Journal Homepage: -www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

Article DOI:10.21474/IJAR01/7985
DOI URL: http://dx.doi.org/10.21474/IJAR01/7985

RESEARCH ARTICLE

SHORT DEVELOPMENT IN FIXED POINT THEORY.

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Manuscript Info

Manuscript History
Received: 02 September 2018
Final Accepted: 04 October 2018
Published: November 2018

Keywords:-
point, Metric Space, Contraction mapping, Benach Contraction principle.

Abstract

In this paper, we study observe progressive result some fixed point theory related to complete. Some the important result from start to present.

Introduction:-

Fixed point theory is one of the most power fruitful and powerful tool of modern mathematics it may be core subject of nonlinear. The presence or absence of fixed point is an intrinsic property of function. However many necessary and sufficient condition for existence of such point involve mixture of algebraic theoretic. Fixed point theory plays a very important and significant role in analysis to solve differential equation. Partial differential equation as well as random differential equation and integral equation. It is also useful in solving the Eigen value problems for the characterization and the compleness of metric space.

Fixed point theory some use application in theory of game, non linear problems in mechanics as well as in integral domain equation and to Topological dynamics, Engineering Mathematics, Physics, Economics, Biology and Chemistry.

Definition :
Let X be a non-empty set. A function T : X→X is called a self map on X . A point z ϵ X is called a fixed point of a self map T: X→X, if T (z) = z

History of fixed point theory :
In 1886, Poincare was first to the work field then Brouwer in 1992, proved fixed point theorem for the solution of equation $f(x) = x$ mean while Benach principle came into existence which was considered as one of the fundamental principle in the field of functional analysis in 1922. Benach proved that a contraction mapping in field of a complete metric space. Possesses unique fixed point. Later on it was developed Kannan. Fixed point theory is an interdisciplinary topic which can be applied in various disciplines of Mathematical economics, optimixation theory approximation theory and variational in equalities.

In 20^{th} century may mathematicians like Brower, Schauder, Benach, Kannan, Kakuntani, Tarski and other.

Brouwer fixed point theorem :
In 1912, Brouwer published his famous fixed point theorem. The theorem states that

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Theorem 1.
If B is a closed unit ball in R^n and if T: B→B is continuous then T has a fixed point in B .

Remark:
The Brouwer's fixed point theorem guarantees the existence of fixed point. But it does not provide any information about the uniqueness and determination of the fixed point.

Schauder's fixed point theorem :
in 1930 Schauder was given The first fixed point theorem in an infinite dimensional Benach space . The theorem is stated below:

Theorem 2
If T: B→B is a continuous function on a compact, convex subset B of a Banach space X then f has a fixed point.

Remark:
The schauder fixed point theorem is very important and has several applications in economics, game theory, approximation theory etc. In the above theorem Schauder imposed a strong condition of compactness on B. Schauder relaxed this condition and established the following classical result

Theorem 3
If B has a closed bounded convex subset of a Banach space X and T: B → B is continuous map such that T(B) is compact, then T has a fixed point.

Tychonoff fixed point theorem
In 1935 The above Schauder's theorem was generalized to locally convex topological vector spaces by Tychonoff is as follows

Theorem 4
If B is a nonempty compact convex subset of a locally convex topological vector space X and T : B → B is a continuous map, then f has a fixed point Further extension of Tychonoff's theorem was given by KyFan A very interesting useful result in fixed point theory is due to Benach known as the Benach contraction principle.

Benach Contraction Principle
In 1922 Benach proved a classical fixed point theorem has many application in the existence and uniqueness problem of differential equation and integral equation. This theorem is also known as Benach Contraction Principle.

Theorem 5
If X is a complete metric space and T : X → X is a contraction map, then f has a unique fixed point or T (x) = x has a unique solution.

While Benach principle came in to existence which was considered as one of the fundamental principle in the field of functional analysis.

In this theorem Benach proved that a contraction mapping in the field of a complete metric space possesses a unique fixed point. Later on it was developed by Kannan.

The fixed point theory (as well as Benach contraction principle) has been studied and generalized in different spaces and various fixed point theorem were developed.

Rothe fixed point theorem
In 1937 Rothe gave a fixed point theorem for non self maps

Theorem 6
If T: B → R^n is a continuous map, such that
$T(\partial B) \subseteq B$ --------------→ (1)
Then T has a fixed point.

The famous fixed point theorem for non expansive maps was given by Browder, Kirk and Gohde independently in 1965.
Further extensions of iteration process due to Mann, Ishikawa , and Rhoades are worth mentioning.

The contraction, contractive and nonexpensive maps have been further extended to densifying, and 1- set contraction maps in 1969.

In 1966, Hartman and Stampacchia gave the following interesting result in variational inequalities.

Theorem 7
If B is a unit ball in R^n and T : B → R^n a continuous function, then there is a y ∈ B such that
$< Ty, x - y > \geq 0$ --------------→ (2) for all x ∈ B .

In 1969 the following result was given by KyFan commonly known as the best approximation theorem.

Theorem 8
If C is a nonempty compact convex subset of a normed linear space X And T: C → X is a continuous function, then there is a y ∈ C such that

$|Ty - y| = \inf|x - Ty|$ --------------→ (3) for all x ∈ C.

If P is a metric projection onto C, then P of has a fixed point if and only if (3) holds.

Recall that $d(x, C) = \inf||x - y||$ for all y ∈ C, x ≠ C.

The Ky Fan's theorem has been widely used in approximation theory, fixed point theory, variational inequalities, and other branches of mathematics.

Theorem 9
If T : B → X is a continuous function and one of the following boundary conditions are satisfied, then f has a fixed point. Here B is a closed ball of radius r and center 0 (∂B stands for the boundary of the ball B).

1. T(∂B) ⊆ B, (Rothe condition)
2. $|Tx - x|^2 \geq |Tx|^2 - |x|^2$, (Altman's condition)
3. If Tx = kx for x ∈ ∂B, then k ≤ 1 (Leray Schauder condition)
4. If T : B → X and Ty ≠ y, then the line segment [y, Ty] has at least two elements of B. (fan's condition).

A vast literature available for the fixed point theorem of multivalved maps. In 1941 Kakutani gave the following generalization of Brouwer fixed point theorem to multivalued maps.

Theorem 10
If T is multivalue map on close bounded Convex ' C' subset R^n such a that 'T' upper semicontinuous with nonempty closed convex values, then 'F' has fixed point

The fixed point is center of vigorous research activity and most of these result deal with commutating mapping. Concept of commutating mapping has useful for generalized theorem is major research activity in fixed point theory and application. Recent research Dolhare U.P. and Khanpate V.B. in this paper we introduce generalized altering distance and prove fixed point theorem for of ten unique fixed point.

The Topological degree theory :
Is a generalization of the winding number of curlles in a complex plane. It can be use estimate noumber of solution of an equation and is closely connected to fixed point theory. When one solution theory can of then be used to prove existence of second, non trival, solution. There are different types of degree for different types of map between Benach space. There is the Brower degree in R^n. Leray Schauder degree compact is normal space. The degree and various other types. The is also a degree for continuous maps between maniholds the Topological theory has application complenuously problem differential equation, differential inclusion and dynamical system.

Application to fixed point theorem

There are so many application of fixed point theorem some of the applications are as follows.

The integral equation :

these equation occur in applied mathematics, engineering and mathematical physics. They also arise representation formulas in the solution of differential equation.

The method of successive Approximation :

The method is very useful in determining solution of integral, differential and Algebric equation.

Some application of fixed point theory in economics and functional analysis.

Conclusion:-

In this fixed point theory Poincare and Brower's Theorem to Ky Fan theorem and fixed point theorem of generalized study of commutating map are have been briefly present. It's implicit function are used Game theory, functional analysis, other applied science and several application in economics mathematics.

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