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RESEARCH ARTICLE

THE INTRINSIC PARITY PHASE FACTORS OF REGULAR GRAPHS

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Abstract

Let r and k be integers such that $1 \leq k < r$, and G an m-edge-connected, r-regular graph with v-vertices where $m \geq 1$. Again it is sharp for infinitely many v and we characterize when equality holds in the bound.

Introduction

Let G = (V,E) be a graph with vertex set V (G) and edge set E(G). Number of vertices of a graph G is called the order of G and is denoted by n. The number of edges of G is called the size of G and is denoted by e. For a vertex v of graph G, the number of edges of G incident to v is called the degree of v in G and is denoted by $d_G(v)$.
For two subsets S, T Ξ V (G),
lete_G(S; T) represent the number of edges of G joining S to T.

Let H be a function associating a subset of Z to each vertex of G.
A spanning subgraph F of graph G is called an H-factor of G if d_F (x) Є H(x) for every vertex x Є V (G). F or spanning subgraph F of G and for a vertex v of G,
define d(H; F, v) = min {$|d_f(v) - i|$ i Є H_v}, and
let d(H; F) = $\sum_{x \in v(G)}$ d(H;F,x).
Thus a spanning subgraph F is an H-factor if and only if d(H;F) = 0.
Let $d_H(G)$ =min { |(H;F) | F are spanning of G }
a spanning subgraph F is called H-optional if d(H;F) = $d_H(G)$.
The H-factor problem is to determine the value $d_H(G)$.
An integer h is called a gap of H(v) if h≠ $H_{(v)}$
but $H_{(v)}$ contains an element less than h and an element greater than h. Lov´asz [11] gave a structural description on the H-factor problem in the case where $H_{(v)}$ has no two consecutive gaps for all v ≠ V (G) and showed that the problem is NP-complete without this restriction. Moreover, he also conjectured that the decision problem of determining whether a graph has an H-factor is polynomial in the case where H(v) has no two consecutive gaps for all v Є V(G). Cornu´ejols [2] proved the conjecture.

Let therefore g,f : V → Z^+ such that g(v) ≤ f(v) and g(v) Ξ f(v) for every v ЄV. Then a spanning sub-graph F of G is called a (g,f) parity factor, if g(v) ≤ $d_f(v)$ ≤ f(v) and

$d_f(v) \equiv f(v)$ for all $v \in V$. A (g,f) parity factor is a special kind of H-factor and it has been shown that the decision problem of determining whether a graph has a (g,f) parity factor is polynomial.

Let a,b be two integers such that $1 \leq a \leq b$ and $a \equiv b$. if $g(v) = a$ and $f(v) = b$ for all $v \in V(G)$, then a (g,f) parity factor is called an (a,b) parity factor. Let $n \geq 1$ be odd. If a=1 and b=n, then an (a,b) parity factor is called an (1,n) odd factor. There is also a special case of the (g,f) factor problem which is called the even factor problem. The problem with $g(v) = 2$, $f(v) \geq |V(G)|$ and $f(v) \equiv g(v)$ for all $v \in V(G)$.
Fleischner suggested a sufficient criteria for a graph to have an even factor in terms of edge connectivtiy.

Theorem 1.(Cui and Kano, [3]). Let $h : V(G) \rightarrow N$ be odd value function. A graph G has a (1; h)-odd factor if and only if $o(G - S) \leq h(S)$ for all subsets $S \subset V(G)$.
Now there are many results on consecutive factors (i.e. (g; f)-factor). In non-consecutive factor problems, (g; f)-parity factors have many similar properties with k-factors. So it is believed that results on k-factors can be extended to (g, f)-factor. In this paper, we will extend a result on k-factors of regular graphs to the (g, f)-parity-factors.

Theorem 2 (Fleischner,[8]; Lov´asz, [1]). If G is a bridgeless graph with $d(G) \geq 3$, then G has an even factor.
For a general graph G and an integer k, a spanning subgraph F such that$d_F(x) = k$ for all $x \in V(G)$ is called a k-factor. Which is also a (k,k) parity factor.

Theorem 3 (Gallai [4). Let r and k be integers such that $1 \leq k < r$, and G an m-edge -connected r-regular graph, where $m \geq 1$. If one of the following conditions is valid, G has a k-factor.
(i) r is even, k is odd, |G| is even, and $r/m \leq k \leq r(1\text{-}1/m)$
(ii) r is odd, k is even and $2 \leq k \leq r(1\text{-}1/m)$
(iii) r and k are both odd and $r/m \leq k$.

Theorem 4. (Bollob´as, Saito andWormald). Let r and k be integers such that $1 \leq k < r$, and G be an m-edge-connected r-regular graph, where $m \geq 1$ is a positive integer. Let $m^* \in \{m,m+1\}$ such that $m \equiv 1$. If one of the following conditions is valid, G has a k-factor.
(i) r is odd, k is even and $2 \leq k \leq r(1\text{-}1/m)$
(ii) r and k are both odd and $r/m \leq k$

Theorem 5.(Lov´asz [7]). G has a (g, f)-parity factor if and only if for all disjoint subsets S and T of V (G),

$D(S,T) = f(S) + \sum d_G(x) - g(T) - e_G(S,T) - t \geq 0.$

Where t denotes the number of components C, called f- odd components of G – (SUT) such that $e_G(V(C),T) + f(V(C)) \equiv 1$. Moreover $d(S,T) \equiv f(V(G))$.

Theorem: let a, b and r be integers such that $1 \leq a \leq b < r$ and $a \equiv b$. Let G be an m-edge-connected r-regular graph with n vertices. Let $m^* \in \{m, m+1\}$ such that $m^* \equiv 1$. If one of the following conditions holds, then G has an (a,b) parity factor.

(i) R is even , a, b, are odd, |G| is even, $r/m \leq b$ and $a \leq r(1\text{-}1/m)$

(ii) R is odd, a,b are even and $a \leq r(1\text{-}1/m^*)$

(iii) R,a,b are odd and $r/m^* \leq b$.

Now we prove (i)

Let $\phi_1 = a/r$ and $\phi_2 = b/r$ .then $0 \leq \phi_1 \leq \phi_2 < 1$. Suppose that G contains no (a,b) parity factors, there exists two disjoint subsets S and T of V(G) such that $S \cup T \neq \phi$, and

$-2 \geq d(S,T) = b|S| + \sum d_G(x) - a|T| - e_G(S,T) - t$

Where t is the number of a-odd components C of G – (SUT). Let C_1, ……..C_t denote a-odd components of G –S-T and D=C_1 U …. U C_t.

Note that

$$-2 \geq d(S,T) = b|S| + \sum d_G(x) - a|T| - e_G(S,T) - t$$

$$= b|S| + (r-a)|T| - e_G(S,T) - t$$

$$= \phi_2 r|S| + (1-\phi_1)r|T| - e_G(S,T) - t$$

$$= \phi_2 \sum d_G(x) + (1-\phi_1)\sum d_G(x) - e_G(S,T) - t$$

$$\geq \phi_2 (e_G(S,T) + \sum_{i=1}^{T} e_G(S,C_i)) + (1-\phi_1)(e_G(S,T) + \sum_{i=1}^{T} e_G(T,C_i)) - e_G(S,T) - t$$

$$= \sum_{i=1}^{T} \phi_2 e_G(S,C_i) + (1-\phi_1)(e_G(T,C_i) - 1) + (\phi_2 - \phi_1) e_G(S,T)$$

$$\geq \sum_{i=1}^{T} \phi_2 e_G(S,C_i) + (1-\phi_1)(e_G(T,C_i) - 1).$$

Since G is connected and $0 < \phi_1 \leq \phi_2 < 1$, so $\phi_2 e_G(S,C_i) + (1-\phi_1) e_G(T,C_i) > 0$ for each C_i . Hence we obtain a contradiction by showing that for every $C=C_i$, $1 \leq I \leq t$, we have

$$\phi_2 e_G(S,C) + (1-\phi_1) e_G(T,C) \geq 1.$$

These inequalities implies,

$$-2 \geq d(S,T) \geq \sum_{i=1}^{T} \phi_2 e_G(S,C_i) + (1-\phi_1)(e_G(T,C_i) - 1)$$

$$> \sum_{i=1}^{T-2} (\phi_2 e_G(S,C_i) + (1-\phi_1)(e_G(T,C_i) - 1) - 2 \geq -2$$, Which is impossible

Since C is an a-odd component of G – (SUT), we have

$a|C| + e_G(T,C) \Xi 1$

moreover ,$r|C| = \sum_{x \in V(C)} d_G(x) = e_G(S\ UT, C) + 2|E(C)|$,

we have,

$$r|C| = e_G(S\ UT, C)$$

It is obvious that the two inequalities $e_G(S , C) \geq 1$ and $e_G(T,C) \geq 1$ imply

$\phi_2 e_G(S,C) + (1-\phi_1)(e_G(T,C) \geq \phi_2 + 1 - \phi_1 = 1$

hence we may assume $e_G(S,C) = 0$ or $e_G(T,C) = 0$

if$e_G(S,C) = 0$, then $e_G(T,C) \geq m$. since $a \leq r(1-1/m)$, then $\phi_1 \leq 1-1/m$ and

so $1 \leq (1-\phi_1)m$. By substituting $e_G(T,C) \geq m$ and $e_G(S,C) = 0$, we have

$$(1-\phi_1) e_G(T,C) \geq (1-\phi_1)m \geq 1.$$

If $e_G(T, C) = 0$, then $e_G(S, C) \geq m$. since $r/m \leq b$, hence $\phi_2 m \geq 1$, and so we obtain

$\phi_2 e_G (S, C) \geq \phi_2 m \geq 1$.

The proof is completed.

Let $r \geq 2$ be an integer ,$a,b \geq 1$ two odd integers and $2 \leq m \leq r - 2$ an even integer such that $b < r/m < a$. since G has an (a,b) parity factor if and only if G has an (r-b, r-a) parity factor, so we can assume $b < r/m$. Let J(r,m) be the complete graph K_{r+1} from which a matching of size m/2 is deleted.Connect each of these vertices to a vertex of degree r-1 of J(r,m). This gives an m-edge – connected – r- regular graph denoted by G. Let S denote the set of m new vertices and $T = \phi$. Let t denote the number of components C, which are called a- odd components of G – (S U T) and e_G (V(C) , T) + a|C| Ξ 1.

Then we have, t = r , and

$\delta (S,T) = b|S| + \sum_{x \in T} d_G - S(x) - a|T| - t(S, T) = bm - r < 0$.

So G contains no (a,b) parity factors.

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