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RESEARCH ARTICLE

A CLASS OF INTEGRAL TRANSFORM.

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Abstract

In this research paper, we discuss new class of integral transform whose kernel involved a product of exponential function and I-function of two variables defined by K. Shantha Kumari, Vasudevan Nambisan and A.K. Rathie [2]. In the last particular cases of two dimensional integral transform are also discussed.

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Introduction:-

The double Mellin Barnes type contour integral occurring in this paper will be defined by [2] K. Shantha Kumari, T.M. Vasudevan Nambisan and Rathie A.K. and represented in the following manner.

$$I^{0,n_1;N_1}_{p_1,q_1;N_2}\left[\begin{matrix} z_1 \\ z_2 \end{matrix}\middle| \begin{matrix} ((P)),((Q)),((R)) \\ ((S)),((T)),((U)) \end{matrix}\right]$$

$$= \frac{1}{-4\pi^2}\int_{\mathcal{L}_s}\int_{\mathcal{L}_t} \phi(s,t)\theta_1(s)\theta_2(t)\, z_1^{s} z_2^{t}\, ds dt \qquad \textbf{(1.1)}$$

Where $\phi(s,t)$, $\theta_1(s)$, $\theta_2(t)$ are given by

$$\phi(s,t)=\frac{\prod_{j=1}^{n_1}\Gamma^{\xi_j}\left(1-a_j+\alpha_j s+A_j t\right)}{\prod_{j=n_1+1}^{p_1}\Gamma^{\xi_j}\left(a_j-\alpha_j s-A_j t\right)\prod_{j=1}^{q_1}\Gamma^{\eta_j}\left(1-b_j+\beta_j s+B_j t\right)}, \qquad \textbf{(1.2)}$$

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$$\theta_1(s)=\frac{\prod_{j=1}^{n_2}\Gamma^{U_j}\left(1-c_j+C_js\right)\prod_{j=1}^{m_2}\Gamma^{V_j}\left(d_j-D_js\right)}{\prod_{j=n_2+1}^{p_2}\Gamma^{U_j}\left(c_j-C_js\right)\prod_{j=m_2+1}^{q_2}\Gamma^{V_j}\left(1-d_j+D_js\right)}, \quad (1.3)$$

$$\theta_2(t)=\frac{\prod_{j=1}^{n_3}\Gamma^{P_j}\left(1-e_j+E_jt\right)\prod_{j=1}^{m_3}\Gamma^{Q_j}\left(f_j-F_jt\right)}{\prod_{j=n_3+1}^{p_3}\Gamma^{P_j}\left(e_j-E_jt\right)\prod_{j=m_3+1}^{q_3}\Gamma^{Q_j}\left(1-f_j+F_jt\right)}, \quad (1.4)$$

Also:

- $z_1 \neq 0, z_2 \neq 0$;
- $i = \sqrt{-1}$;
- an empty product is interpreted as unity;
- $\mathcal{L}_s$ and $\mathcal{L}_t$ are suitable contours of Mellin-Barnes type. Moreover, the contour $\mathcal{L}_s$ is in the complex s-plane and runs from $\sigma_1 - i\infty$ to $\sigma_1 + i\infty$, (σ_1 real) so that all the singularities of $\Gamma^{V_j}(d_j - D_js)(j=1,......,m_2)$ lies to the right of $\mathcal{L}_s$ and all the singularities of $\Gamma^{U_j}(1-c_j+C_js)(j=1,......,n_2)$, $\Gamma^{\xi_j}(1-a_j+\alpha_js+A_jt)(j=1,......,n_1)$ lie to the left of $\mathcal{L}_s$;
- The contour $\mathcal{L}_t$ is in the complex t-plane and runs from $\sigma_2 - i\infty$ to $\sigma_2 + i\infty$, (σ_2 real) so that all the singularities of $\Gamma^{Q_j}(f_j - F_jt)(j=1,......,m_3)$ lie to the right of $\mathcal{L}_t$ and all the singularities of $\Gamma^{P_j}(1-e_j+E_jt)(j=1,......,n_3)$, $\Gamma^{\xi_j}(1-a_j+\alpha_js+A_jt)(j=1,......,n_1)$ lie to the left of $\mathcal{L}_t$.

For the condition of existence and condition on the various parameters of I-function of two variables $I[Z_1, Z_2]$ we refer to [2,5] in (**1.1**) and that follows, we use the following notations for the sake of brevity.

$\mathrm{N}_1 \equiv m_2, n_2 : m_3, n_3;\quad \mathrm{N}_2 \equiv p_2, q_2 : p_3, q_3;$

And sets of parameters are

$((P)) \equiv \left(a_j, \alpha_j, A_j; \xi_j\right)_{1,p_1}, ((Q)) \equiv \left(c_j, C_j; U_j\right)_{1,p_2}, ((R)) \equiv \left(e_j, E_j; P_j\right)_{1,p_3}$

$((S)) \equiv \left(b_j, \beta_j, B_j; \eta_j\right)_{1,q_1}, ((T)) \equiv \left(d_j, D_j; V_j\right)_{1,q_2}, ((U)) \equiv \left(f_j, F_j; Q_j\right)_{1,q_3}$

Following the results of Braaksma [1, p.378] and Rathie [2, 7, 10, 11], it can easily be shown that the function defined in (**1.1**) is analytic function of Z_1 and Z_2 if R < 0 and S < 0. Where

$$R=\sum_{j=1}^{p_1}\xi_j\alpha_j+\sum_{j=1}^{p_2}U_jC_j-\sum_{j=1}^{q_2}\eta_j\beta_j-\sum_{j=1}^{q_2}V_jD_j, \quad (1.5)$$

$$S=\sum_{j=1}^{p_1}\xi_jA_j+\sum_{j=1}^{p_3}P_jE_j-\sum_{j=1}^{q_1}\eta_jB_j-\sum_{j=1}^{q_3}Q_jF_j, \quad (1.6)$$

And the integral (1.1) is convergent if,

$$\Delta_1 > 0, \Delta_2 > 0, |\arg(z_1)| < \frac{1}{2}\Delta_1\pi, |\arg(z_2)| < \frac{1}{2}\Delta_2\pi,$$

Where,

$$\Delta_1 = \left[\sum_{j=1}^{n_1}\xi_j\alpha_j - \sum_{j=n_1+1}^{p_1}\xi_j\alpha_j - \sum_{j=1}^{q_1}\eta_j\beta_j + \sum_{j=1}^{n_2}U_jC_j - \sum_{j=n_2+1}^{p_2}U_jC_j + \sum_{j=1}^{m_2}V_jD_j - \sum_{j=m_2+1}^{q_2}V_jD_j\right], \tag{1.7}$$

$$\Delta_2 = \left[\sum_{j=1}^{n_1}\xi_jA_j - \sum_{j=n_1+1}^{p_1}\xi_jA_j - \sum_{j=1}^{q_1}\eta_jB_j + \sum_{j=1}^{n_3}P_jE_j - \sum_{j=n_3+1}^{p_3}P_jE_j + \sum_{j=1}^{m_3}Q_jF_j - \sum_{j=m_3+1}^{q_3}Q_jF_j\right], \tag{1.8}$$

Integral (1.1) is convergent absolutely if

$$\Delta_1 \geq 0, \Delta_2 \geq 0, |\arg(z_1)| = \frac{1}{2}\Delta_1\pi, |\arg(z_2)| = \frac{1}{2}\Delta_2\pi$$

Transform for I-function of two variables with respect to 2-D:-

In this section we shall define the 2-D integral transform of the function $f(x,y)$ whose kernel involving a product of the exponential function and I-function of two variables defined by K. Shantha Kumari, Nambisan Vasudevan and Rathie, A.K. [2], which will be defined as follows:

$$\phi[f;s,t] = \int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} I^{0,n_1;N_1}_{p_1,q_1;N_2}\left[\begin{matrix}\alpha sx\\ \beta ty\end{matrix}\middle|\begin{matrix}((P)),((Q)),((R))\\ ((S)),((T)),((U))\end{matrix}\right] f(x,y)\,dxdt \tag{2.1}$$

Where, $0 < a < \infty$, $0 < b < \infty$, $s, t \neq 0$ and $f(x,y)$ is real and complex valued function of two variables x and y.

((P)), ((Q)), ((R)), ((S)), ((T)) and ((U)) are sets of parameters given in the previous section and a, b lies between $(0,\infty)$ such that the product $x^\alpha . y^\beta . f(x,y)$ is integrable over the finite region,

$$D:(D_1,D_2): 0 \leq x \leq D_1, 0 \leq y \leq D_2, D_1 > 0, D_2 > 0,$$

Where

$$\alpha = \min_{0\leq j\leq m_2}\left[\operatorname{Re}\left(\frac{d_jV_j}{D_j}\right)\right], \quad \text{and} \quad \beta = \min_{0\leq j\leq m_3}\left[\operatorname{Re}\left(\frac{f_jQ_j}{F_j}\right)\right],$$

We shall denote the two dimensional I-function transform of the function $f(x,y)$ by

$$I^{0,n_1;N_1}_{p_1,q_1;N_2}\{f(x,y);s,t\};$$

Special cases:

Case 1:

If all the exponents $\xi_j(j=1,......,p_1), \eta_j(j=1,......,q_1), U_j(j=1,......,p_2), V_j(j=1,......,q_2),$

$P_j(j=1,......,p_3), Q_j(j=1,......,q_3)$ are equal to unity and $n_1 = 0$,then **(2.1)** will reduce to the H-function of two variables defined by Mittal and Gupta [4] is as follows:

$$\phi[f;s,t] = \int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} I^{0,0;m_2,n_2:m_3,n_3}_{p_1,q_1;p_2,q_2:p_3,q_3}\left[\begin{matrix}\alpha sx \\ \beta ty\end{matrix}\middle| \begin{matrix}((a_j:\alpha_j,A_j;1));((c_j,C_j;1));((e_j,E_j;1)) \\ ((b_j:\beta_j,B_j;1));((d_j,D_j;1));((f_j,F_j;1))\end{matrix}\right] f(x,y)dxdy$$

$$\phi[f;s,t] = \int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} H^{0,0;m_2,n_2:m_3,n_3}_{p_1,q_1;p_2,q_2:p_3,q_3}\left[\begin{matrix}\alpha sx \\ \beta ty\end{matrix}\middle| \begin{matrix}(a_j:\alpha_j,A_j);(c_j,C_j);(e_j,E_j) \\ (b_j:\beta_j,B_j);(d_j,D_j);(f_j,F_j)\end{matrix}\right] f(x,y)dxdy, \tag{2.2}$$

Provided the double integral transform on the right hand side of (2.2) exists and defines the double H-function transform of the given function $f(x,y)$ under the following conditions.

1. $0 < a < \infty, 0 < b < \infty,\ s,t \neq 0$
2. All the convergence conditions of H-function of two variables x and y defined in the region $D: 0 \leq x \leq \infty$, $0 \leq y \leq \infty$ such that the product $x^\alpha . y^\beta . f(x,y)$ is integrable over the finite region.

$$D:(D_1,D_2):0 \leq x \leq D_1, 0 \leq y \leq D_2, D_1 > 0, D_2 > 0,$$

Where

$$\alpha = \min_{0 \leq j \leq m_2}\left[\operatorname{Re}\left(\frac{d_j}{D_j}\right)\right], \qquad \beta = \min_{0 \leq j \leq m_3}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right], \tag{2.3}$$

Case 2:-

In (2.2), taking $a = \infty, b = \infty$, then we get new two dimensional H-function transform [8, eq.(3), p.93].

$$\phi[f;s,t] = \int_0^\infty\int_0^\infty H^{0,0;m_2,n_2:m_3,n_3}_{p_1,q_1;p_2,q_2:p_3,q_3}\left[\begin{matrix}\alpha sx \\ \beta ty\end{matrix}\middle| \begin{matrix}(a_j:\alpha_j,A_j);(c_j,C_j);(e_j,E_j) \\ (b_j:\beta_j,B_j);(d_j,D_j);(f_j,F_j)\end{matrix}\right] f(x,y)dxdy, \tag{2.4}$$

Provided

$$\alpha = \min_{0 \leq j \leq m_2}\left[\operatorname{Re}\left(\frac{d_j}{D_j}\right)\right], \qquad \beta = \min_{0 \leq j \leq m_3}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right],$$

Case 3:-

If we take $p_1 = q_1 = n_1 = 0$ in (2.1) then transform degenerates into product of two I-function of one variable introduced by Rathie A.K. [7] as,

$$\phi[f;s,t] = \int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} I^{m_2,n_2}_{p_2,q_2}\left[\alpha sx\middle| \begin{matrix}(c_j,C_j;U_j)_{1,p_2} \\ (d_j,D_j:V_j)_{1,q_2}\end{matrix}\right] \times I^{m_3,n_3}_{p_3,q_3}\left[\beta ty\middle| \begin{matrix}(e_j,E_j;P_j)_{1,p_3} \\ (f_j,F_j:Q_j)_{1,q_3}\end{matrix}\right] f(x,y)dxdy, \tag{2.5}$$

Transform (2.5) is valid under condition obtainable and those with [8, Eq. (xvi), p. 90] hold.

Case 4:-

If $p_1 = q_1 = 0$ in (2.2) then transform degenerates into product of two H-function of one Variable given by Singhe, Gir-Raj [9], as,

$$\phi[f;s,t]=\int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} H^{0,0;m_2,n_2:m_3,n_3}_{0,0;p_2,q_2:p_3,q_3}\left[\begin{matrix}\alpha sx\\ \beta ty\end{matrix}\middle|\begin{matrix}\_,(c_j,C_j),(e_j,E_j)\\ \_,(d_j,D_j),(f_j,F_j)\end{matrix}\right] f(x,y)dxdy$$

$$\phi[f;s,t]=\int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} H^{m_2,n_2}_{p_2,q_2}\left[\alpha sx\middle|\begin{matrix}((c_j,C_j))\\ ((d_j,D_j))\end{matrix}\right]\times H^{m_3,n_3}_{p_3,q_3}\left[\beta ty\middle|\begin{matrix}((e_j,E_j))\\ ((f_j,F_j))\end{matrix}\right] f(x,y)dxdy, \quad (2.6)$$

Provided

$$\alpha=\min_{0\le j\le m_2}\left[\operatorname{Re}\left(\frac{d_j}{D_j}\right)\right], \qquad \beta=\min_{0\le j\le m_3}\left[\operatorname{Re}\left(\frac{f_j}{F_j}\right)\right],$$

Case 5:- In (2.1) putting

$\alpha_j = A_j = \xi_j = C_j = U_j = E_j = P_j = \beta_j = B_j = \eta_j = D_j = V_j = F_j = Q_j = 1$

And using the relation [3, p. 297 (45)], we get transform for G-function of two variables as,

$$\phi[f;s,t]=\int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} I^{0,n_1;m_2,n_2:m_3,n_3}_{p_1,q_1;p_2,q_2:p_3,q_3}\left[\begin{matrix}\alpha sx\\ \beta ty\end{matrix}\middle|\begin{matrix}(a_j:1,1;1)_{1,p_1};(c_j,1;1)_{1,p_2};(e_j,1;1)_{1,p_3}\\ (b_j:1,1;1)_{1,q_1};(d_j,1;1)_{1,q_2};(f_j,1;1)_{1,q_3}\end{matrix}\right] f(x,y)dxdy$$

$$\phi[f;s,t]=\int_0^\infty\int_0^\infty e^{\frac{-sx}{a}} e^{\frac{-ty}{b}} G^{0,n_1;m_2,n_2:m_3,n_3}_{p_1,q_1;p_2,q_2:p_3,q_3}\left[\begin{matrix}\alpha sx\\ \beta ty\end{matrix}\middle|\begin{matrix}(a_{p_1});(c_{p_2});(e_{p_3})\\ (b_{q_1});(d_{q_2});(f_{q_3})\end{matrix}\right] f(x,y)dxdy, \quad (2.7)$$

The sets of conditions mentioned with (2.2) and [8, Eq. 12.3, p.7] will satisfy.

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