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RESEARCH ARTICLE

Smg-interior and Smg-Closure.

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Abstract

The aim of this paper is to introduce the notion of Smg-interior, Smg- closure in topological spaces and new class of quasi Smg open and quasi closed function on spaces with minimal structures.

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Introduction:-

In 1970 the notion of generalized closed (briefly g-closed) sets were introduced and investigated by LEVINE [2]. MAKI [1] introduced the notions of minimal structures and minimal spaces. Recently many modifications of g closed sets have defined and investigated.

Valeiru Popa [3] introduced the notions of m_X-closed sets on spaces with minimal structures. In [4], Noiri introduced and studied the concepts of mg-closed sets on spaces with minimal structures. In [5], Pushpalatha and Subha introduced and investigated the notions of strongly minimal g-closed sets (briefly, smg-closed sets).

In this paper, the notion of smg-interior is defined and some of its basic prop- erties are studied. Also we introduce the concept of smg-closure in topological spaces using the notions of smg-closed sets.

Preliminaries:-

Definition 2.1:- [3] Let (X, τ) be a topological space. Then the families τ, SO(X) and αO(X) are all m-structures on X.

Remark 2.2:- [3] Let (X, τ) be a topological space and A a subset of X. If $m_X = \tau$ (resp. SO(X), PO(X), αO(X)) then we have

1. m_X-cl(A) = cl(A) (resp. scl(A), pcl(A), αcl(A)).
2. m_X-int(A) = int(A) (resp. sint(A), pint(A), αint(A)).

Lemma 2.3:- [3] Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then $x \in m_X$-cl(A) if and only if $U \cap A \neq \varphi$ for every $U \in m_X$ containing x.

Definition 2.4:- [4] Let (X, m_X) be an m-space. A subset A of X is said to be mg-closed if m_X-cl(A) $\subseteq$ G whenever $A \subseteq G$ and G is m_X-open. The complement of a mg-closed set is said to be a mg-open set.

Definition 2.5:- A minimal structure m_X on a nonempty set X is said to have the the property $\mathcal{B}$ if the union of any family of subsets belonging to m_X belongs to m_X .

Definition 2.6:- [5] Let (X, m_X) be an m-space. A subset A of X is said to be strongly minimal generalized closed (briefly smg-closed) if m_X-cl(A) ⊆ G whenever A ⊆ G and G is mg-open. The complement of a smg-closed set is called a smg-open set in (X, m_X).
We denote the set of all smg-closed (resp. smg-open) sets in X by SMGC(X)(resp. SMGO(X)).

Remark 2.7: [6] Let X be a non-empty set with a minimal structure m_X satisfying the property $\mathcal{B}$. Then every m_X-closed set is smg-closed but not conversely.

Lemma 2.8: [3] Let (X, m_X) be an m-space and m_X satisfy property $\mathcal{B}$. . Then for subset A of X, the following properties hold:

1. A ∊ m_X if and only if m_X-int(A) = A,
2. A is m_X-closed if and only if m_X-cl(A) = A,
3. m_X-int(A) ∊ m_X and m_X-cl(A) is m_X-closed.

Remark 2.9: [5] Let X be a non-empty set with a minimal structure m_X satisfying the property $\mathcal{B}$. We have the following implications.
m_X-closed set → smg-closed set → mg-closed set.
The reverse implications are not true.

Example 2.10: Consider X = {a, b, c} with minimal structures satisfying the property $\mathcal{B}$.. We have m_X = {φ, X, {b}, {a, b}, {a, c}}. Here A = {b, c} is smg-closed set but not $m_X$closed.

Example 2.11: Consider X = {a, b, c} with minimal structures satisfying the property$\mathcal{B}$. We have m_X = {φ, X, {a}, {b, c}}. Here A ={a, b} is mg-closed set but not smg-closed.

Definition 2.12: [3] A function f : (X, m_X) → (Y, m_Y), where (X, m_X) and (Y, m_Y) are nonempty sets X and Y with minimal structures m_X and m_Y, respectively, is said to be M-continuous if for each x ∊ X and each V ∊ my containing f(x), there exists U ∊ m_X containing x such that f(U) ⊆ V.

Definition :2.13: Let X be a non-empty set with a minimal structure m_X satisfying the property $\mathcal{B}$. A function f : (X, m_X) → (Y, my) is said to be strongly minimal g-continuous (briefly, smg-continuous) if f^{-1}(V) is smg-closed in (X, m_X) for every m_Y-closed set V in
(Y, m_Y).

Smg-interior and Smg-Closure:-

Definition 3.1: Let X be a topological space and let x ∊ X. A subset N of X is said to be smg-neighbourhood (briefly smg-nbhd) of x if there exists a smg-open set G such that x ∊ G ⊆ N.

Definition 3.2: Let A be a subset of X. A point x ∊ A is said to be smg-interior point of A if A is a smg-nbhd of x. The set of all smg-interior points of A is called the smg-interior of A and is denoted by smg-int(A).

Theorem 3.3: If A be a subset of X. Then smg-int(A) = ∪ { G : G is smg-open, G ⊆ A}.

Theorem 3.4: Let A and B be subsets of X. Then

1. smg-int(X) = X and smg-int(φ) = φ.
2. smg-int(A) ⊆ A.

3. If B is any smg-open set contained in A, then B ⊆ smg-int(A).
4. If A ⊆ B, then smg-int(A) ⊆ smg-int(B)
5. smg-int(smg-int(A)) = smg-int(A).

Theorem 3.5: If a subset A of a space X is smg-open, then smg-int(A) = A.
Example .3.6: Let X = {a, b, c, d} with minimal structure m_X = {φ, X, {c}, {a, d}}. If A= {a, c}, then smg-int(A) = {a, c} = A but it is not smg-open.

Theorem 3.7: If A and B are subsets of X, then smg-int(A) ∪ smg-int(B) ⊆ smg-int(A∪B).

Proof. We know that A ⊆ A ∪ B and B ⊆ A ∪ B. We have, by Theorem 3.4(4), smg-int(A) ⊆ smg-int(A ∪ B) and smg-int(B) ⊆ smg-int(A ∪ B). This implies that smg-int(A) ∪ smg-int(B) ⊆ smg-int(A ∪ B).
Containment relation in the above Theorem 3.7 may be proper as seen from the following example.
Example 3.8: Let X = {a, b, c, d} with minimal structure m_X = {φ, X, {d}, {a, b, c}, {a, c, d}}. If we take A = {a} and B = {b, c}, then smg-int(A) ∪ smg-int(B) = {a, c} ⊆ {a, b, c} = smg-int(A ∪ B) and smg-int(A) ∪ smg-int(B) ≠ smg-int(A ∪ B).

Theorem 3.9: If A and B are subsets of X, then smg-int(A ∩ B) = smg-int(A) ∩smg-int(B).
Proof: We know that A ∩ B ⊆ A and A ∩ B ⊆ B. We have, by Theorem 3.4(4), smg-int(A ∩ B) ⊆ smg-int(A) and smg-int(A ∩ B) ⊆ smg-int(B). This implies that smg-int(A ∩ B) ⊆ smg-int(A) ∩ smg-int(B) → (1). Again, let x ∈ smg-int(A) ∩ smg-int(B). Then x ∈ smg-int(A) and x ∈ smg-int(B). Hence x is a smg-interior point of each of sets A and B. It follows that A and B are smg-nbhds of x, so that their intersection A ∩ B is also a smg-nbhds of x. Hence x ∈ smg-int(A ∩ B). Thus x ∈ smg-int(A) ∩ smg-int(B) implies that x ∈ smg-int(A ∩ B). Therefore smg-int(A)∩ smg-int(B) ⊆ smg-int(A ∩ B) → (2). From (1) and (2), we get smg-int(A ∩ B) = smg-int(A) ∩ smg-int(B).

Theorem 3.10: If A is a subset of a space X, then m_X-int(A) ⊆ smg-int(A)

Remark 3.11: Containment relation in the above Theorem3.10 may be proper as seen from the following example.
Example 3.12: In Example 3.6, if we take A = {b}, we have smg-int(A) = {b} and m_X-int(A) = φ. It follows that m_X-int(A) ⊆ smg-int(A) and m_X-int(A) ≠ smg-int(A).
Analogous to closure in a space X, we define smg-closure in a space X as follows.

Definition 3.13: Let A be a subset of a space X. We define the smg-closure of A to be the intersection of all smg-closed sets containing A. In symbols, smg-cl(A) = ∩{F : A ⊆ F, F is smg-closed set}.

Theorem 3.14: Let A and B be subsets of X. Then
1. smg-cl(X) = X and smg-cl(φ) = φ.
2. A ⊆ smg-cl(A).
3. If B is any smg-closed set containing A, then smg-cl(A) ⊆ B.
4. If A ⊆ B, then smg-cl(A) ⊆ smg-cl(B).
5. smg-cl(A) = smg-cl(smg-cl(A)).

Theorem 3.15: If A ⊆ X is smg-closed, then smg-cl(A) = A.
Example 3.16: In Example 3.6, if we take, A = {a, b}, we have smg-cl(A) = {a, b} = A but it is not smg-closed.
Theorem 3.17: If A and B are subsets of a space X, then smg-cl(A ∩ B) ⊆ smg-cl(A) ∩

smgcl(B).

Example 3.18: In Example 3.6, if we take A = {b} and B = {c}, we have smg-cl(A ∩ B) = φ ⊆ {b} = smg-cl(A) ∩ smg-cl(B) and smg-cl(A ∩ B) ≠ smg-cl(A) ∩ smg-cl(B).

Theorem 3.19: If A and B are subsets of a space X, then smg-cl(A ∪ B) ⊆ smg-cl(A) ∪ smg-cl(B).

Lemma 3.20: Let X be a nonempty set with a minimal structure m_X and A a subset of X. Then x ∈ smg-cl(A) if and only if U ∩ A ≠ φ for every smg-open set U containing x.

Theorem 3.21: If A is a subset of a space X, then smg-cl(A) ⊆ m_X-cl(A).

proof. Let A be a subset of a space X. By the definition of m_X-closure, m_X-cl(A)= ∩{F: A ⊆ F, X − F ∈ m_X}. If A ⊆ F is m_X-closed, then A ⊆ F ∈ SMGC(X), because every m_X-closed set is smg-closed. That is smg-cl(A) ⊆ F. Therefore smg-cl(A) ⊆ ∩{F: A ⊆ F, X − F ∈ ∈ m_X}= m_X-cl(A). Hence smg-cl(A) ⊆ m_X-cl(A).

Remark 3.22: Containment relation in the above Theorem 3.21 may be proper as seen from the following example.

Example 3.23: In Example 3.6, if we take A = {a}, we have smg-cl(A) = {a, b} and $m_X$cl(A) = X. It follows that smg-cl(A) ⊆ m_X-cl(A) and smg-cl(A) ≠ m_X-cl(A).

Theorem 3.24:- Let A be any subset of X. Then

1. $(\text{smg-int}(A))^c$ = smg-cl(A^c).
2. smg-int(A) = $(\text{smg-cl}(A^c))^c$.
3. smg-cl(A) = $(\text{smg-int}(A^c))^c$.

Proof: (1) Let x ∈ $(\text{smg-int}(A))^c$. Then x ∉ smg-int(A). That is every smg-open set U containing x is such that U ⊈ A. That is every smg-open set U containing x is such that U ∩ A^c ≠ φ. By Lemma 3.20, x ∈ smg-cl(A^c) and therefore $(\text{smg-int}(A))^c$ ⊆ smg-cl(A^c). Conversely, let x ∈ smg-cl(A^c). Then by Lemma 3.20, every smg-open set U containing x is such that U ∩ A^c ≠ φ. That is every smg-open set U containing x is such that U ⊈ A This implies by definition of smg-interior of A, x ∉ smg-int(A). That is x ∈ $(\text{smg-int}(A))^c$ and smg-cl(A^c) ⊆ $(\text{smg-int}(A))^c$. Thus $(\text{smg-int}(A))^c$ = smg-cl(A^c).

(2) Follows by taking complements in (1). (3) Follows by replacing A by A^c in (1).

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