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RESEARCH ARTICLE

A NONLINEAR MIXED CONVECTIVE OSCILLATORY FLOW OVER A SEMI-INFINITE VERTICAL PLATE THROUGH POROUS MEDIUM UNDER UNIFORM MAGNETIC FIELD

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Abstract

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The objective of the present work is to study the MHD mixed convection flow of an unsteady two-dimensional, laminar, viscous, incompressible fluid along a semi-infinite porous plate embedded in a porous medium with the presence of pressure gradient, thermal radiation field and uniform vertical magnetic field. The governing equations are obtained for the above physical configuration using conservation of mass, momentum and energy. The natures of these equations are non-linear and coupled with each other. Approximate solutions for nonlinear partial differential equations are solved using an analytical approach by double regular perturbation technique. During simplification it is assumed that the free stream consists of a mean velocity and temperature over which are superimposed an exponentially varying with time. The behavior of the velocity and temperature are discussed in detail for various non-dimensional parameters present in the problem. The values are found to be in good agreement with known results.

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Introduction:-

The study of heat and mass transfer of a fluid flow has considerable number of applications especially the science and engineering, biomechanical problems like the flow of blood in a tube, wind power, geo-thermal reservoirs, thermal insulation, drying of porous bed, many catalytic reactors, nuclear reactor cooling and the transport of energy in the underground etc. The study of oscillatory flow is very important from the technological viewpoint and such physical phenomenon was first studied by Lighthill (1954) for some two dimensional fluid flow. He was the one who first considered a two – dimensional flow of an incompressible viscous fluid by assuming that a regular fluctuating flow is superimposed on the mean steady boundary layer flow, which is completely solved by momentum method.

A comprehensive review of literature regarding the subject mentioned was well documented in a book written by Ingham and Pop (2005), Nield and Bejan (2006). The study of boundary-layer phenomena is of great importance in recent times owing to their wide applications in several engineering fields. The boundary-layer zone can be considered to be an interface region where fluid flow and heat transfer characteristics of two different, porous media and a fluid or of porous and impermeable media are adjusted to one another. To give a specific example, one can consider flow from petroleum reservoirs, wherein the oil flow encounters different layers of sand, rock, shale, lime stone, etc. Vafai and Thiyagaraja (1987) analyzed the flow and heat transfer at the interface region of porous medium. The analysis of natural convection about a vertical plate embedded in a porous medium was examined by

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Kim and Vafai (1989). Vafai and Kim (1990) obtained an exact solution for the interface region between a porous medium and a fluid layer.

Studies on the response of a laminar boundary-layer flow due to free-stream oscillations are of prime importance in many industrial and aerodynamic flow problems. Typical problems arise in the study of aircraft response to atmospheric gusts, aerofoil lift hysteresis at the stall, flutter phenomena involving wing, panel, and stalling flutter, as well as the prediction of flow over helicopter rotor blades and through turbo-machinery blade cascades. Stuart (1955) extended this idea to study a two-dimensional flow past an infinite porous plate when the free stream oscillates in time about a constant mean. Along with the unsteady velocity field, Stuart also studied the unsteady temperature field by assuming that there is no heat transfer between the plate and the fluid. The physical situation discussed by Stuart by one of the possible cases. Another physical phenomenon will be that if the difference between the plate temperature $T_w^{'}$, (temperature of the plate) and the free stream temperature $T_\infty^{'}$, (temperature of the fluid in the free stream) namely "$T_w^{'}$- $T_\infty^{'}$" is appreciably large causing the free convection currents to flow in the boundary layer and the free stream velocity is also oscillating in time about a constant mean in the direction of flow, then how is the flow field near a porous infinite vertical plate with constant suction affected by the free convective currents? An attempt in this direction was made by Soundalgekar (1973), who assumed that: (i) The plate temperature oscillates in time about a constant mean, (ii) the free convective currents are present in the boundary layer and (iii) the flow is very slow and hence viscous dissipative effects are negligible. This problem, governed by coupled linear differential equations was solved by Soundalgekar (1973), and it was observed by the author that the temperature field was not at all affected by the free convective currents. This is not always true. In subsonic flow of an incompressible fluid, the heat due to viscous dissipation is present in a number of physical phenomenon. Also, in the case of fluids with high Prandtl number, viscous dissipative heat is always present even in slow motions. Hence it is interesting and also important from the practical point of view to study the effects of the free convection currents on the oscillatory type of boundary layer flow. Now, in Stuart's case, it was observed that the mean flow was not affected by the frequency of the oncoming oscillatory flow.

In many practical problems, porous media have been used to provide an effective cooling device and it has been demonstrated using potential nature of Darcy equation. Rudraiah (1984), Kim and Vafai (1989) by considering non-Darcy equation with boundary and inertia effects that the heated vertical plate embedded in a porous medium significantly influences the flow and heat transfer characteristics. The works mentioned above are concerned with the study of steady convection when an impermeable vertical plate is embedded in a porous medium. It has been observed that a suction or injection at the plate controls heating by controlling the boundary layer. Later, this problem has been extended by Rudraiah to convective flow through a porous medium bounded by an infinite vertical porous plate with uniform free stream velocity away from the plate. Goma and Taweel (2005) examined the effects of oscillatory motion on heat transfer at vertical surfaces and developed a model that predicted both transient and time average heat-transfer rates. Effects of unsteady mixed convection boundary-layer flow along a symmetric wedge with variable surface temperature were investigated by Hossain et al. (2006).

Patil (2008) studied effects of free convection on the oscillatory flow of a polar fluid through a porous medium in the presence of variable wall heat flux. Girinath Reddy et al. (2017) investigated the combined effects of Soret and Dufour and variable fluid properties like viscosity and porosity on mixed double diffusive convective flow over an accelerating surface. Basavaraj et al. (2018) examined the effects of thermal radiation on casson fluid flow over a stretched surface of variable thickness in the presence of magnetic field. Recently Basavaraj (2019) investigated stability of the flow through porous media by considering uniform vertical magnetic field and the similar effect is considered in this problem also.However, for an effective convective cooling, heat pipes, direct contact heat exchangers and so on, it is advantageous to study the unsteady mixed convection on a heated permeable vertical plate embedded in a high porosity porous medium with fluctuating free stream and suction velocities without disturbing the uniform temperature maintained at the vertical plate. In spite of its importance this problem has not been given any attention to our knowledge.

Nomenclature:

a	Width of the rectangular channel	U	Dimensional free stream velocity
β	Thermal expansion coefficient	R_a	Thermal Rayleigh number

C_p	Specific heat at constant pressure	t	time
g	acceleration due to gravity	T	Dimensional temperature
Gr	Grashof number	T_w	Temperature at the plate
Pr	Prandtl number	T_∞	Fluid temperature far away from the plate
N	Frequency	u_0	Mean velocity
h	Height of the rectangular channel	u_1	Fluctuating part of the velocity
κ	Thermal diffusivity	v_0	Suction velocity
k	Permeability of the porous medium	U_0	constant velocity
K	Thermal conductivity	Q	Rate of heat flux
E	Eckert number	θ_1	Fluctuating part of the temperature
p	pressure	θ	Dimensionless temperature
u, v	velocity components along x and y direction	θ_0	Mean temperature
β_0	Co-efficient of thermal expansion	ε	perturbation parameter
M	Hartmann number	μ	Coefficient of viscosity
(x, y)	space co-ordinates	ρ_0	reference density
υ	kinematic viscosity	ψ	stream function
ρ	density of the fluid		

Mathematical formulation

We consider a two-dimensional, unsteady Boussinesq viscous fluid through a porous medium bounded by a vertical infinite porous plate. We assume an oscillatory suction velocity and the free stream velocity away from the porous plate oscillates about a mean constant value in a direction parallel to the *x*-axis. The *x*-axis is taken along the porous plate with direction opposite to the direction of gravity and the *y*-axis is the direction normal to the porous plate. The temperature of the vertical porous plate is maintained at a constant temperature. Since, the flow extends to infinity in the *x*-direction, the flow variables except the pressure *p* are functions of *y* and *t* only (see Fig. 1). Under these approximations, the basic equations of motion are:

$$\frac{\partial v}{\partial y} = 0. \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\bar{\upsilon}}{k}u - \frac{\sigma B_0{}^2 u}{\rho_0} + \beta\vec{g}(T - T_\infty), \tag{2}$$

$$\rho_0\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} - \frac{\bar{\upsilon}}{k}v, \tag{3}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho_0 C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho_0 C_p}\left[\frac{\partial u}{\partial y}\right]^2 + \frac{\mu_p}{\rho_0 C_p k}u^2 - \frac{\sigma B_0{}^2}{\rho_0 C_p}u^2 \tag{4}$$

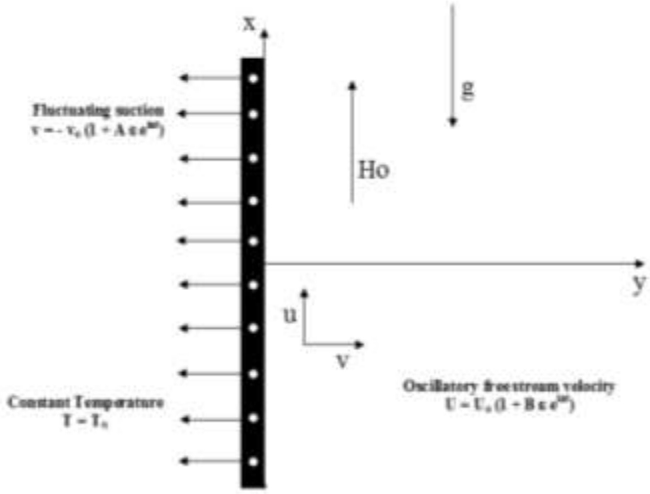

Fig.1:- Physical Configuration.

We assume the following assumptions to find an approximate analytical solution for the above governing equations. (i) x varies between $-\infty$ to $+\infty$, all the physical quantities are independent of x except for pressure, (ii) the Boussinesq approximation i.e., density is constant throughout the momentum equation except for body force, (iii)Incompressible fluid, density is constant. (iv) $B_0 = \mu H_0$, (v)From the equation of the state we consider density (ρ) is a function of temperature only, hence $\rho = \rho_0 \left[1-\beta(T-T_\infty)\right]$, (vi) We assume the fluctuating free-stream and suction velocities respectively as,

$$U(t) = U_0\left(1 + A\varepsilon e^{\mathrm{int}}\right), \tag{5}$$

$$v(t) = -v_0\left(1 + B\varepsilon e^{\mathrm{int}}\right). \tag{6}$$

where the negative sign indicates that the suction is directed towards porous plate. All the physical quantities are defined in the nomenclature. Based on the physical configuration which is as shown in Fig.1, the boundary conditions for the system takes the form:

$$\begin{aligned} &y = 0: \quad u = 0, \quad T = T_w. \\ &y \to \infty: \quad u \to U(t), \quad T \to T_\infty. \end{aligned} \tag{7}$$

These boundary conditions are derived on the assumption that the free stream velocity is fluctuating with time and maintaining uniform temperature away from the plate as well as at the plate.

The governing free stream velocity equation is,

$$\frac{dU}{dt} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} - \frac{\bar{\upsilon}}{k}U - \frac{\sigma B_0^2 U}{\rho_0}. \tag{8}$$

We solve the above governing non-linear partial differential equations by eliminating the pressure gradient between (2) and (8), we get

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{dU}{dt} + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\bar{\upsilon}}{k}(u-U) - \frac{\sigma B_0{}^2 U}{\rho_0}(u-U) + \beta\vec{g}(T-T_\infty)\ . \tag{9}$$

Making these equations dimensionless using,

$$y^* = \frac{\nu_o y}{\nu},\ u^\bullet = \frac{u}{U_o},\ \ t^* = \frac{\nu_o^2 t}{\nu},\ \ n^* = \frac{\nu n}{\nu_o^2},\ v^* = \frac{v}{\nu_o},\ \ k^* = \frac{\nu^2 k}{\nu_o^2},\ \ \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$Pr = \frac{\nu}{k},\ Gr = \frac{\beta \nu g (T_w - T_\infty)}{U_o^2 \nu_o^2},\ M = \frac{\sigma B_o^2 \nu}{\rho_o \nu_o^2},\ E = \frac{U_o^2}{C_p (T_w - T_\infty)} \tag{10}$$

and for simplicity neglecting the asterisks (*), we get

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + \frac{\lambda_1}{k}(U-u) + M(U-u) + Gr\,\theta, \tag{11}$$

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + E\left[\left(\frac{\lambda_1}{k} - M\right)u^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]. \tag{12}$$

The corresponding boundary conditions in dimensionless form are,

$$y = 0: \quad u = 0, \quad \theta = 1.$$
$$y \to \infty: \quad u \to (1 + \varepsilon A e^{\mathrm{int}}), \quad \theta \to 0. \tag{13}$$

Method of Solution:-

In order to solve the non-linear coupled partial differential equations (11) and (12) we will make use of the double regular perturbation method(one for ε and another for *E)*, the solutions of the form

$$u(y,t) = u_0(y) + \varepsilon u_1(y) e^{\mathrm{int}} + O(\varepsilon^2), \tag{14}$$

$$\theta(y,t) = \theta_0(y) + \varepsilon \theta_1(y) e^{\mathrm{int}} + O(\varepsilon^2). \tag{15}$$

Where ε is the perturbation parameter, which is a very small quantity.

Substituting equations (14) and (15) into equations (11) and (12) and equating the like powers of ε on both sides, we get the following system of equations.

The zeroth order steady equations in ε^0

$$u_0''(y) + u_0'(y) - (M + \frac{\lambda_1}{k}) u_0(y) = -Gr\,\theta_0 - (M + \frac{\lambda_1}{k}), \tag{16}$$

$$\theta_0''(y) + \Pr \theta_0'(y) = -E \Pr\left[\left(\frac{\lambda_1}{k} - M\right) u_0{}^2 + u_0'(y)\right]. \tag{17}$$

The corresponding boundary conditions are,

$$y = 0: \quad u_0 = 0, \quad \theta_0 = 1,$$
$$y \to \infty: \quad u_0 \to 1, \quad \theta_0 \to 0. \tag{18}$$

The first order unsteady equations in ε^1 are,

$$u_1''(y) + u_1'(y) - (in + M + \frac{\lambda_1}{k}) + u_1(y) = -Bu_0'(y) - \theta_1 Gr - (M + \frac{\lambda_1}{k})A, \tag{19}$$

$$\theta_1''(y) + Pr\,\theta_1'(y) - in\,Pr\,\theta_1(y) = B\theta_1'(y) - 2E\left[\left(\frac{\lambda_1}{k} - M\right)u_o(y)u_1(y) + u'_o(y)u'_1(y)\right] \tag{20}$$

The corresponding boundary conditions are,

$$y = 0: \quad u_1 = 0, \quad \theta_1 = 1,$$
$$y \to \infty: \quad u_1 \to A, \quad \theta_1 \to 0. \tag{21}$$

where the primes denote the differentiation with respect to *y*.

Equations (16), (17), (19) and (20) are coupled equations in u_0, θ_0, u_1 and θ_1.To solve these equations, we assume that heat due to viscous dissipation is superimposed on the motion. Mathematically this can be done by expanding the velocity and temperature in powers of E, because in the case of incompressible fluids, E is very small. Therefore we assume,

$$u_0(y)=u_{01}(y)+E\,u_{02}(y)+o(E^2)\,, \tag{22}$$

$$u_1(y)=u_{11}(y)+E\,u_{12}(y)+o(E^2)\,, \tag{23}$$

$$\theta_0(y)=\theta_{01}(y)+E\,\theta_{02}(y)+o(E^2)\,, \tag{24}$$

$$\theta_1(y)=\theta_{11}(y)+E\,\theta_{12}(y)+o(E^2)\,. \tag{25}$$

Substituting equations (22) and (23) in (16) and (19), and equating the co-efficients of the like powers of E, we get
The zeroth order steady equations on E^0 are,

$$u_{01}''(y)+u_{01}'(y)-(M+\frac{\lambda_1}{k})u_{01}(y)=-Gr\,\theta_{01}(y)-(M+\frac{\lambda_1}{k})\,, \tag{26}$$

$$\theta_{01}''(y)+\Pr\theta_{01}'(y)=0\,. \tag{27}$$

and the corresponding boundary conditions are,

$$\begin{aligned} y=0: &\quad u_{01}=0, \quad \theta_{01}=1,\\ y\to\infty: &\quad u_{01}\to 1, \quad \theta_{01}\to 0. \end{aligned} \tag{28}$$

The first order steady equations in E^1 are,

$$u_{02}''(y)+u_{02}'(y)-(M+\frac{\lambda_1}{k})u_{02}(y)=-Gr\,\theta_{02}(y)\,, \tag{29}$$

$$\theta_{02}''(y)+Pr\,\theta_{02}'(y)=-\left[\left(\frac{\lambda_1}{k}-M\right)u_{01}{}^2(y)+\left(u_{01}'(y)\right)^2\right]. \tag{30}$$

and the corresponding boundary conditions are,

$$\begin{aligned} y=0: &\quad u_{02}=0, \quad \theta_{02}=0,\\ y\to\infty: &\quad u_{02}\to 0, \quad \theta_{02}\to 0\,. \end{aligned} \tag{31}$$

Similarly, substituting equations (24) and (25) in (17) and (20), we get the following system of unsteady equations:
The zeroth order unsteady equations in E^0 are,

$$u_{11}''(y)+u_{11}'(y)-(in+M+\frac{\lambda_1}{k})u_{11}(y)=-Bu_{01}'(y)-Gr\,\theta_{11}(y)-(M+\frac{\lambda_1}{k})A, \tag{32}$$

$$\theta_{11}''(y)+\Pr\theta_{11}'(y)-in\Pr\theta_{11}(y)=B\theta_{01}'(y)\,. \tag{33}$$

and the corresponding boundary conditions are,

$$\begin{aligned} y=0: &\quad u_{11}=0, \quad \theta_{11}=0,\\ y\to\infty: &\quad u_{11}\to A, \quad \theta_{11}\to 0. \end{aligned} \tag{34}$$

The first order unsteady equations in E^1 are,

$$u_{12}''(y)+u_{12}'(y)-(in+M+\frac{\lambda_1}{k})u_{12}(y)=-Bu_{02}'(y)-Gr\,\theta_{12}(y)\,, \tag{35}$$

$$\begin{aligned}\theta_{12}''(y)+\Pr\theta_{12}'(y)-in\Pr\theta_{12}(y)= B\Pr\theta_{02}'(y)+\Pr u_{01}'(y)u_{11}'(y)-\\ 2\Pr\left[(\frac{\lambda_1}{k}-M)u_{01}(y)u_{11}(y)\right]\end{aligned} \tag{36}$$

and the corresponding boundary conditions are,

$$\begin{aligned} y=0: &\quad u_{12}=0, \quad \theta_{12}=0,\\ y\to\infty: &\quad u_{12}\to 0, \quad \theta_{12}\to 0. \end{aligned} \tag{37}$$

Solving the differential equations (26) to (36) using the corresponding boundary conditions and with suitable simplification, we get

$$u(y,t)=\left[\begin{pmatrix} A_1\,e^{-R_2 y}+\\ A_2\,e^{-\Pr y}+1\end{pmatrix}+E\begin{pmatrix} A_3\,e^{-2R_2 y}+A_4\,e^{-R_8 y}+A_5\\ e^{-2\Pr y}+A_6\,e^{-(R_2+\Pr)y}+\\ A_7(y+1)\,e^{-\Pr y}+A_8\end{pmatrix}\right]$$

$$+\varepsilon\left[\begin{matrix}\left(A_9\,e^{-R_2 y}+A_{10}\,e^{-R_4 y}+A_{11}\,e^{-R_6 y}+A_{12}\,e^{-\Pr y}+A_{13}\right)\\ +E\begin{pmatrix} A_{14}\,e^{-R_2 y}+A_{16}\,e^{-2R_2 y}+A_{17}\,e^{-R_6 y}+\\ A_{18}\,e^{-R_8 y}+A_{19}\,e^{-R_{10} y}+A_{20}\,(y+1)e^{-\Pr y}\\ +A_{21}\,e^{-2\Pr y}+A_{22}\,e^{-(R_2+\Pr)y}+A_{23}\,e^{-(R_2+R_6)y}\\ +A_{24}\,e^{-(R_2+R_4)y}+A_{25}\,e^{-(R_4+\Pr)y}+\\ A_{26}\,e^{-(R_6+\Pr)y}+A_{27}\end{pmatrix}\end{matrix}\right]e^{\mathrm{int}} \tag{38}$$

$$\theta(y,t)=\left[\left(e^{-\Pr y}\right)+E\begin{pmatrix} B_1e^{-R_2 y}+B_2e^{-2R_2 y}+B_3(y+1)e^{-R_2 y}+\\ B_4e^{-2\Pr y}+B_5e^{-(R_2+\Pr)y}\end{pmatrix}\right]$$

$$+\varepsilon\left[\begin{matrix}\left(B_6e^{-R_4 y}+B_7e^{-\Pr y}\right)+\\ E\begin{pmatrix} B_8e^{-R_2 y}+B_9e^{-2R_2 y}+B_{10}e^{-R_4 y}+B_{11}e^{-R_6 y}+\\ B_{12}e^{-R_{10} y}+B_{13}(y+1)e^{-\Pr y}+B_{14}e^{-2\Pr y}+\\ B_{15}e^{-(R_2+\Pr)y}+B_{16}e^{-(R_2+R_6)y}+B_{17}e^{-(R_2+R_4)y}\\ +B_{18}e^{-(R_6+\Pr)y}+B_{19}e^{-(R_4+\Pr)y}+B_{20}\end{pmatrix}\end{matrix}\right]e^{\mathrm{int}} \tag{39}$$

where

$$R_{1,2}=R_{7,8}=\frac{-1\pm\sqrt{1+4K_1}}{2}\qquad R_{3,4}=\frac{-\Pr\pm\sqrt{\Pr^2+4\Pr in}}{2}$$

$$R_{5,6}=\frac{-1\pm\sqrt{1+4K_2}}{2}\qquad R_{9,10}=\frac{-\Pr\pm\sqrt{\Pr^2-4K_4}}{2}\ .$$

The constants A_i (i=1 to 27) and B_i (i=1 to 20) are the functions of non-dimensional parameters involved in the problem. For the want of space the expressions for them are omitted here but given in appendix. However, they are numerically computed and used in computing u and θ.

Results And Discussion:-

The analytical solutions for the velocity and temperature are found using double perturbation technique. The numerical computation is performed for velocity and temperature for different values of non-dimensionalised parameters that are involved in the solution. The graphical representation for velocity and temperature are depicted in Fig.2 to Fig. 21. Fig.2 to Fig.7 shows the variation of velocity and temperature for positive and negative values of Grashof number *Gr* and also for Eckert number E = 0.01 and E = 0.03. Fig.2 and Fig.4 shows the mean velocity for air increases due to more cooling of the vertical permeable wall by the free convection currents when E is constant. Physically this can be explained as follows:

In the process of cooling the plate by free convection currents are carried away from the plate to the free stream and as the free stream is in the upward direction, the free convection currents induce the mean velocity to increase. In order that these results may be useful for experimental verification similarly when the value of E at 0.03 the same behavior can be seen which is depicted in Fig. 4. Fig.6 shows that variation of velocity for different negative values of Grashof number *Gr*. This means the mean velocity for air is negative and decrease due to more heating of the plate and increases due to more addition of viscous dissipation of heat. Physically this is possible because the flow of air moving in the upward direction both near and away from the vertical permeable wall, is now being opposed by the free convection currents traveling towards the vertical permeable wall and hence the mean velocity decreases. Thus the mean flow of air is of reversed type when the vertical permeable wall is being heated by the free convection currents.

Fig.3 & Fig.5 represents the plot of temperature versus y for different positive values of Grashof number Gr and Eckert number E = 0.01 & E = 0.03 which shows the increasing due to more cooling of the vertical permeable wall decreases the temperature from the plate and away from the plate. Similarly an opposite behavior can be seen for temperature versus y for different negative values of Grashof number Gr and for the Eckert number E = 0.03 which is depicted in the Fig.7. Physically this represents for more heating of the vertical permeable wall. To understand the mean velocity and temperature variation for different fluids, which represents the increase in the Prandtl number Pr which is depicted in the Fig.8 to Fig. 11.

Fig. 8 and Fig. 10 represents the velocity variation for different fluids from air to mercury. For small Prandtl number (0.71 to 3) the variation of velocity are large for cooling of the vertical permeable wall. For higher or large values of Prandtl number (Pr>3), the variation of velocity are very less because due to viscous dissipation. Fig.9 and Fig.11 represents the behavior of temperature for different fluids (Pr=0.71 to 7) due to more cooling of the vertical permeable wall (Gr>0) for different values of Eckert number E=0.01 and E=0.03. From Fig.9 the temperature decreases near the vertical permeable wall for higher values of Prandtl number for the constant value E=0.01 and the same behavior can be seen for the Eckert number E=0.03 in Fig.11.

Fig.12 to Fig.15 shows the velocity and temperature for different values of permeable parameter k of the porous media, for the constant values of E=0.01 and E=0.03. If the permeability of the porous media increases there is increase in variation for the velocity of the fluid but much variation is not seen for the temperature of the fluids. In Fig. 16 and Fig.17 we observe the variation of velocity and temperature for different values of viscous ratio λ due to more cooling of the vertical permeable wall (Gr>0), with the increase in the viscous ratio there is a decrease in the velocity of the fluid and increase in the temperature can be seen in Fig.16 and Fig.17 respectively.

From the Fig.18 to Fig.21 gives the variation of velocity and temperature versus y for different values of Hartmann number M which is a measure of Laurent's force to viscous force in a finitely conducting fluid. As the Hartmann number M increases due to more cooling of the vertical permeable wall (Gr>0) the velocity of the fluid decreases due to an increase of Laurent's force to viscous force. An opposite behavior are been observed for the temperature of the fluid for the increase of Laurent's force to viscous force for different constant values of Eckert number E=0.01 to E=0.03. Fig.22 and Fig.23 represents the variation of velocity and temperature versus y for different values of frequency parameter n. As the frequency of the suction velocity at the vertical permeable wall increases there is a decrease of velocity as well as temperature are been observed and the same is depicted in the figures. Similarly an opposite behavior is been observed for the velocity and temperature due to heating of the vertical permeable wall for different values of Pr, λ , M and E.

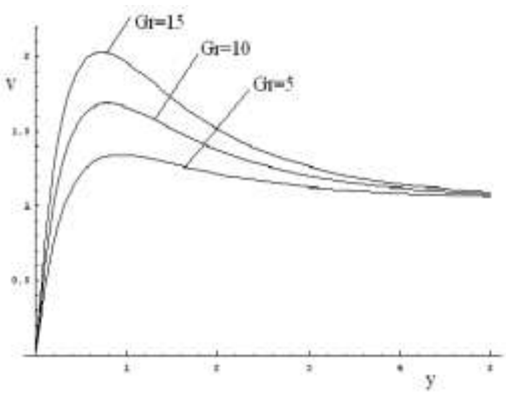

Fig. 2: Plot of velocity versus y for different values of Gr.
(Pr=0.71; M=5; k=0.5; E =0.01; ε = 0.1)

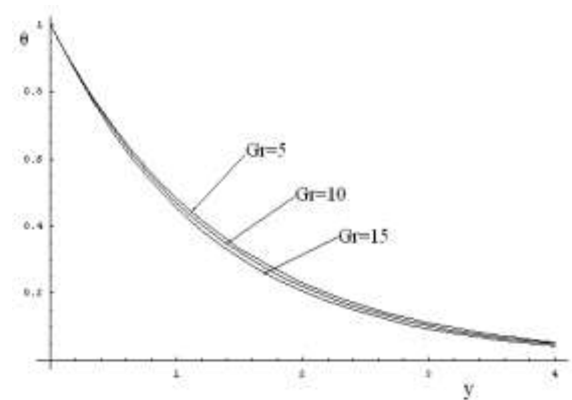

Fig. 3: Plot of temperature versus y for different values of Gr.
(Pr=0.71; M=5; k=0.5; E =0.01; ε = 0.1)

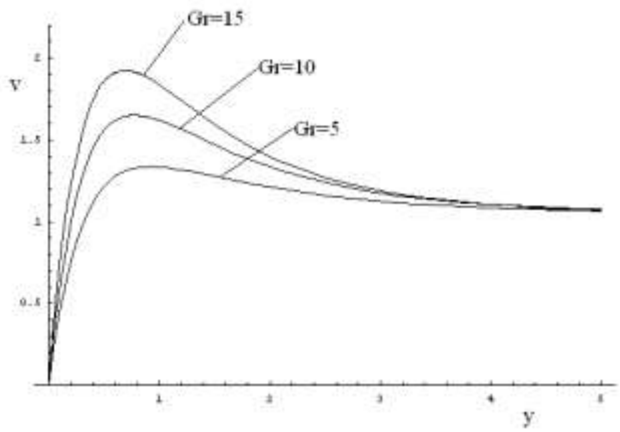

Fig. 4: Plot of velocity versus *y* for different values of *Gr*.
(Pr=0.71; M=5; k=0.5; E =0.03; ε = 0.1)

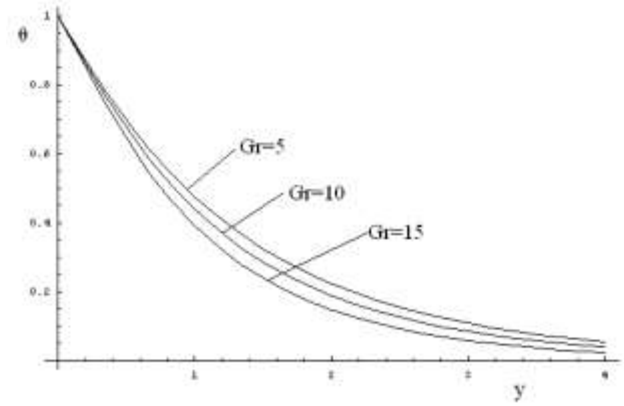

Fig. 5: Plot of temperature versus *y* for different values of *Gr*.
(Pr=0.71; M=5; k=0.5; E =0.03; ε = 0.1)

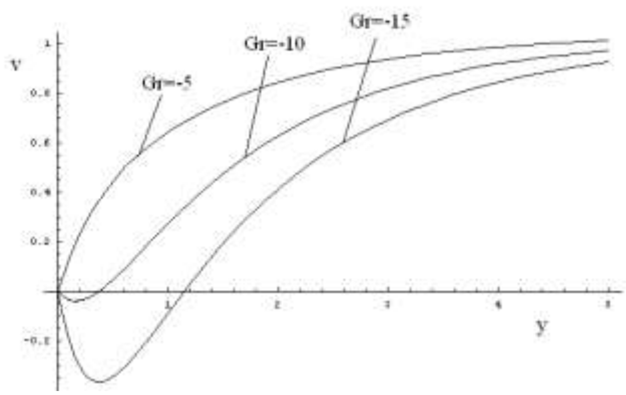

Fig. 6: Plot of velocity versus *y* for different negative values of *Gr*.
(Pr=0.71; M=5; k=0.5; E =0.03; ε = 0.1)

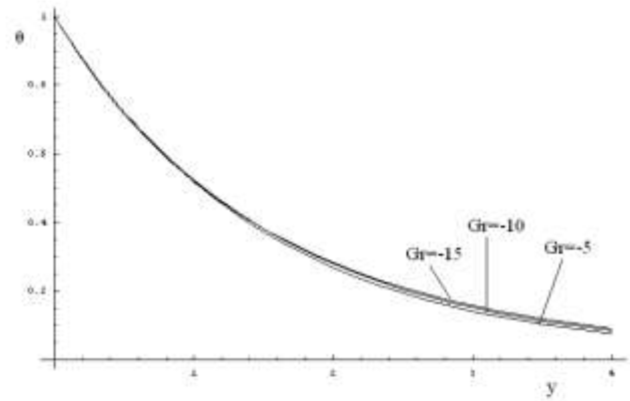

Fig. 7: Plot of temperature versus *y* for different negative values of *Gr*.
(Pr=0.71; M=5; k=0.5; E =0.03; ε = 0.1)

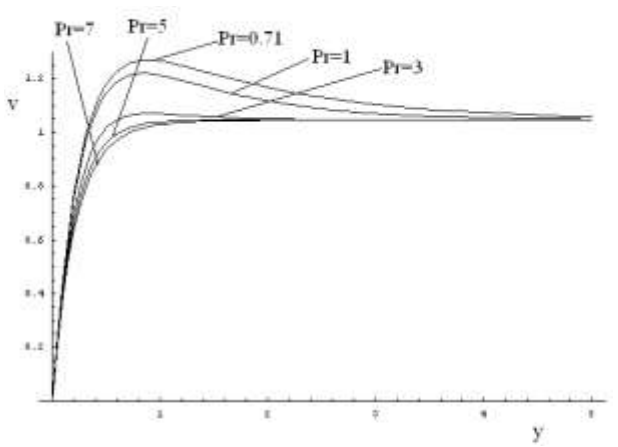

Fig. 8: Plot of velocity versus *y* for different values of *Pr*.
(Gr=5; M=5; k=0.5; E =0.01; ε = 0.1)

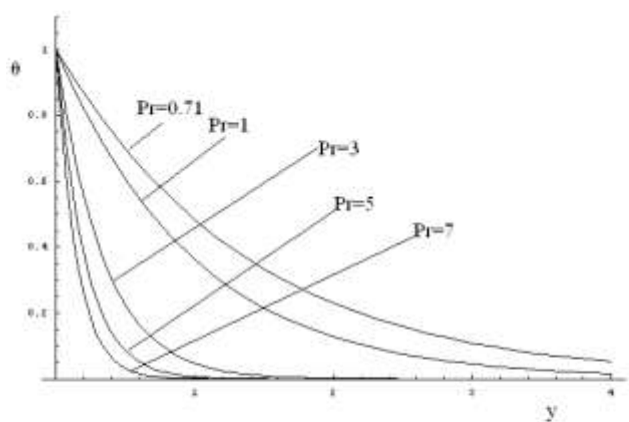

Fig. 9: Plot of temperature versus *y* for different values of *Pr*.
(Gr=5; M=5; k=0.5; E =0.01; ε = 0.1)

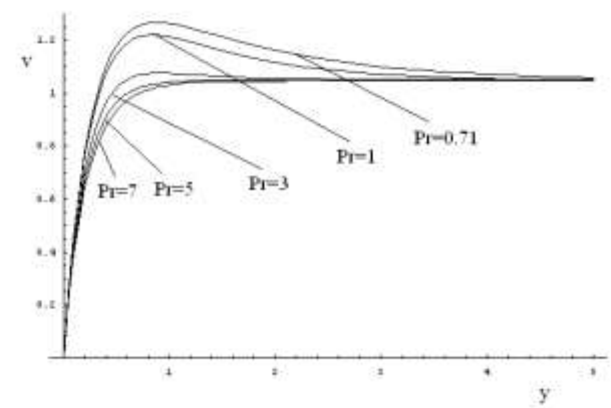

Fig. 10: Plot of velocity versus *y* for different values of *Pr*.
(Gr=5; M=5; k=0.5; E =0.03; ε = 0.1)

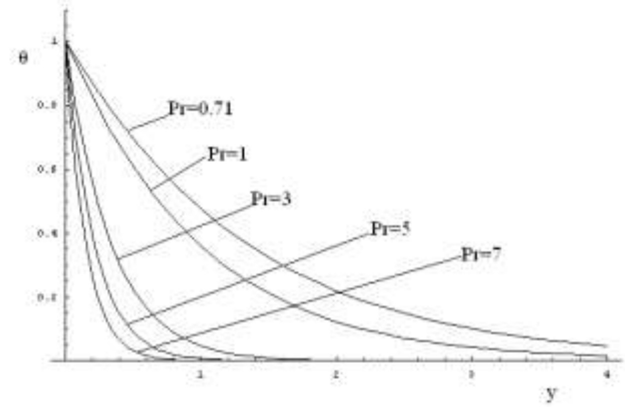

Fig. 11: Plot of temperature versus *y* for different values of *Pr*.
(Gr=5; M=5; k=0.5; E =0.03; ε = 0.1)

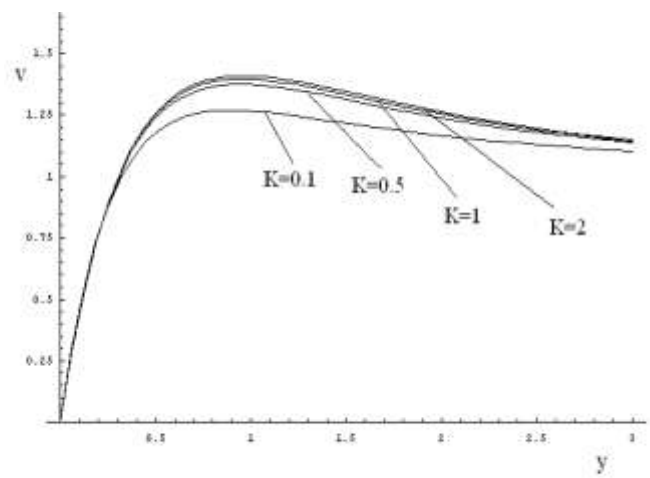

Fig.12: Plot of velocity versus y for different values of *k*.
(Pr=0.71; Gr=5; M=5; E =0.01; ε = 0.1)

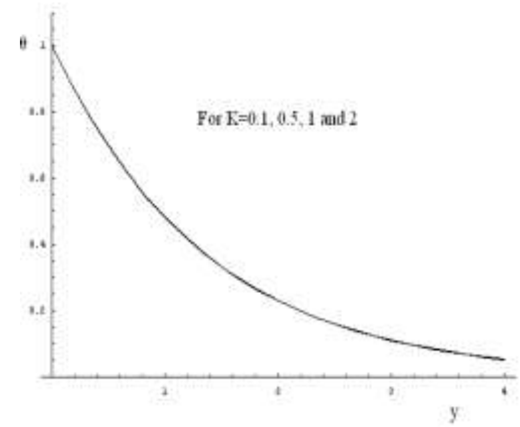

Fig.13: Plot of temperature versus y for different values of *k*.
(Pr=0.71; Gr=5; M=5; E =0.01; ε = 0.1).

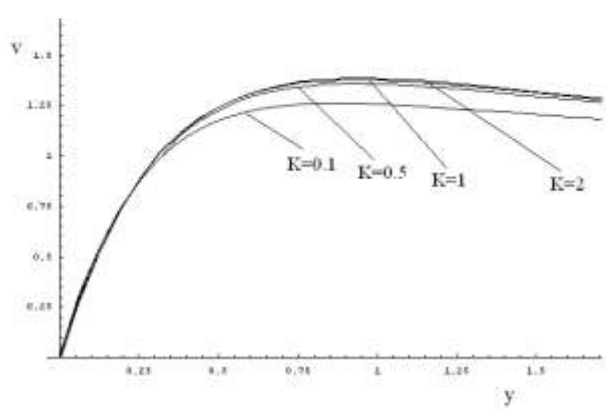

Fig.14: Plot of velocity versus *y* for different values of *k*.
(Pr=0.71; Gr=5; M=5; E =0.03; ε = 0.1).

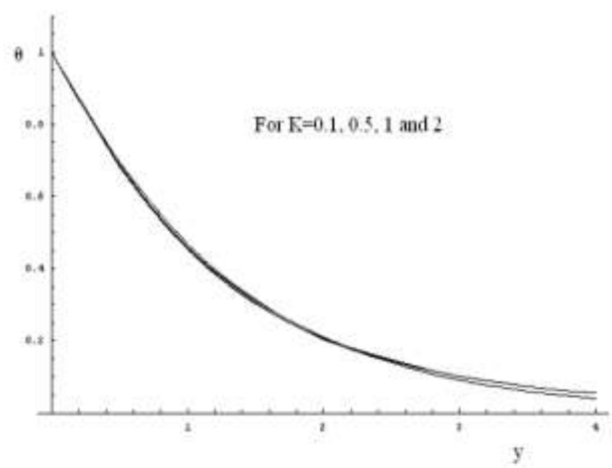

Fig.15: Plot of temperature versus *y* for different values of *k*

(Pr=0.71; Gr=5; M=5; E =0.03; ε = 0.1)

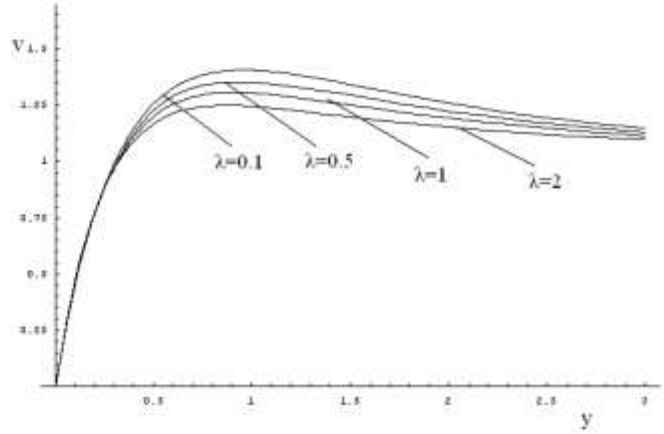

Fig.16: Plot of velocity versus *y* for different values of λ.
(Gr=5; M=5; k=0.5; E =0.03; ε = 0.1)

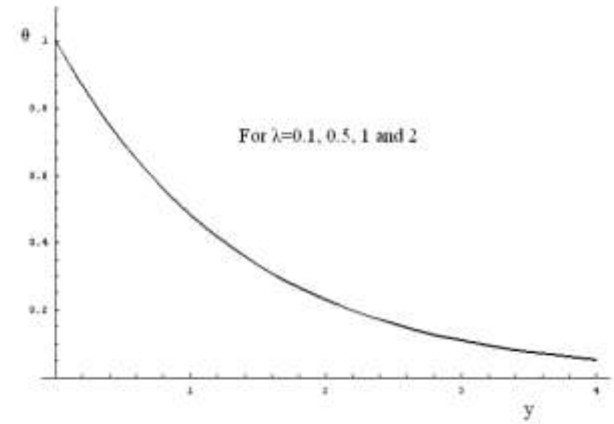

Fig.17: Plot of temperature versus *y* for different values of λ
(Pr=0.71; Gr=5; M=5; E =0.03; ε = 0.1)

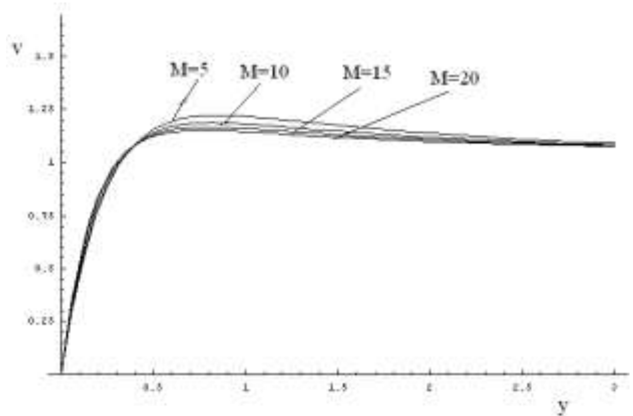

Fig.18: Plot of velocity versus *y* for different values of *M*.
(Pr=0.71; Gr=5; M=5; E =0.01; ε = 0.1).

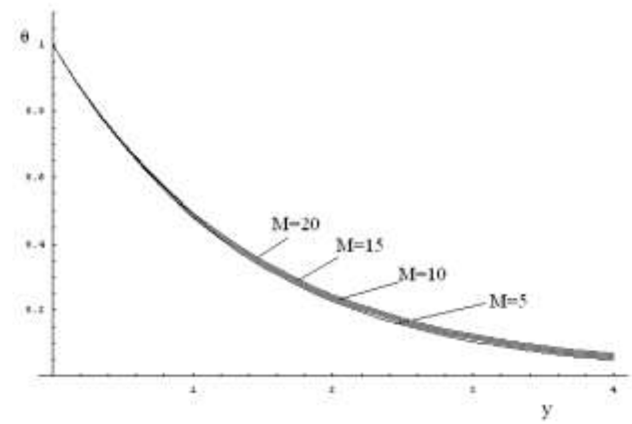

Fig.19: Plot of temperature versus *y* for different values of *M*.
(Pr=0.71; Gr=5; M=5; E =0.01; ε = 0.1).

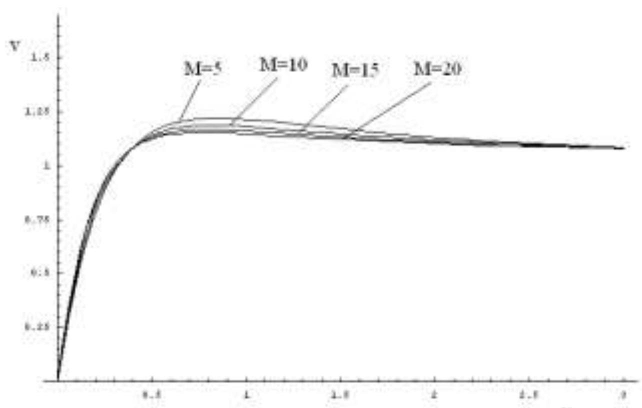

Fig.20: Plot of velocity versus *y* for different values of *M*.
(Pr=0.71; Gr=5; M=5; E =0.03; ε = 0.1).

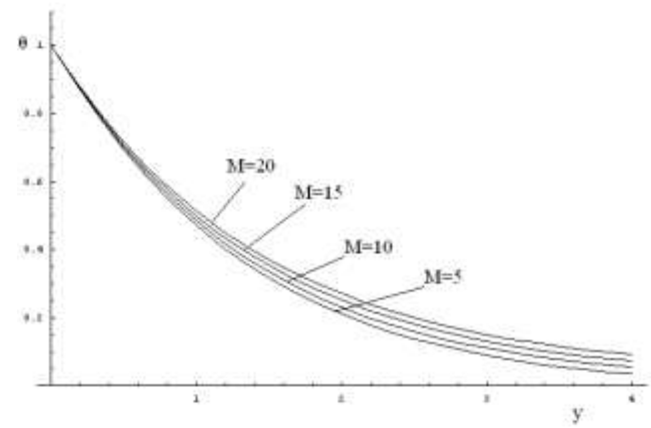

Fig.21: Plot of temperature versus *y* for different values of *M*.
(Pr=0.71; Gr=5; M=5; E =0.03; ε = 0.1).

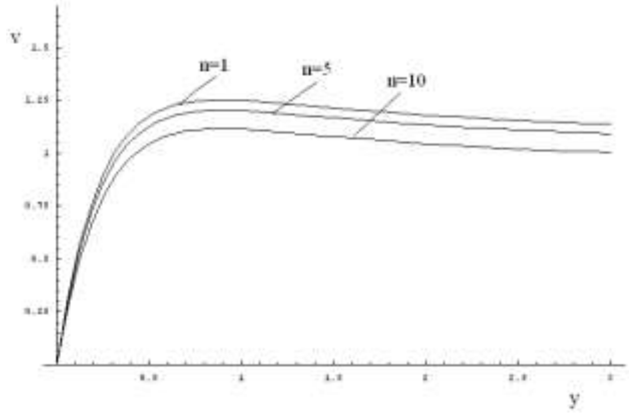

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Fig.22: Plot of velocity versus *y* for different values of "*n*".
(Pr=0.71; Gr=5; M=5; E =0.01; ε = 0.1)

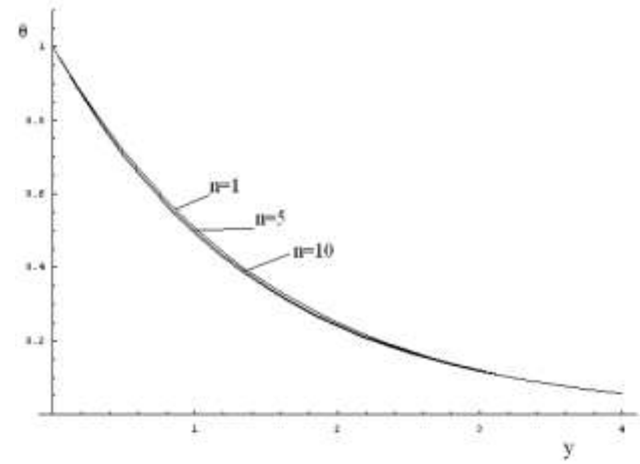

Fig.23: Plot of temperature versus *y* for different values of "*n*".
(Pr=0.71; Gr=5; M=5; E =0.01; ε = 0.1)

Conclusions:-

This study deals with the effect of magnetic field and porosity of the porous media on the oscillatory flow of mixed convection in a semi-infinite vertical porous wall. Using the double regular perturbation technique, the governing equations are transformed into a set of coupled linear ordinary differential equations in the non-dimensional form and these equations are solved numerically. The results computed are good approximation with the existing results. The numerical results are presented to analyze the mass flow and heat transfer characteristics of the fluid under the influence of various physical parameters present in the problem. the following conclusions can be drawn from the study:

1. An increase in the strength of the magnetic field is to decrease the velocity of the fluid flow and this may be due to increase of the Laurentz force on the fluid particles.
2. When the values of the Prandtl number (0.71 to 3) the velocity variation is large but for the large values of Prandtl number (*Pr* >3), the velocity variation is very less.
3. The flow velocity increases with an increase in the porosity of the porous media but the dependence of this parameter on temperature is weak.
4. Velocity is increasing function of Grashof number but the trend gets reversed in the case of temperature.
5. It is observed that the velocity and the temperature are both decreasing functions of the the frequency of the suction velocity.

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Appendix:

$A_{16} = g72$, $A_{17} = g81 + g85$, $A_{18} = g68$, $A_{19} = g74$, $A_{20} = g69 + y\,g83$, $A_{21} = g70$, $A_{22} = g73$, $A_{23} = g76$,
$A_{24} = g77$, $A_{25} = g79$, $A_{26} = g78$, $A_{27} = g82$, $B_1 = g14$, $B_2 = g110$, $B_3 = g15 + g13\,y$, $B_4 = g11$, $B_5 = g12$,
$B_6 = -g2$, $B_7 = g2$, $B_8 = g61$, $B_9 = g60$, $B_{10} = g46$, $B_{11} = g43$, $B_{12} = g67$, $B_{13} = g59 + g56\,y$, $B_{14} = g59$,
$B_{15} = g62$, $B_{16} = g63$, $B_{17} = g64$, $B_{18} = g65$, $B_{19} = g66$, $B_{20} = g47$, $K_1 = M + \frac{la}{k}$, $K_2 = in + M + \frac{la}{k}$,

$K_3 = -\Pr\left(\frac{la}{k} - M\right)$, $K_4 = -in + \Pr$, $L_1 = 2g1$, $L_2 = -2(g1+1)$, $g261 = -(g21 + g22 + g23 + g24 + g26)$,

$g301 = \frac{B\Pr g13}{K_4}$, $g1 = \frac{-Gr}{\Pr^2 - \Pr - K_1}$, $g2 = \frac{B\Pr}{i\,n}$, $g3 = \frac{-B(g1+1)R_2}{R_2{}^2 - R_2 - K_2}$, $g4 = \frac{B\Pr g1 + g2\,Gr}{\Pr^2 - \Pr - K2}$,

$g5 = \frac{g2\,Gr}{R_4{}^2 - R_4 - K_2}$, $g6 = \frac{K_1 A}{K_2}$, $g7 = -(g3 + g4 + g5 + g6)$, $g701 = K_3(g1+1)^2 - \Pr R_2^2 (g1+1)^2$,

$g8 = g1^2\left(K_3 - \Pr^3\right)$,

$g9 = -2K_3\,g1(g1+1) + 2g1\Pr^2 R_2 (g1+1)$, $g10 = \frac{g701}{4R_2^2 - 2R_2 - \Pr}$, $g11 = \frac{g8}{\Pr^2}$, $g12 = \frac{g9}{(R_2+\Pr)^2 - \Pr(R_2+\Pr)}$,

$g13 = \frac{-L_1}{\Pr}$, $g14 = \frac{L_{12}}{R_2^2 - \Pr R_2}$, $g15 = -(g10 + g11 + g12 + g14)$, $g16 = -Gr\,g15$, $g17 = -Gr\,g10$, $g18 = -Gr\,g11$

$g19 = -Gr\,g12$, $g20 = -Gr\,g13$, $g21 = \frac{g16}{\Pr^2 - \Pr - K_1}$, $g22 = \frac{g17}{4R_2{}^2 - 2R_2 - K_1}$, $g23 = \frac{g18}{4\Pr^2 - 2\Pr - K_1}$,

$g24 = \frac{g19}{(R_2+\Pr)^2 - (R_2+\Pr) - K_1}$, $g25 = \frac{g20}{\Pr^2 - \Pr - K_1}$, $g26 = \frac{2\Pr - 1}{\left(\Pr^2 - \Pr - K_1\right)^2}$, $g27 = \frac{-B\Pr^2 g15}{K_4}$,

$g28 = \frac{-2B\Pr R_2\,g10}{4R_2{}^2 - 2\Pr R_2 + K_4}$, $g29 = \frac{-2B\Pr^2 g11}{2\Pr^2 + K_4}$, $g30 = \frac{-B\Pr(R_2+\Pr)g12}{(R_2+\Pr)^2 - \Pr(R_2+\Pr) + K_4}$, $g31 = -B\Pr^2 g13$,

$g32 = \frac{-B\Pr R_2\,g14}{R_2{}^2 - \Pr R_2 + K_4}$, $g33 = \frac{-2K_3 g7(g1+1)}{(R_2+R_6)^2 - \Pr(R_2+R_6) + K_4}$, $g34 = \frac{-2K_3\,g3(g1+1)}{4R_2{}^2 - 2R_2\Pr + K_4}$,

$g35=\frac{-2K_3\,g4(g1+1)}{(R_2+\Pr)^2-\Pr(R_2+\Pr)+K_4}$, $g36=\frac{-2K_3\,g5(g1+1)}{(R_2+R_4)^2-\Pr(R_2+R_4)+K_4}$, $g37=\frac{-2K_3\,g6(g1+1)}{R_2^{\,2}-R_2\Pr+K_4}$,

$g38=\frac{2K_3\,g1g7}{(R_6+\Pr)^2-\Pr(R_6+\Pr)+K_4}$, $g39=\frac{2K_3\,g1g3}{(R_2+\Pr)^2-\Pr(R_2+\Pr)+K_4}$, $g40=\frac{2K_3\,g4\,g1}{2\Pr^2+K_4}$,

$g41=\frac{2K_3\,g1g5}{(R_4+\Pr)^2-\Pr(R_4+\Pr)+K_4}$, $g42=\frac{2K_3\,g6\,g1}{K_4}$, $g43=\frac{2\,g7K_3}{R_6^{\,2}-\Pr R_6+K_4}$, $g44=\frac{2\,g3K_3}{R_2^{\,2}-\Pr R_2+K_4}$,

$g45=\frac{2\,g4K_3}{K_4}$, $g46=\frac{2\,g5K_3}{R_4^{\,2}-\Pr R_4+K_4}$, $g47=\frac{2\,g6K_3}{K_4}$, $g48=\frac{2\Pr R_2\,R_6\,g7(g1+1)}{(R_2+R_6)^2-\Pr(R_2+R_6)+K_4}$,

$g49=\frac{2\Pr R_2^{\,2}g3(g1+1)}{4R_2^{\,2}-2R_2\Pr+K_4}$, $g50=\frac{2\Pr^2 R_2\,g4(g1+1)}{(R_2+\Pr)^2-\Pr(R_2+\Pr)+K_4}$, $g51=\frac{2\Pr R_2\,R_4\,g5(g1+1)}{(R_2+R_4)^2-\Pr(R_2+R_4)+K_4}$,

$g52=\frac{-2\Pr^2\,R_6\,g7g1}{(R_6+\Pr)^2-\Pr(R_6+\Pr)+K_4}$, $g53=\frac{-2\Pr g1\,R_2\,g3\Pr}{(R_2+\Pr)^2-\Pr(R_2+\Pr)+K_4}$, $g54=\frac{-2\Pr^3 g1g4}{2\Pr^2+K_4}$,

$g55=\frac{-2\Pr^2 g5\,R_4\,g1}{(R_4+\Pr)^2-\Pr(R_4+\Pr)+K_4}$, $g56=\frac{g31}{K_4}$, $g57=\frac{g31\Pr}{K_4^2}$, $g58=(g27+g38+g42+g45+g301)$,

$g59=(g29+g40+g54)$, $g60=(g28+g34+g49)$, $g61=(g32+g37+g44)$,

$g62=(g30+g35+g39+g50+g53)$, $g66=(g41+g55)$, $g63=(g33+g48)$, $g64=(g36+g51)$,

$g65=(g38+g52)$, $g68=\frac{B\,R_8\,g261}{R_8^{\,2}-R_8-K_2}$

$g67=-(g58+g59+g60+g61+g62+g63+g64+g65+g66+g46+g43+g47)$,, $g69=\frac{B\,g21\Pr-B\,g25-Gr\,g58}{\Pr^2-\Pr-K_2}$, $g70=\frac{2B\,g23\Pr-Gr\,g59}{4\Pr^2-2\Pr-K_2}$,

$g71=B\,g25\Pr-Gr\,g56$, $g72=\frac{2B\,g22\,R_2-Gr\,g60}{4R_2^{\,2}-2R_2-K_2}$, $g74=\frac{-g67\,Gr}{R_{10}^{\,2}-R_{10}-K_2}$, $g73=\frac{B\,g24(R_2+\Pr)-Gr\,g62}{(R_2+\Pr)^2-(R_2+\Pr)-K_2}$,

$g75=\frac{-g61\,Gr}{R_2^{\,2}-R_2-K_2}$, $g76=\frac{-Gr\,g63}{(R_2+R_6)^2-(R_2+R_6)-K_2}$, $g77=\frac{-Gr\,g64}{(R_2+R_4)^2-(R_2+R_4)-K_2}$,

$g78=\frac{-Gr\,g65}{(R_6+\Pr)^2-(R_6+\Pr)-K_2}$.