omega_{ss}\omega_{is}(\omega_{ss}+\omega_{is})}{c^2r^2} - \frac{81}{20\pi}\frac{Gm_{ss}m_{is}}{r}\frac{G\hbar}{c^3r^2} - \frac{23}{3\pi}\frac{G\omega_{ss}\omega_{is}}{r}\frac{G\hbar}{c^3r^2} - \frac{G({\omega_{ss}}^2{m_{is}}^2 + {\omega_{is}}^2 + {m_{ss}}^2)}{6\pi^2 m_{ss}m_{is}r}\frac{G\hbar}{c^3r^2}. (3.57)$$

It can be found that the gravitational interactions involving exchanges of either gravitons or scalar particles contribute to both classical relativistic post-Newtonian corrections and quantum corrections. The first two terms are modified Newtonian potential for a scalarized binary system, which represent the lowest order interactions of the two stars and dominate the potential at low energies. The next three terms denote the classical relativistic corrections to the gravitational potential, which are the leading post-Newtonian corrections in general relativity with the scalar charged stars. The classical relativistic corrections arise from just pure particle exchanges. The pure graviton exchanges contribute to the corrections to Newtonian piece, while the pure scalar exchanges produce the contributions to the scalar modified part of the potential. The last three terms represent the leading 1-loop corrections to gravitational potential of scalarized binaries from a quantum point of view. Looking at the quantum corrections, we notice that the contributions arising from graviton exchanges combine into one term, while that resulting from the exchanges involving scalar fields are split up into two terms. There is one term where the two scalar charges are multiplied together, which comes from the combined contributions of pure scalar exchanges. The last term originates from the mixed graviton-scalar field exchanges in 1PR diagrams FIG. 5(c) and FIG. 5(d), in which the two scalar charges are squared and separated, because of different compactness of two components, and the dependence of the scalar charges on the stars' compactness [47], even in DNS systems.

Summary And Discussions:-

The gravitational waves radiated from wide in-spiraling NS binaries, with orbital periods in units of days, locate in the typical low-frequency band of around 10^{-4} Hz. The amplitudes are of the order of 10^{-24}. So it is very unlikely to be detected currently by LIGO. However, the first space-based gravitational-wave observatory, LISA, is expected to detect space-born low-frequency gravitational waves, whose sensitivity can be reduced to 10^{-24}. Therefore, we would expect the gravitational waves and scalar counterparts from inspiraling scalarized DNS and NS-WD binaries to be detected and constrained potentially by LISA/eLISA in the near future. In addition, the masses of gravitational quanta from these systems depend on the binding energy of stars, which is the mission of NICER.

By considering the dependence of scalar charges on the stars' masses, the scalar charges, which are parameterized by the “sensitivities” ($s = 0.2$ for NSs and $s = 10^{-4}$ for white dwarfs), range from $10^{-4} - 0.1$ [46]. Therefore, the gravitational scalar effect of the potential is $\frac{G\omega_{ss}\omega_{is}}{r} \sim 10^{-18} - 10^{-13}\ \frac{m}{r}$ in scalarized DNS systems, which is smaller in NS binaries with white dwarf or main sequence companion stars. In the SI units, we can estimate the effect of quantum corrections in Eq. (3.57), i.e. $\frac{G\hbar}{c^3 r^2} \sim 10^{-70} m^2$.

Accordingly, the quantum effects on the gravitational potential are indeed small and seemingly impossible to be detected by astronomical observations. However, the quantum effects are on long distance. By taking the rotational effects of compact NSs into consideration, which give rise to space-time ripples, we expect that the detections and constraints on stochastic gravitational waves from pulsar timing array can give us some indications. In addition, the rotation of compact NS may disturb the space-time in its vicinity and lead to some virtual processes in the view of quantum field theory, which can be a potential tool to investigate the nature of dark energy. So we expect the cosmological observations for dark energy can give verifications and constraints to such quantum corrections NS binaries.

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