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Cosmological models with Magnetized Anisotropic Dark Energy in Lyra Geometry

V. R. Chirde, S. H. Shekh*

Department of Mathematics, G. S. G. Mahavidyalaya, Umarkhed-445206.India

Department of Mathematics, Dr. B.N.College of Engg.& Tech., Yavatmal-445001.India.

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Corresponding Author

Dr. Arathi Rao

Abstract

We have studiedanisotropic and homogenous Bianchi Type VI_0 space-time under the assumption of anisotropy of the fluid within the frame work of Lyra manifold in the presence and absence of magnetism. A special form of deceleration parameter which gives an early deceleration and late time accelerating cosmological models has been utilized to solve the field equations. The physical and geometrical behaviors of the model are also discussed.

INTRODUCTION

One of the most intriguing modifications of general relativity is proposed by Weyl [1], invented to unify gravitation and electromagnetism by means of fundamental changes in Riemannian geometry. Unfortunately the Weyl theory suffers from non integrability of length which is physically unacceptable. However being interesting from mathematical point of view, it may still have the germs of a future fruitful theory. Later, Lyra [2] modified Riemannian geometry and removed non integrability of length transfer by introducing a gauge function into the structure-less manifold as a result of which a displacement field arises naturally. Subsequently, Sen et al. [3, 4] proposed a new scalar tensor theory of gravitation. They constructed an analog of the Einstein Field Equation based on Lyra's geometry. Halford [5, 6] showed that the scalar-tensor treatment based on Lyra's geometry predicts some effects within observational limitsas in Einstein's theory. Several authors Sen and Vanstone [7], Bhamra [8], Karade and Borikar [9], Kalyanshetti and Wagmode [10], Reddy and Innaiah [11], Beesham [12], Reddy and Venkateswarlu [13], Soleng [14] had studied cosmological models based on Lyra's manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one for convenience and there is no priori reason for it. Beesham [15] have considered Friedmann-Robertson-Walker (FRW) models with time dependent displacement field. Singh et al. [16−[20] had studied Bianchi-type I, III, Kantowaski-Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Robertson-Walker models with constant deceleration parameter in Einstein's theory with cosmological term in the cosmological theory based on Lyra's geometry. Soleng [14] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector φ will either include as creation field and be equal to Hoyle's creation field cosmology [21−23] or contain a special vacuum field, which together with the gauge vector term may be considered as a cosmological term. Sri Ram and Singh [24] have obtained exact solutions of the field equations in vacuum and in the presence of stiff-matter for anisotropic Bianchi type V cosmological models in the normal gauge with a time-dependent displacement vector field. Singh [25] considered flat FRW model in Lyra's geometry by using varying adiabatic equation of state and solved the field equations for the early phases of evolution of universe. Pradhan and Vishwakarma [26] have investigated a class of LRS Bianchi type I models in the cosmological theory based on Lyra's geometry by considering a time-dependent displacement field for constant deceleration parameter of the universe. Rahaman et al. [27] have obtained exact solutions for a spatially homogeneous and LRS Bianchi type-I model with constant deceleration parameter in Lyra's geometry. Kumar and Singh [28] have presented Bianchi type -I models in Lyra's geometry. Recently, Singh et al. [29] have obtained a new class of Bianchi type-I cosmological models in Lyra's geometry. Ram et al. [30] have obtained exact solutions for anisotropic Bianchi type V perfect fluid cosmological models in Lyra's geometry. Very recently, Chaubey [31] has obtained exact solutions for Kantowski-Sachs cosmological model in Lyra's geometry.

Complementary cosmological observations of supernovae of the type Ia (SNIa) [32, 33], cosmic microwave background radiation (CMBR) [34,40], large-scale structure (LSS) [41,42] and other cosmic phenomena have firmly established the picture of the accelerated expansion of the contemporary universe [43,44]. The present accelerating phase of the expansion of the universe and its onset at a relatively low redshift ($z \approx 1$) represent one of the most intriguing and most studied problems in modern cosmology. A lucid introduction and nice review of the work on dark energy model in general relativity is given by Farooq et al. [45] and Pradhan et al. [46]. Pradhan and Amihaschi [47,48], Amirhashchi et al. [49], Pradhan et al. [50] have discussed dark energy models in anisotropic Bianchi type space times with variable EoS parameter. Very recently, Naidu et al. [51-53] have presented Bianchi type-II and III dark energy models in Saez-Ballester [54] scalar-tensor theory of gravitation. Recently, Adhav K.S. [55] studied LRS Bianchi type – I universe with anisotropic dark energy in Lyra Geometry. V.K. Shchigolev[56] investigated cosmological models with a varying Λ – term in Lyra's geometry. Scalar Field Cosmology in Lyra's Geometry studied by V. K. Shchigolev et al. [57]. HoavoHova [58, 59] obtained a dark energy model in Lyra manifold and Vacuum expansion in arbitrary gauge Lyra geometry. Quintessence Cosmology with an Effective Λ-Term in Lyra Manifold investigated by M. Khurshudyan et al. [60]. Also Toy models of Universe with an Effective varying Λ-Term in Lyra Manifold investigated by [61].

Motivating with above research works we have studiedanisotropic and homogenous Bianchi Type VI_0 space-time under the assumption on the anisotropy of the fluid within the frame work of Lyra manifold in the presence and absence of Magnetism. A special form of deceleration parameter (q) which gives an early deceleration and late time accelerating cosmological models has been utilized to solve the field equations. The physical and geometrical behaviors of the model are also discussed.

2. Metric and field equations:

The spatially homogeneous and anisotropic Bianchi type – VI_0 metric is in the form

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 e^{-2qx} dy^2 - a_3^2 e^{2qx} dz^2, \tag{1}$$

where a_1, a_2, a_3 be the functions of cosmic time t only and q is the non zero constant.

The Einstein's field equation for Lyra geometry is given by

$$R_{ij} - \frac{1}{2} R g_{ij} + \frac{3}{2}\phi_i \phi_j - \frac{3}{4} g_{ij} \phi_\alpha \phi^\alpha = -T_{ij}, \tag{2}$$

where ϕ_i is the displacement field vector defined as $\phi_i = (0, 0, 0, \lambda(t))$, the symbols have their usual meaning i.e. R_{ij} is Ricci tenser, R is the Ricci scalar and T_{ij} is the usual stress energy momentum tensor of the matter.

The simplest generalization of EoS parameter of perfect fluid may be to determine the EoS parameter separately on each spatial axis by preserving the diagonal form of the energy momentum tensor in a consistence way with the considered metric. Therefore, the energy momentum tensor of fluid can be written most generally in anisotropic diagonal form as follows,

$$T_j^i = diag\left[T_1^1, T_2^2, T_3^3, T_4^4\right], \tag{3}$$

Allowing for anisotropy in the pressure of the fluid and thus in its EoS parameter gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor given in (3) as follows:

$$\begin{aligned} T_j^i &= diag\left[-p_x, -p_y, -p_z, \rho\right], \\ &= diag\left[-w_x, -w_y, -w_z, 1\right]\rho, \\ &= diag\left[-w\rho, -(w+\delta)\rho, -(w+\delta)\rho, \rho\right]. \end{aligned} \tag{4}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressure and w_x, w_y, w_z are the directional EoS parameter along the x, y, z axis respectively, δ be the deviations from the free EoS parameter (hence the deviation free pressure) on x axis and y axis.

In a co-moving co-ordinate system the field equations for the metric (1) with the help of equations (2) and (4) can be written as

$$\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3}+\frac{q^2}{a_1^2}+\frac{3}{4}\lambda^2=-w\rho \quad , \tag{5}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_3}{a_3}+\frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3}-\frac{q^2}{a_1^2}+\frac{3}{4}\lambda^2=-(w+\delta)\rho , \tag{6}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}-\frac{q^2}{a_1^2}+\frac{3}{4}\lambda^2=-(w+\delta)\rho , \tag{7}$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3}+\frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3}-\frac{q^2}{a_1^2}-\frac{3}{4}\lambda^2=\rho , \tag{8}$$

$$\frac{\dot{a}_2}{a_2}-\frac{\dot{a}_3}{a_3}=0 . \tag{9}$$

The conservation equation $T^{ij}_{;j}=0$ gives,

$$\dot{\rho}+(1+w)\rho\left(\frac{\dot{a}_1}{a_1}+2\frac{\dot{a}_2}{a_2}\right)+2\delta\rho\frac{\dot{a}_2}{a_2}=0 . \tag{10}$$

From equation (9).We get,

$a_2=\vartheta a_3$,

where ϑ is integration constant.

Without loss of generality, we consider $\vartheta=1$. We obtain,

$$a_2=a_3 . \tag{11}$$

Also from the field equation (2),

$$\left(R_{ij}-\frac{1}{2}R\,g_{ij}\right)_{;j}+\left(\frac{3}{2}\phi_i\,\phi_j\right)_{;j}-\left(\frac{3}{4}g_{ij}\,\phi_\alpha\,\phi^\alpha\right)_{;j}=-\left(T_{ij}\right)_{;j} ,$$

above equation leads to

$$\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j}+\phi^l\Gamma^j_{lj}\right]+\frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial\phi^j}+\phi_l\Gamma^l_{ij}\right]-\frac{3}{4}g^j_i\,\phi_k\left[\frac{\partial\phi^k}{\partial x^j}+\phi^l\Gamma^k_{lj}\right]-\frac{3}{4}\phi^k\left[\frac{\partial\phi_k}{\partial\phi^j}+\phi_l\Gamma^l_{kj}\right]=0$$

above equation is identically satisfied for $i=1,2,3$ .For $i=4$ reduces to

$$\frac{3}{2}\beta\left[\frac{\partial}{\partial x^4}\left(g^{44}\phi_4\right)+\phi^4\Gamma^j_{4j}\right]+\frac{3}{2}g^{44}\phi_4\left[\frac{\partial\phi_4}{\partial t}-\phi_4\Gamma^4_{44}\right]-\frac{3}{4}g^4_4\phi_4\left[\frac{\partial\phi_4}{\partial x^4}+\phi^4\Gamma^4_{44}\right]$$

$$-\frac{3}{4}g^4_4\,g^4_4\,\phi_4\left[\frac{\partial\phi_4}{\partial t}-\phi^4\;\Gamma^4_{44}\right]=0,$$

which leads to

$$\frac{3}{2}\lambda\lambda_4+\frac{3}{2}\lambda^2\left[\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right]=0 . \tag{12}$$

Using equation (11), equations (5) - (8), (10) and equation (12) become

$$2\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_2^2}{a_2^2}+\frac{q^2}{a_1^2}+\frac{3}{4}\lambda^2=-w\rho , \tag{13}$$

$$\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\dot{a}_1\dot{a}_2}{a_1a_2}-\frac{q^2}{\dot{a}_1^2}+\frac{3}{4}\lambda^2=-(w+\delta)\rho , \tag{14}$$

$$2\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2^2}{a_2^2}-\frac{q^2}{a_1^2}-\frac{3}{4}\lambda^2=\rho , \tag{15}$$

$$\frac{3}{2}\lambda\lambda_4+\frac{3}{2}\lambda^2\left[\frac{\dot{a}_1}{a_1}+2\frac{\dot{a}_2}{a_2}\right]=0 , \tag{16}$$

$$\dot{\rho}+(1+w)\rho\left(\frac{\dot{a}_1}{a_1}+2\frac{\dot{a}_2}{a_2}\right)+2\delta\rho\frac{\dot{a}_2}{a_2}=0 . \tag{17}$$

We define mean Hubble parameter for the Bianchi type–VI_0model which are important in cosmological observations

$$H = (\ln a)^{\bullet} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a}_1^2}{a_1} + 2\frac{\dot{a}_2^2}{a_2}\right), \tag{18}$$

with a being the average scale factor of Bianchi type-VI_0 model and can be expressed as

$$a = \left(a_1 a_2^2\right)^{\frac{1}{3}}. \tag{19}$$

The spatial volume (V) is given by

$$V = a^3 = \left(a_1 a_2^2\right). \tag{20}$$

The expansion scalar (θ), the shear scalar (σ^2) and the mean anisotropy parameter A_m are defined as

$$\theta = 3H = u^i_{;i} = \left(\frac{\dot{a}_1^2}{a_1} + 2\frac{\dot{a}_2^2}{a_2}\right), \tag{21}$$

$$\sigma^2 = \frac{3}{2}A_m H^2, \tag{22}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{23}$$

where the mean Hubble parameter and $H_i\left(i = 1, 2, 3\right)$ represent the directional Hubble parameters in the directions of x, y and z axes respectively.

The deceleration parameter q , defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{24}$$

3. Solution of field equations:

Since the field equations (13) - (16) are highly non linear, supply only four equations in six unknowns a_1 , a_2 , w , ρ , δ , and λ .In order to obtain exact solutions of the Einstein's field equations, we normally assume a form for the matter content or suppose that the space-time admits killing vector symmetries. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter, which was first proposed by Berman [62] in FRW models and that yields a constant value of deceleration parameter (DP). Cunha and Lima [63] favors recent acceleration and past deceleration with high degree of statistical confidence level by analyzing three SNe type Ia samples. In order to match this observation, Singh and Debnath [64], Adhav et al. [55] has defined a special form of deceleration parameter for FRW metric as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{\alpha}{1 + a^{\alpha}}, \tag{25}$$

where $\alpha > 0$ is a constant and a is mean scale factor of the universe.

After solving (25) one can obtain the mean Hubble parameter Has

$$H = \frac{\dot{a}}{a} = k(1 + a^{-\alpha}), \tag{26}$$

where$k$is a constant of integration.

On integrating (26), we obtain the mean scale factor as

$$a = \left(e^{\alpha kt} - 1\right)^{\frac{1}{\alpha}}. \tag{27}$$

Here we discussed two cases:

3.1 Case-I: Bianchi type VI_0 model for $a_2 = V^2$. (28)

Using equations (16), (19), (27) and (28).we get,

$$a_1 = \left(e^{\alpha kt} - 1\right)^{\frac{-9}{\alpha}}, \tag{29}$$

$$a_2 = a_3 = \left(e^{\alpha kt} - 1\right)^{\frac{6}{\alpha}}, \tag{30}$$

$$\lambda = \frac{c}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} \quad . \tag{31}$$

Therefore the metric (1) reduces to

$$ds^2 = dt^2 - \left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}} dx^2 - \left(e^{\alpha kt}-1\right)^{\frac{12}{\alpha}} e^{-2qx} dy^2 - \left(e^{\alpha kt}-1\right)^{\frac{-12}{\alpha}} e^{2qx} dz^2 . \tag{32}$$

Using equations (15) and (29-31), we get

Energy Density as

$$\rho = \frac{-72k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2} - \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}} - \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} . \tag{33}$$

Figure (1): demonstrate the behavior of the density parameter verses time in the evolution of universe as representative case with appropriate choice of constants of integration and other physical parameters, the figure (1) figure depicts that the energy density is decreasing function of time t.

Using equations (13) and (29-31).We get,

EoS parameter as

$$w = \frac{\left\{ \frac{12\alpha k^2 e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\left(1+\frac{6-\alpha}{\alpha}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\right) + \frac{36k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2} + \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}} + \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} \right\}}{\left\{ \frac{-72k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2} - \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}} - \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} \right\}} \quad . \tag{34}$$

The skewness parameters δ (i.e. deviation from w along y-axis and z-axis) are computed as

$$\delta = -\frac{\left\{ \frac{-15\alpha k^2 e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)} + \frac{15(\alpha+3)k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)} - \frac{90k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2} - \frac{2q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}} \right\}}{\left\{ \frac{-72k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2} - \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}} - \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} \right\}} . \tag{35}$$

3.2 Case-II: Bianchi type VI_0 model for $a_1 = V^{\frac{1}{2}}$. (36)

Using equations (16), (19), (27) and (36). We get,

$$a_1 = \left(e^{\alpha kt}-1\right)^{\frac{3}{2\alpha}} , \tag{37}$$

$$a_2 = a_3 = \left(e^{\alpha kt}-1\right)^{\frac{3}{4\alpha}} , \tag{38}$$

$$\lambda = \frac{c}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} . \tag{39}$$

Therefore the metric (1) reduces to

$$ds^2 = dt^2 - \left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}} dx^2 - \left(e^{\alpha kt}-1\right)^{\frac{3}{2\alpha}} e^{-2qx} dy^2 - \left(e^{\alpha kt}-1\right)^{\frac{3}{2\alpha}} e^{2qx} dz^2 . \tag{40}$$

From equations (15) and (37-39), we get

Energy density as

$$\rho = \frac{45k^2 e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2} - \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} - \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} . \tag{41}$$

Figure (2) depict energy density ρ versus time t. The energy density ρ are always decreasing function of time t and approach to constant value at $t \to \infty$. The following plot showing the positive decreasing function of time t.

From equations (13) and (37-39), we get
EoS parameter as

$$w=-\frac{\left\{\frac{3\alpha k^2 e^{\alpha kt}}{2\left(e^{\alpha kt}-1\right)}\left(1+\frac{3-4\alpha}{4\alpha}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\right)+\frac{9k^2e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2}+\frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}+\frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}}\right\}}{\left\{\frac{45k^2e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2}-\frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}-\frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}}\right\}}. \tag{42}$$

Figure (3) demonstrate the EoS parameter verses time t. as we know the different forms of dynamically changing dark energy with an effective equation of state (EoS), $w=\left(\frac{p}{\rho}\right)<-\frac{1}{3}$, were proposed instead of the constant vacuum energy density. Other possible forms of DE include quintessence ($w>-1$) Steinhardt et al.[65], phantom ($w<-1$) Caldwell [66] etc. While the possibility $w=-1$ is ruled out by current cosmological data from SN Ia (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) Riess et al.[67]; Astier et al.[68], CMBR (WMAP, BOOMERANG) (Eisentein et al.[69]; MacTavish et al.[70] and large scale structure (Sloan Digital Sky Survey) data Komatsu et al.[71], from the figure (3) we obtain $w<-1$ i.e. phantom field.
The skewness parameters δ (i.e. deviation from w along y-axis and z-axis) are computed as

$$\delta=-\frac{\left\{\frac{3\alpha k^2 e^{\alpha kt}}{2\left(e^{\alpha kt}-1\right)}\left(\frac{1}{2}+\frac{9-4\alpha}{8\alpha}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\right)+\frac{9k^2e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2}-\frac{2q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{2\alpha}}}\right\}}{\left\{\frac{45k^2e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2}-\frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}-\frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}}\right\}}. \tag{43}$$

4. Field equation with magnetized anisotropic dark energy

Allowing for anisotropy in the presence of the fluid and thus in EoS parameter gives rise to new possibilities for the evolution of the energy source. To see this, we first parameterize the energy momentum tensor for magnetized dark energy given in (3) as follows,

$$T_i^J=dia[\rho+\rho_B,-px+\rho_B,-p_y-\rho_B,-p_z-\rho_B],$$
$$=diag[-w\rho+\rho_B,-(w+\delta)\rho-\rho_B,-(w+\delta)\rho-\rho_B,\rho+\rho_B], \tag{44}$$

where ρ is the energy density of the fluid p_x, p_y, p_z are the pressure and w_x, w_y, w_z are the directional EoS parameter along the *x, y, z* axis respectively, δ be the deviations from the free EoS parameter (hence the deviation free pressure) on x axis and y axis and ρ_B stands for energy density of magnetic field
In a commoving co-ordinate system, the field equations for the metric (1) with the help of equations (2), (44) and (11) can be written as

$$2\frac{\ddot a_2}{a_2}+\frac{\dot a_2^2}{a_2^2}+\frac{q^2}{a_1^2}+\frac{3}{4}\lambda^2=-w\rho+\rho_B, \tag{45}$$

$$\frac{\ddot a_1}{a_1}+\frac{\ddot a_2}{a_2}+\frac{\dot a_1\dot a_2}{a_1a_2}-\frac{q^2}{\dot a_1^2}+\frac{3}{4}\lambda^2=-(w+\delta)\rho-\rho_B, \tag{46}$$

$$2\frac{\dot a_1\dot a_2}{a_1a_2}+\frac{\dot a_2^2}{a_2^2}-\frac{q^2}{a_1^2}-\frac{3}{4}\lambda^2=\rho+\rho_B, \tag{47}$$

$$\frac{3}{2}\lambda\lambda_4+\frac{3}{2}\lambda^2\left[\frac{\dot a_1}{a_1}+2\frac{\dot a_2}{a_2}\right]=0. \tag{48}$$

The conservation equation $T^{ij}_{;j}$ leads to

$$\dot{\rho}+(1+w)\rho\left(\frac{\dot{a}_1}{a_1}+2\frac{\dot{a}_2}{a_2}\right)+2\delta\rho\frac{\dot{a}_2}{a_2}=0, \tag{49}$$

$$\dot{\rho}_B+\rho_B\left(4\frac{\dot{a}_2}{a_2}\right)=0. \tag{50}$$

5. Solution of field equations:

5.1 Case-I: Bianchi type VI$_0$ model for $a_2=V^2$**.** (51)

Since the field equations (45)-(48) are highly non linear supply only four equations in seven unknowns a_1, a_2 w, ρ, ρ_B δ, and λ. Proceeding just as in Section 3.1 we again obtain the expression for the model (1) with ρ_B, ρ, w and δ as

$$\rho_B=\frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{24}{\alpha}}}. \tag{52}$$

Energy Density

$$\rho=\frac{-72k^2e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}-\frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}}-\frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}}-\frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{24}{\alpha}}}. \tag{53}$$

Figure (1): demonstrate the behavior of the density of magnetized anisotropic dark energy parameter verses time in the evolution of universe as representative case with appropriate choice of constants of integration and other physical parameters.

EoS parameter

$$w=-\frac{\left\{\frac{12\alpha k^2e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\left(1+\frac{6-\alpha}{\alpha}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\right)+\frac{36k^2e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}+\frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}}+\frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}}-\frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{24}{\alpha}}}\right\}}{\left\{\frac{-72k^2e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}-\frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}}-\frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}}-\frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{24}{\alpha}}}\right\}}. \tag{54}$$

The skewness parameters δ (i.e. deviation from w along y-axis and z-axis) are computed as

$$\delta=-\frac{\left\{\frac{-15\alpha k^2e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}+\frac{15(\alpha+3)k^2e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)}-\frac{90k^2e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}-\frac{2q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}}+\frac{2\beta}{\left(e^{\alpha kt}-1\right)^{\frac{24}{\alpha}}}\right\}}{\left\{\frac{-72k^2e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}-\frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{-18}{\alpha}}}-\frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}}-\frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{24}{\alpha}}}\right\}}. \tag{55}$$

5.2 Case-II: Bianchi type VI$_0$ model for $a_1=V^{\frac{1}{2}}$**.** (56)

Since the field equations (45)-(48) are highly non-linear supply only four equations in seven unknowns a_1, a_2 w, ρ, ρ_B, δ, and λ. Proceeding just as in Section 3.2 we again obtain the expression for the model (1) with ρ_B, ρ, w and δ as

$$\rho_B=\frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}. \tag{57}$$

Therefore the metric (1) reduces to

$$ds^2=dt^2-\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}dx^2-\left(e^{\alpha kt}-1\right)^{\frac{3}{2\alpha}}e^{-2qx}dy^2-\left(e^{\alpha kt}-1\right)^{\frac{3}{2\alpha}}e^{2qx}dz^2. \tag{58}$$

Energy density

$$\rho = \frac{45k^2 e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2} - \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} - \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} - \frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} . \tag{59}$$

Figure (2): demonstrate the behavior of energy density of magnetized anisotropic dark energy parameter verses time in the evolution of universe as representative case with appropriate choice of constants of integration and other physical parameters. From this figure we conclude that for $\alpha \geq 1$ the energy density is positive decreasing function of time *t*.

EoS parameter

$$w = -\frac{\left\{\frac{3\alpha k^2 e^{\alpha kt}}{2\left(e^{\alpha kt}-1\right)}\left(1+\frac{3-4\alpha}{4\alpha}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\right) + \frac{9k^2 e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2} + \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} + \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} - \frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}\right\}}{\left\{\frac{45k^2 e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2} - \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} - \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} - \frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}\right\}} . \tag{60}$$

Figure (3): demonstrate the EoS parameter verses time t. As we know the different forms of dynamically changing dark energy with an effective equation of state (EoS), $w = \left(\frac{p}{\rho}\right) < -\frac{1}{3}$ were proposed instead of the constant vacuum energy density. Other possible forms of dark energy include quintessence ($w > -1$) Steinhardt et al. [65], phantom ($w < -1$) Caldwell [66] etc. While the possibility $w = -1$ is ruled out by current cosmological data from SN Ia (Supernovae Legacy Survey, Gold sample of Hubble Space Telescope) Riess et al. [67]; Astier et al. [68], CMBR (WMAP, BOOMERANG) (Eisentein et al. [69]; MacTavish et al. [70] and large scale structure (Sloan Digital Sky Survey) data Komatsu et al. [71], from the figure (3) we obtain $w < -1$ i.e. phantom field.

The skew-ness parameters δ (i.e. deviation from *w* along *y*-axis and *z*-axis) are computed as

$$\delta = -\frac{\left\{\frac{3\alpha k^2 e^{\alpha kt}}{2\left(e^{\alpha kt}-1\right)}\left(\frac{1}{2}+\frac{9-4\alpha}{8\alpha}\frac{e^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}\right) + \frac{9k^2 e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2} - \frac{2q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{2\alpha}}} + \frac{2\beta}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}\right\}}{\left\{\frac{45k^2 e^{2\alpha kt}}{16\left(e^{\alpha kt}-1\right)^2} - \frac{q^2}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}} - \frac{3}{4}\frac{c^2}{\left(e^{\alpha kt}-1\right)^{\frac{6}{\alpha}}} - \frac{\beta}{\left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}}\right\}} . \tag{61}$$

6. Some Physical and Geometrical Properties of the Model:

Equation (49) represents Bianchi type VI_0 dark energy cosmological model in Lyra geometry. The physical properties that are important in cosmology are average scale factor *a*, spatial volume V, anisotropy expansion parameter A_m, Hubble parameter H ,expansion scalar θ, shear scalar σ^2 and the deceleration parameter which is describe from the equations (18) – (24) as

The average scale factor: $a = \left(e^{\alpha kt}-1\right)^{\frac{1}{\alpha}}$. (62)

The spatial volume: $V = \left(e^{\alpha kt}-1\right)^{\frac{3}{\alpha}}$. (63)

The anisotropy expansion parameter:

For case-I $A_m = 50$, for case-II $A_m = \frac{1}{8}$. (64)

The Hubble parameters: $H = \frac{ke^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}$. (65)

The expansion scalar: $\theta = \frac{3ke^{\alpha kt}}{\left(e^{\alpha kt}-1\right)}$. (66)

The shear scalar:

For Case-I $\sigma^2 = \frac{3}{2}\frac{k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}$, for case-II $\sigma^2 = \frac{3}{16}\frac{k^2 e^{2\alpha kt}}{\left(e^{\alpha kt}-1\right)^2}$. (67)

The deceleration parameter: $q = -1 + \frac{\alpha}{e^{\alpha kt}}$. (68)

Equations (62) and (63) represents that average scale factor *a* and Spatial volume *V*. The universe starts with big bang at $t = 0$. The average scale factor and spatial volume increases with time *t* i.e. when $t \to \infty$ then spatial volume $V \to \infty$. Thus inflation is possible in Bianchi type VI_0 mode. From the above results, we observe that spatial volume is zero at $t = 0$ and it increases with increase of *t*. This shows that the universe starts evolving with zero volume at $t = 0$and expands with cosmic time *t*. It can be seen that the Hubble's parameter and the scalar expansion is constant as $t \to \infty$. Also it is a positive decreasing function of time *t*.The shear scalar σ^2 diverge at $t = 0$ while they become constant as $t \to \infty$.
Figure (4) is the evidence for deceleration parameter $q = -1$. The sign of *q* indicates whether the model inflates or not. The negative sign of *q* indicates inflation. Also, recent observations of SNeIa, expose that the present universe is accelerating and the value of deceleration parameter lies on some place in the range $-1 \le q \le 0$. It follows that in our derived model, one can choose the value of deceleration parameter consistent with the observation. Figure (4) depicts the deceleration parameter (*q*) versus time (*t*) which gives the behavior of *q* from decelerating to accelerating phase for different values of α which is consistent with recent observations of Type Ia supernovae (Perlmutter et al. [33]; Riess et al. [32, 72]; Tonry et al. [73]; Clocchiatti et al. [74]).H. Weyl, Sitz. ber. PreussAkad. Wiss., [1]. Pradhan et al. (46).

7. Conclusion:

In this paper we have studied a spatially homogeneous and anisotropic Bianchi type-VI_0 space-time within the framework of the scalar-tensor theory of gravitation proposed by Lyra [2]. To find the deterministic solution, we have considered a special form of deceleration parameter which yields time dependent deceleration parameters. As we know that Universe decelerating in past and accelerating at present. Hence the DP must show signature flipping [75-77]. This scenario is consistent with observations (Perlmutter et al. [33]; Riess et al. [32, 67]; Tonry et al. [73]; Clocchiatti et al. [74]).Our whole discussions have been concentrated by putting $\alpha = 1, 2, 3$. By this choice, we find the present value of deceleration parameter in derived model as $q = -1$. This value is very near to the observed value of DP i.e. $-1 \le q \le 0$.

Thus, the solutions demonstrated in this paper may be useful for better understanding of the evolution of the universe in Bianchi type-VI_0 space-time within the framework of Lyra geometry. The solutions presented here can be one of the potential candidates to describe the observed universe.

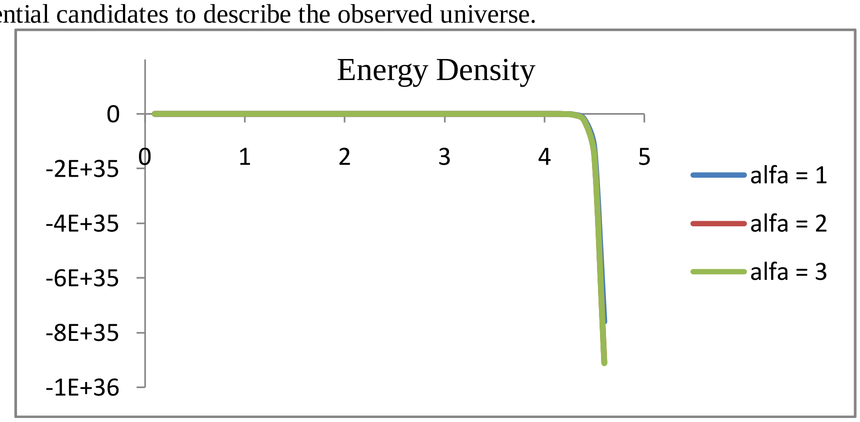

Figure (1): Energy density verses time

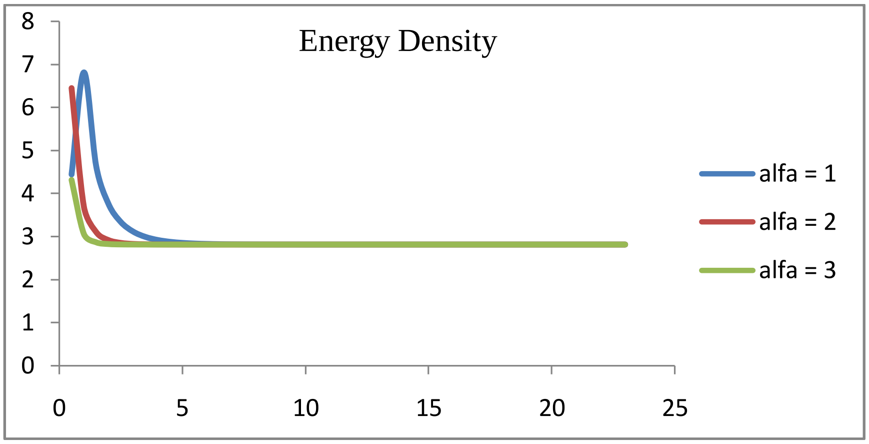

Figure (2): Energy density verses time

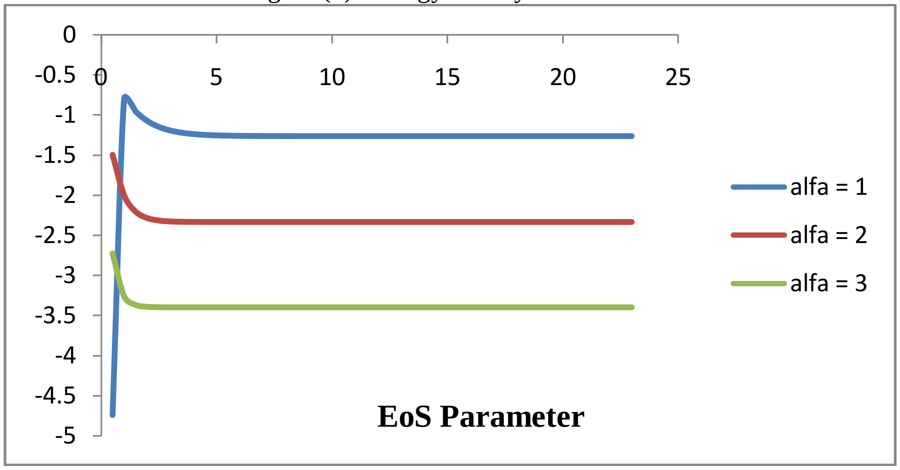

Figure (3): EoS parameter verses time

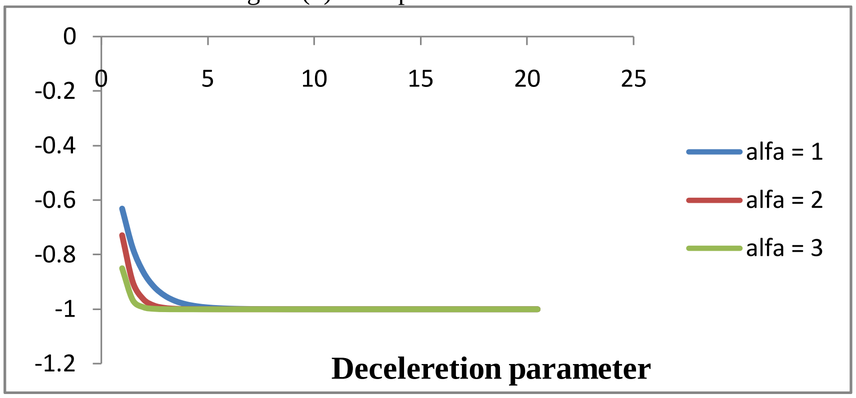

Figure (4): Deceleration Parameter verses time.

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