RESEARCH ARTICLE

Smg-interior and Smg-Closure.

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Abstract

The aim of this paper is to introduce the notion of Smg-interior, Smg-closure in topological spaces and new class of quasi Smg-open and quasi closed function on spaces with minimal structures.

Key words:

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Introduction:-

In 1970 the notion of generalized closed (briefly g-closed) sets were introduced and investigated by LEVINE [2]. MAKI [1] introduced the notions of minimal structures and minimal spaces. Recently many modifications of g closed sets have defined and investigated.

Valeiru Popa [3] introduced the notions of \(m_X\)-closed sets on spaces with minimal structures. In [4], Noiri introduced and studied the concepts of mg-closed sets on spaces with minimal structures. In [5], Pushpalatha and Subha introduced and investigated the notions of strongly minimal g-closed sets (briefly smg-closed sets).

In this paper, the notion of smg-interior is defined and some of its basic properties are studied. Also we introduce the concept of smg-closure in topological spaces using the notions of smg-closed sets.

Preliminaries:-

**Definition 2.1:** [3] Let \((X, \tau)\) be a topological space. Then the families \(\tau, \text{SO}(X)\) and \(\alpha\text{O}(X)\) are all \(m\)-structures on \(X\).

**Remark 2.2:** [3] Let \((X, \tau)\) be a topological space and \(A\) a subset of \(X\). If \(m_X = \tau\) (resp. \(\text{SO}(X), \alpha\text{O}(X)\)) then we have

1. \(m_X\text{-cl}(A) = \text{cl}(A)\) (resp. \(\text{scl}(A), \text{pcl}(A), \text{uccl}(A)\)).
2. \(m_X\text{-int}(A) = \text{int}(A)\) (resp. \(\text{sint}(A), \text{pint}(A), \text{aint}(A)\)).

**Lemma 2.3:** [3] Let \(X\) be a nonempty set with a minimal structure \(m_X\) and \(A\) a subset of \(X\). Then \(x \in m_X\text{-cl}(A)\) if and only if \(U \cap A \neq \emptyset\) for every \(U \in m_X\) containing \(x\).

**Definition 2.4:** [4] Let \((X, m_X)\) be an \(m\)-space. A subset \(A\) of \(X\) is said to be mg-closed if \(m_X\text{-cl}(A) \subseteq G\) whenever \(A \subseteq G\) and \(G\) is \(m_X\)-open. The complement of a mg-closed set is said to be a mg-open set.
Definition 2.5:- A minimal structure \( m_X \) on a nonempty set \( X \) is said to have the property \( \mathcal{B} \) if the union of any family of subsets belonging to \( m_X \) belongs to \( m_X \).

Definition 2.6:- [5] Let \((X, m_X)\) be an m-space. A subset \( A \) of \( X \) is said to be strongly minimal generalized closed (briefly smg-closed) if \( m_X\text{-cl}(A) \subseteq G \) whenever \( A \subseteq G \) and \( G \) is mg-open. The complement of a smg-closed set is called a smg-open set in \((X, m_X)\).

We denote the set of all smg-closed (resp. smg-open) sets in \( X \) by SMGC(X)(resp. SMGO(X)).

Remark 2.7: [6] Let \( X \) be a non-empty set with a minimal structure \( m_X \) satisfying the property \( \mathcal{B} \). Then every \( m_X \)-closed set is smg-closed but not conversely.

Lemma 2.8: [3] Let \((X, m_X)\) be an m-space and \( m_X \) satisfy property \( \mathcal{B} \). Then for subset \( A \) of \( X \), the following properties hold:
1. \( A \in m_X \) if and only if \( m_X\text{-int}(A) = A \).
2. \( A \) is \( m_X \)-closed if and only if \( m_X\text{-cl}(A) = A \).
3. \( m_X\text{-int}(A) \in m_X \) and \( m_X\text{-cl}(A) \) is \( m_X \)-closed.

Remark 2.9: [5] Let \( X \) be a non-empty set with a minimal structure \( m_X \) satisfying the property \( \mathcal{B} \). We have the following implications:
\( m_X \)-closed set \( \rightarrow \) smg-closed set \( \rightarrow \) mg-closed set.

The reverse implications are not true.

Example 2.10: Consider \( X = \{a, b, c\} \) with minimal structures satisfying the property \( \mathcal{B} \). We have \( m_X = \{\emptyset, X, \{b\}, \{a, b\}, \{a, c\}\} \). Here \( A = \{b, c\} \) is smg-closed set but not \( m_X \)-closed.

Example 2.11: Consider \( X = \{a, b, c\} \) with minimal structures satisfying the property \( \mathcal{B} \). We have \( m_X = \{\emptyset, X, \{a\}, \{b, c\}\} \). Here \( A = \{a, b\} \) is mg-closed set but not smg-closed.

Definition 2.12: [3] A function \( f : (X, m_X) \rightarrow (Y, m_Y) \), where \((X, m_X)\) and \((Y, m_Y)\) are nonempty sets \( X \) and \( Y \) with minimal structures \( m_X \) and \( m_Y \), respectively, is said to be \( M \)-continuous if for each \( x \in X \) and each \( V \in m_Y \) containing \( f(x) \), there exists \( U \in m_X \) containing \( x \) such that \( f(U) \subseteq V \).

Definition 2.13: Let \( X \) be a non-empty set with a minimal structure \( m_X \) satisfying the property \( \mathcal{B} \). A function \( f : (X, m_X) \rightarrow (Y, m_Y) \) is said to be strongly minimal g-continuous (briefly, smg-continuous) if \( f^{-1}(V) \) is smg-closed in \((X, m_X)\) for every \( m_Y \)-closed set \( V \) in \((Y, m_Y)\).

Smg-interior and Smg-Closure:-

Definition 3.1: Let \( X \) be a topological space and let \( x \in X \). A subset \( N \) of \( X \) is said to be smg-neighborhood (briefly smg-nbhd) of \( x \) if there exists a smg-open set \( G \) such that \( x \in G \subseteq N \).

Definition 3.2: Let \( A \) be a subset of \( X \). A point \( x \in A \) is said to be smg-interior point of \( A \) if \( A \) is a smg-nbhd of \( x \).

The set of all smg-interior points of \( A \) is called the smg-interior of \( A \) and is denoted by \( \text{smg-int}(A) \).

Theorem 3.3: If \( A \) be a subset of \( X \). Then \( \text{smg-int}(A) = \bigcup \{G : G \text{ is smg-open, } G \subseteq A\} \).

Theorem 3.4: Let \( A \) and \( B \) be subsets of \( X \). Then
1. \( \text{smg-int}(X) = X \) and \( \text{smg-int}(\emptyset) = \emptyset \).
2. \( \text{smg-int}(A) \subseteq A \).
3. If $B$ is any smg-open set contained in $A$, then $B \subseteq \text{smg-int}(A)$.

4. If $A \subseteq B$, then $\text{smg-int}(A) \subseteq \text{smg-int}(B)$

5. $\text{smg-int}(\text{smg-int}(A)) = \text{smg-int}(A)$.

**Theorem 3.5:** If a subset $A$ of a space $X$ is smg-open, then $\text{smg-int}(A) = A$.

**Example 3.6:** Let $X = \{a, b, c, d\}$ with minimal structure $m_X = \{\varnothing, X, \{c\}, \{a, d\}\}$. If $A = \{a, c\}$, then $\text{smg-int}(A) = \{a, c\} = A$ but it is not smg-open.

**Theorem 3.7:** If $A$ and $B$ are subsets of $X$, then $\text{smg-int}(A) \cup \text{smg-int}(B) \subseteq \text{smg-int}(A \cup B)$.

**Proof.** We know that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. We have, by Theorem 3.4(4), $\text{smg-int}(A) \subseteq \text{smg-int}(A \cup B)$ and $\text{smg-int}(B) \subseteq \text{smg-int}(A \cup B)$. This implies that $\text{smg-int}(A) \cup \text{smg-int}(B) \subseteq \text{smg-int}(A \cup B)$.

Containment relation in the above Theorem 3.5 may be proper as seen from the following example.

**Example 3.8:** Let $X = \{a, b, c, d\}$ with minimal structure $m_X = \{\varnothing, X, \{d\}, \{a, b, c\}, \{a, c, d\}\}$. If we take $A = \{a\}$ and $B = \{b, c\}$, then $\text{smg-int}(A) \cup \text{smg-int}(B) = \{a, c\} \subseteq \{a, b, c\} = \text{smg-int}(A \cup B)$ and $\text{smg-int}(A) \cup \text{smg-int}(B) \neq \text{smg-int}(A \cup B)$.

**Theorem 3.9:** If $A$ and $B$ are subsets of $X$, then $\text{smg-int}(A \cap B) = \text{smg-int}(A) \cap \text{smg-int}(B)$.

**Proof:** We know that $A \cap B \subseteq A$ and $A \cap B \subseteq B$. We have, by Theorem 3.4(4), $\text{smg-int}(A \cap B) \subseteq \text{smg-int}(A)$ and $\text{smg-int}(A \cap B) \subseteq \text{smg-int}(B)$. This implies that $\text{smg-int}(A \cap B) \subseteq \text{smg-int}(A \cap B) \rightarrow (1)$.

Again, let $x \in \text{smg-int}(A) \cap \text{smg-int}(B)$. Then $x \in \text{smg-int}(A)$ and $x \in \text{smg-int}(B)$. Hence $x$ is a smg-interior point of each of sets $A$ and $B$. It follows that $A$ and $B$ are smg-nbds of $x$, so that their intersection $A \cap B$ is also a smg-nbhd of $x$. Hence $x \in \text{smg-int}(A \cap B)$. Thus $x \in \text{smg-int}(A) \cap \text{smg-int}(B)$ implies that $x \in \text{smg-int}(A \cap B)$. Therefore $\text{smg-int}(A \cap B) \subseteq \text{smg-int}(A \cap B) \rightarrow (2)$. From (1) and (2), we get $\text{smg-int}(A \cap B) = \text{smg-int}(A) \cap \text{smg-int}(B)$.

**Theorem 3.10:** If $A$ is a subset of a space $X$, then $m_X^{-1}(A) \subseteq \text{smg-int}(A)$

**Remark 3.11:** Containment relation in the above Theorem 3.10 may be proper as seen from the following example.

**Example 3.12:** In Example 3.6, if we take $A = \{b\}$, we have $\text{smg-int}(A) = \{b\}$ and $m_X^{-1}(A) = \varnothing$. It follows that $m_X^{-1}(A) \subseteq \text{smg-int}(A)$ and $m_X^{-1}(A) \neq \text{smg-int}(A)$.

Analogous to closure in a space $X$, we define smg-closure in a space $X$ as follows.

**Definition 3.13:** Let $A$ be a subset of a space $X$. We define the smg-closure of $A$ to be the intersection of all smg-closed sets containing $A$. In symbols, $\text{smg-cl}(A) = \bigcap \{F : A \subseteq F, F \text{ is smg-closed set}\}$.

**Theorem 3.14:** Let $A$ and $B$ be subsets of $X$. Then

1. $\text{smg-cl}(X) = X$ and $\text{smg-cl}(\varnothing) = \varnothing$.
2. $A \subseteq \text{smg-cl}(A)$.
3. If $B$ is any smg-closed set containing $A$, then $\text{smg-cl}(A) \subseteq B$.
4. If $A \subseteq B$, then $\text{smg-cl}(A) \subseteq \text{smg-cl}(B)$.
5. $\text{smg-cl}(A) = \text{smg-cl}(\text{smg-cl}(A))$.

**Theorem 3.15:** If $A \subseteq X$ is smg-closed, then $\text{smg-cl}(A) = A$.

**Example 3.16:** In Example 3.6, if we take $A = \{a, b\}$, we have $\text{smg-cl}(A) = \{a, b\} = A$ but it is not smg-closed.

**Theorem 3.17:** If $A$ and $B$ are subsets of a space $X$, then $\text{smg-cl}(A \cap B) \subseteq \text{smg-cl}(A) \cap \text{smg-cl}(B)$.
smgcl(B).

**Example 3.18:** In Example 3.6, if we take \( A = |b| \) and \( B = |c| \), we have \( \text{smg-cl}(A \cap B) = \varnothing \subseteq |b| = \text{smg-cl}(A) \cap \text{smg-cl}(B) \) and \( \text{smg-cl}(A \cap B) \not\subseteq \text{smg-cl}(A) \cap \text{smg-cl}(B) \).

**Theorem 3.19:** If \( A \) and \( B \) are subsets of a space \( X \), then \( \text{smg-cl}(A \cup B) \subseteq \text{smg-cl}(A) \cup \text{smg-cl}(B) \).

**Lemma 3.20:** Let \( X \) be a nonempty set with a minimal structure \( m_X \) and \( A \) a subset of \( X \). Then \( x \in \text{smg-cl}(A) \) if and only if \( U \cap A = \varnothing \) for every smg-open set \( U \) containing \( x \).

**Theorem 3.21:** If \( A \) is a subset of a space \( X \), then \( \text{smg-cl}(A) \subseteq m_X \cdot \text{cl}(A) \).

**proof.** Let \( A \) be a subset of a space \( X \). By the definition of \( m_X \)-closure, \( m_X \cdot \text{cl}(A) = \bigcap \{ F : A \subseteq F, X - F \in m_X \} \). If \( A \subseteq F \) is \( m_X \)-closed, then \( A \subseteq F \in \text{SMGC}(X) \), because every \( m_X \)-closed set is smg-closed. That is smg-cl(A) \( \subseteq F \). Therefore \( \text{smg-cl}(A) \subseteq \bigcap \{ F : A \subseteq F, X - F \in m_X \} = m_X \cdot \text{cl}(A) \). Hence \( \text{smg-cl}(A) \subseteq m_X \cdot \text{cl}(A) \).

**Remark 3.22:** Containment relation in the above Theorem 3.21 may be proper as seen from the following example.

**Example 3.23:** In Example 3.6, if we take \( A = |a| \), we have \( \text{smg-cl}(A) = |a|, b \) and \( m_X \cdot \text{cl}(A) = X \). It follows that \( \text{smg-cl}(A) \subseteq m_X \cdot \text{cl}(A) \) and \( \text{smg-cl}(A) \not\subseteq m_X \cdot \text{cl}(A) \).

**Theorem 3.24:** Let \( A \) be any subset of \( X \). Then

1. \( (\text{smg-int}(A))^C = \text{smg-cl}(A^C) \).
2. \( \text{smg-int}(A) = (\text{smg-cl}(A^C))^C \).
3. \( \text{smg-cl}(A) = (\text{smg-int}(A^C))^C \).

**Proof:** (1) Let \( x \in (\text{smg-int}(A))^C \). Then \( x \notin \text{smg-int}(A) \). That is every smg-open set \( U \) containing \( x \) is such that \( U \not\subseteq A \). That is every smg-open set \( U \) containing \( x \) is such that \( U \cap A^C = \varnothing \). By Lemma 3.20, \( x \in \text{smg-cl}(A^C) \) and therefore \( \text{smg-int}(A))^C \subseteq \text{smg-cl}(A^C) \). Conversely, let \( x \in \text{smg-cl}(A^C) \). Then by Lemma 3.20, every smg-open set \( U \) containing \( x \) is such that \( U \cap A^C = \varnothing \). That is every smg-open set \( U \) containing \( x \) is such that \( U \not\subseteq A \). This implies by definition of smg-interior of \( A \), \( x \notin \text{smg-int}(A) \). That is \( x \in (\text{smg-int}(A))^C \) and \( \text{smg-cl}(A^C) \subseteq (\text{smg-int}(A))^C \). Thus \( (\text{smg-int}(A))^C = \text{smg-cl}(A^C) \).

(2) Follows by taking complements in (1). (3) Follows by replacing \( A \) by \( A^C \) in (1).

**References:**