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RESEARCH ARTICLE

g-α-IRRESOLUTE HOMEOMORPHISM AND g-α-IRRESOLUTE QUOTIENT MAP.

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Abstract

In this paper g-α-irresolute homeomorphism, generalized quotient topology and g-α-irresolute quotient topology have been introduced. Moreover, some of their characterization have also studied.

Introduction:-

Császár [2] initiated the theory of generalized topological space, and studied the elementary character of these classes. Especially, he introduced the notion of continuous function on generalized topological spaces, and investigated characterization of generalized continuous functions (g, g')-continuous functions in [3]. In [4], [5], [6] W. K. Min introduced the notion of weak (g, g')-continuity, almost (g, g')-continuity, (α, g')-continuity, (ρ, g')-continuity, (β, g')- continuity on generalized topological spaces.

In [1] S. J. Bai and Y. P. Zuo introduced the notion of g-α-irresolute functions and investigated their properties and relationship between (g, g')-continuity [3] (resp. almost (g, g')-continuity [5], weak (g, g')-continuity [4], (α, g')-continuity [6], (ρ, g')-continuity [6], (β, g')- continuity [6]).

In the present paper, using the concept of of g-α-irresolute functions which is initiated by S. J. Bai and Y. P. Zuo [cf. 1], g-α-irresolute homeomorphism, generalized quotient topology and g-α-irresolute quotient topology have bee introduced. Moreover, some of their characterization have been studied and verify some results.

1.2 Preliminaries

The following definitions and the concepts are required for establishing the assertions of the present paper:

Definition 1.1

[2] Let X be a nonempty set and g be a collection of subsets of X. Then **g** is called a **generalized topology** (briefly **GT** on X if $\phi \in g$ and $G_i \in g$ for $i \in J$ ($\neq \phi$ and is an index set) implies $G = \cup_{i\in J} G_i \in g$. We say **g** is **strong** if $X \in g$; and we call the pair (X, g) a generalized topological space on X. The elements of g are called **g-open sets** and their complements are called **g-closed sets**.

Example 1.1

Let X = {a, b, c} then, we may define following generalized topologies on X. $g_1 = \{\phi, \{c\}\}$, $g_2 = \{\phi, \{b\}, \{c\}, \{b, c\}\}$, $g_3 = \{\phi, X, \{b, c\}, \{a, b\}\}$. In this Example

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g_1, g_2 are the generalized topologies but not strong generalized topologies (cf. Definition 1.1) whereas g_3 is not a topology but it is a strong generalized topology.

Now some important operators in the generalized topological space have been discussed.

Definition 1.2
Let A be the subset of X, then the generalized closure [2] of the set A is the **intersection of all g-closed sets containing A** and it is denoted by $\mathbf{c_g(A)}$.

Example 1.2
Let X = {a, b, c} be a space with the strong generalized topology $g = \{\phi, X, \{a, b\}, \{b, c\}\}$, $F = \{X, \phi, \{c\}, \{a\}\}$. Consider, subsets of X viz. A_1= {b, c}, A_2 = {a}, A_3= {a, b}, A_4 = {b}, A_5 = {c}. Then, the generalized closure of these sets are:
1. $c_g(A_1) = c_g\{b, c\} = X$
2. $c_g(A_2) = c_g\{a\} = \{a\}$
3. $c_g(A_3) = c_g\{a, b\} = X$
4. $c_g(A_4) = c_g\{b\} = X$
5. $c_g(A_5) = c_g\{c\} = \{c\}$

Definition 1.3
Let A be the subset of X, then the generalized interior [2] of the set A is the **union of all g-open sets contained in A** and it is denoted by $i_g(A)$.

Example1.3
Let X = {a, b, c} be a space with the strong generalized topology $g = \{\phi, X, \{a, b\}, \{b, c\}\}$. Consider subsets of X viz. A_1 = {b, c}, A_2 = {a}, A_3 = {a, b}, A_4 = {b}, A_5 = {c}, Then, the generalized interior of these sets are:
1. $i_g(A_1) = i_g\{b, c\} = \{b, c\}$
2. $i_g(A_2) = i_g\{a\} = \phi$
3. $i_g(A_3) = i_g\{a, b\} = \{a, b\}$
4. $i_g(A_4) = i_g\{b\} = \phi$
5. $i_g(A_5) = i_g\{c\} = \phi$

Remark 1.1
Following relations hold for the generalized interior and generalized closure of a subset A of a topological space:
1. $i_g(A) = X - c_g(X-A)$.
2. $c_g(A) = X - i_g(X-A)$.

In view of generalized topological space, some wider concepts of generalized open sets have been introduced and it is considered as follows:

Definition 1.4
Let (X, g) be a generalized topological space and $A \subseteq X$. Then A is said to be
1. **g-semi-open** [3] if $A \subseteq c_g(i_g(A))$.
2. **g-pre-open** [3] if $A \subseteq i_g(c_g(A))$.
3. **gr-open** [5] if $A = i_g(c_g(A))$.
4. **g-α-open** [3] if $A \subseteq i_g(c_g(i_g(A)))$.
5. **g-β-open** [3] if $A \subseteq c_g(i_g(c_g(A)))$.

The complement of a g-semi-open (respectively g-pre-open, g-α-open, g-β-open) set is called **g-semi-closed** (respectively **g-pre-closed,g-α-closed**, **g-β-closed**) set. The class of all g-semi-open sets (respectively g-pre-open sets, g-α-open sets, g-β-open sets) is denoted by **σ(g)** (respectively **ρ(g)**, **α(g)**, **β(g)**).

Remark 1.2
The following inclusions are the direct consequences of the definitions given above:
1. $g \subseteq \alpha(g) \subseteq \sigma(g)$
2. $g \subseteq \alpha(g) \subseteq \sigma(g) \subseteq \beta(g)$.

Example1.4
Let X = {a, b, c} with the strong generalized topology g = {X, ϕ, {a, c},{b, c}}, F = {ϕ, X, {b},{a}}. Consider A = {b, c} is a subset of X. Then we see that $i_g\{b, c\} = \{b, c\}$, $c_g(i_g\{b, c\}) = c_g\{b, c\} = X$ i.e. $A = \{b, c\} \subseteq c_g(i_g\{b, c\}) = c_g(i_g(A))$. Hence, A = {b, c} is **g-semi-open** set.

Example1.5
Let X = {a, b, c} with the generalized topology g = {ϕ, X, {a, b},{b, c}}, F = {X, ϕ,{c},{a}}. Consider A = {b} is a subset of X. Then, we see that $c_g\{b\} = X$, $i_g(c_g\{b\}) = i_gX = X$ i.e. $A = \{b\} \subseteq i_g(c_g\{b\}) = i_g(c_g(A))$. Hence, A = {b} is **g-pre-open** set.

Example1.6
Let X = {a, b, c} with the generalized topology g = {ϕ,{a},{b},{a, b}}, F = {X,{b, c},{a, c},{c}}. Consider A = {a, b} is a subset of X. Then, we have $c_g\{a, b\} = X$, $i_g(c_g\{a, b\}) = i_g(X) = \{a, b\}$ i.e. $A = \{a, b\} = i_g(c_g\{a, b\}) = i_g(c_g(A))$. Hence, A = {a, b} is **gr-open** set.

Example1.7
Let X = {a, b, c} with the generalizedtopology g = {ϕ, {a}}, F = {X,{b, c}}. ConsiderA = {a} is a subset of X. Then, we have $i_g\{a\} = \{a\}$,$c_g(i_g\{a\}) = c_g\{a\} = X$, $i_g(c_g(i_g\{a\})) = i_g(X) = \{a\}$ i.e. $A = \{a\} \subseteq i_g(c_g(i_g\{a\})) = i_g(c_g(i_g(A)))$. Hence, A = {a} is **g-α-open** set.

Example1.8
Let X = {a, b, c} with the generalized topology g = {ϕ, {a, c}}, F = {X, {b}}. Consider A= {a} is a subset of X. Then, we have $c_g\{a\} = X$,$i_g(c_g\{a\}) = i_gX = \{a, c\}$, $c_g(i_g(c_g\{a\})) = c_g\{a, c\} = X$, i.e. $A = \{a\} \subseteq c_g(i_g(c_g\{a\})) = c_g(i_g(c_g(A)))$. Hence, A= {a} is **g-β-open** set.

The formal definition of generalized continuity in generalized topological space is as follows:

Definition 1.1
Let (X, g_X) and $(X', g_X{}')$ be two generalized topological spaces. Then a function $f: X \to X'$ is said to be $(g_X, g_X{}')$-continuous [2] if $f^{-1}(V)$ is g-open set in X for every g-open set V in X'.

Example 1.1
Let X = {a, b, c} and X' = {1, 2, 3} are two generalized topological spaces and their respective generalized topologies are, $g_X = \{\phi, \{a\},\{a, b\}\}$, $g'_X = \{\phi,\{1\}\}$. Consider a function $f: X \to X'$; defined as:
f(a) = 1, f(b) = f(c) = 2

Claim:
f is $(\mathbf{g_X}, \mathbf{g'_X})$**-continuous.**
Since there are two g-open sets of X' viz. ϕ, {1} and their corresponding pre-images are ϕ, {a} in X which are g-open in X. Hence, **f** is $(\mathbf{g_X}, \mathbf{g'_X})$**-continuous.**

1.3 Generalization of Continuity in Generalized Topological Space
Considering the definition of $(g_X, g_X{}')$-continuity, several generalized versions have been discussed in [1].

Definition 1.3
[1] Let (X, g_X) and $(X', g_X{}')$ be generalized topological spaces. Then, a function $f: X \to X'$is said to be
1. $(\boldsymbol{\alpha}, \mathbf{g_X}')$**-continuous** if $f^{-1}(V)$ is **g-α-open** in X for every g-open set V in X'.
2. $(\boldsymbol{\sigma}, \mathbf{g_X}')$**-continuous** if $f^{-1}(V)$ is **g-semi-open** in X for every g-open set V in X'.
3. $(\boldsymbol{\rho}, \mathbf{g_X}')$**-continuous** if $f^{-1}(V)$ is **g-pre-open** in X for every g-open set V in X'.

4. $(\boldsymbol{\beta}, \mathbf{g_X}')$**-continuous** if $f^{-1}(V)$ is **g-β-open** in X for every g-open set V in X'.
5. **g-α-irresolute** if $f^{-1}(V)$ is **g-α-open** in X for every g-α-open set V in X'.

1.3 g-α-Irresolute Homeomorphism and Generalized Quotient Topology

Definition 1.3.1

Let (X, g_X) and (X', g'_X) be generalized topological spaces. Then, a function $f: X \to X'$ is said to be **g-α-irresolute homeomorphism** if

1. f is one–one .
2. f is onto.
3. f is g-α-irresolute**.**
4. f^{-1} is g-α-irresolute**.**

Example 1.3.1

Let X = {a, b, c, d} and X' = {1, 2, 3, 4} are two generalized topological spaces and their respective generalized topologies are, g_X = {ϕ , X, {a}, {a, b, c}, {a, b, d}}, $g_X{}'$ = {ϕ, X′, {1}, {1, 3, 4}, {1, 2, 4}}. Consider a function $f: X \to X'$ defined as: f(a) = 1, f(b) = 2, f(c) = 3, f(d) = 4.

1. It may seen that g_X and $g_X{}'$ are not a topology, hence we can not define a homeomorphism.
2. It may also seen that f is bijective.
3. It is easily check that the only g-α-open sets of X′ are {1},{1, 2}, {1, 3}, {1, 4}, {1, 2, 3},{1, 3 ,4} {1, 2, 4} and their respective pre-images are {a}, {a, b}, {a, c}, {a, d}, {a, b, c},{a, c, d},{a, b, d}, which are also g-α-open sets in X. Hence f is g-α-irresolute function.
4. Similarly we can see that f^{-1} is also g-α-irresolute function. Thus f is g-α-irresolute-homeomorphism.

Definition 1.3.2

Let (X, g_X) and (X', g'_X) be generalized topological spaces. Let $p: X \to X'$ be a surjective map. The map p is said to be a generalized quotient map, provided a subset U of X′ is g-open in X′ if and only if $p^{-1}(U)$ is g-open in X.

Example 1.3.2

Let X = {a, b, c, d} and X' = {1, 2, 3} are two generalized topological spaces and their respective generalized topologies are, g_X = {ϕ , X, {a}, {a, b},{b, c},{a, b, c}}, $g_X{}'$ = {ϕ, X′, {1}, {2}, {1, 2}}. Now define a function $f: X \to X'$ as: f(a) = 1, f(b) = f(c) = 2, f(d) = 3.

1. It is easy to see that f is surjective.
2. It can be easily seen that there are only five g-open sets of X′ are ϕ,{1},{2}, {1, 2}, X′ and their respective pre-images are ϕ, {a}, {b, c}, {a, b, c}, X which are also g-open sets in X. Hence f is $(g_X, g_X{}')$-continuous function. Similarly it is easily check that f^{-1} is also $(g_X, g_X{}')$-continuous.

Definition 1.3.3

If X is a space and A is a set and if $p: X \to A$ is a surjective map, then there exists exactly one generalized topology g_A on A relative to which p is a quotient map; it is called the quotient topology induced by p.

Remark 1.3.1

In the above example we see that the given function $f: X \to A$ is asurjective map, then there exists exactly one generalized topology g_A = {ϕ, {1},{2}, {1,2}}on A relative to which f is a quotient map; it is called the quotient topology induced by f..

Definition 1.3.4

Let X and X′ be generalized topological spaces. Let $p: X \to X'$ be a surjective map. The map p is said to be a g-α-irresolute quotient map, provided a subset U of X′ is g-α-open in X′ if and only if $p^{-1}(U)$ is g- α-open in X.

Example 1.3.4

Let X = {a, b, c, d} and X' = {1, 2, 3} are the generalized topological spaces and their respective generalized topologies are, g_X = {ϕ , X, {a}, {a, b, d},{a, b, c}}, $g_X{}'$ = {ϕ, X′, {1}, {1, 2}, {1, 3}}. Now define a function $f: X \to X'$ as: f (a) = 1, f (b) = f (c) = 2, f(d) = 3.

1. It is easy to see that f is surjective.

2. It can be easily seen that there are only five g- α-open sets of X′ are ϕ,{1},{1, 2}, {1, 3}, X′ and their respective pre-images are ϕ, {a}, {a, b, c}, {a, d}, X which are also g- α-open sets in X. Hence f is $(\alpha, g_{X}{}')$-continuousfunction. Similarly it is easily check that f^{-1} is also $(\alpha, g_{X}{}')$-continuous. Hence f is g-α-irresolute quotient map.

Definition 1.3.5
If X is a space and A is a set and if $p: X \rightarrow A$ is a surjective map, then there exists exactly one g-α-irresolute topology $g_{\alpha}A$ on A relative to which p is a generalized quotient map; it is called g-α-irresolute quotient topology induced by p.

Remark 1.3.2
In the above example we see that the given function $f: X \rightarrow A$ is asurjective map, then there exists exactly one g-α-irresolute topology $g_{\alpha}A$ = {ϕ, {1}, {1, 2}, {1, 3}, X'} on A relative to which f is a generalized quotient map i.e. g-α-irresolute quotient topology induced by f.

Theorem 1.3.1
Let $p: X \rightarrow Y$ be a generalized quotient map. Let Z be a space and $g: X \rightarrow Z$ be a g-continuous map that is constant on each set $p^{-1}(y)$, for $y \in Y$. Then g induces a g-continuous map $f: Y \rightarrow Z$ such that $fop = g$.

Proof:
For each $y \in Y$, the set $g\left(p^{-1}(y)\right)$ is a one point set in Z (since g is contant on $p^{-1}(y)$). If it is assume that $f(y)$ denote this point, then define a map $f: Y \rightarrow Z$ such that for each $x \in X$, $f\left(p(x)\right) = g(x)$.
To show that f is g-continuous, let V be a g-open set in Z. Continuity of g implies that $g^{-1}(V) = p^{-1}(f^{-1}(V))$ is g-open in X. Because p is a generalized quotient map, $f^{-1}(V)$ must be g-open in Y.

Conclusion:-
The concept of g-α-irresolute-continuity is the lighter concept of g-continuity which assigns g-α-open set of the domain to the g-α-open sets of the range. The situation where it is not possible to define g-homeomorphism, the g-α-irresolute-continuity may be applied to define the corresponding lighter form of g-α-irresolute-homeomorphism. A similar concept of homeomorphism can be extended for other continuous functions also. Most of the problems of Quantum Physics are dealt with the idea of homeomorphism and this concept can cover the wider range of such problems.

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