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RESEARCH ARTICLE

Using Irreducible Characters Table for Cyclic Groups C_2 and C_{2v} in Transformation

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Abstract

In this paper, we introduce a new method to transform image by using irreducible character table for point group C_2 , C_{2v} by considering character table as square matrix of size 2×2, 4×4, and designing an algorithm for it, which includes the transformation matrix of the image to the sets of matrices square of size 2×2, 4×4

1- Introduction

Image processing refers to the various operations , Purpose of transformation is to convert the data into a form where compression is easier. This transformation will transform the pixels which are correlated into a representation where they are decorrelated. The new values are usually smaller on average than the original values. The net effect is to reduce the redundancy of representation. For lossy compression, the transform coefficients can now be quantized according to their statistical properties, producing a much compressed representation of the original image data , in this paper we presented a new method to cipher and anti-cipher .[3],[4],[5]

point group(1.1):[6]

Particularly we will consider the following point groups which molecules can belong toThe group Cn. A molecule belongs to the group Cn if it has a n-fold axis. C_2 group as it has the elements E and C_2. The group C_{nv}. A molecule belongs to the group C_{nv} if in addition to the identity E and a Cn axis, it has n vertical mirror planes σv.

Character table(1.2):[6]

C_2	I	(12)
χ_1	1	1
χ_2	1	-1

Character table C_2

C_{2v}	I	(12)	σ_V	σ_V
χ_1	1	1	1	1
χ_2	1	-1	1	-1
χ_3	1	1	-1	-1
χ_4	1	-1	-1	1

Character table C_{2v}

We can see the character table $\equiv (C_2v) = (\equiv C_2 \otimes \equiv C_2)$

2- Algorithm :

In this section will divide the matrix into the blocks matrix of the same order 2x2 or 4x4.

Definition(2.1) : [4],[3]

Let an matrix f be represented as an nxn matrix of integer numbers $f = \begin{bmatrix} f_1 & \cdots & f_m \\ \vdots & \ddots & \vdots \\ f_n & \cdots & f_k \end{bmatrix}$, where f_i are blocks matrix of order ixi

General transform $F = P\ f\ Q$, If P and Q are non-singular (non-zero determinants), inverse matrices exist and

$$f = P^{-1}\ F\ \ Q^{-1}$$

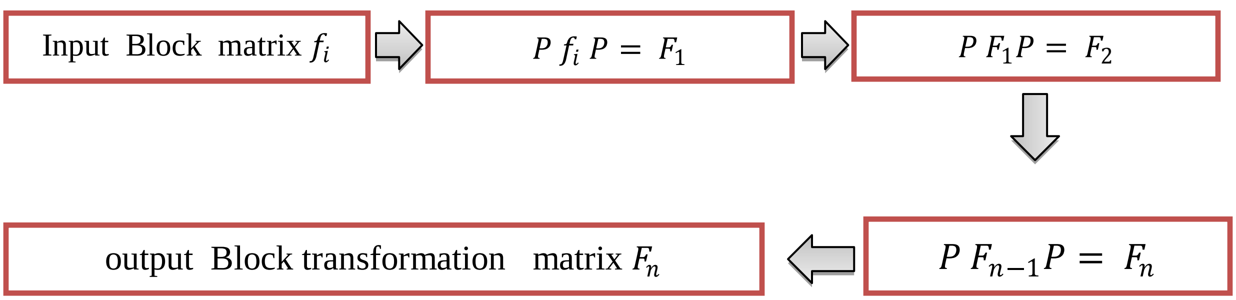
Rule (2.2):

1- Let P = Q

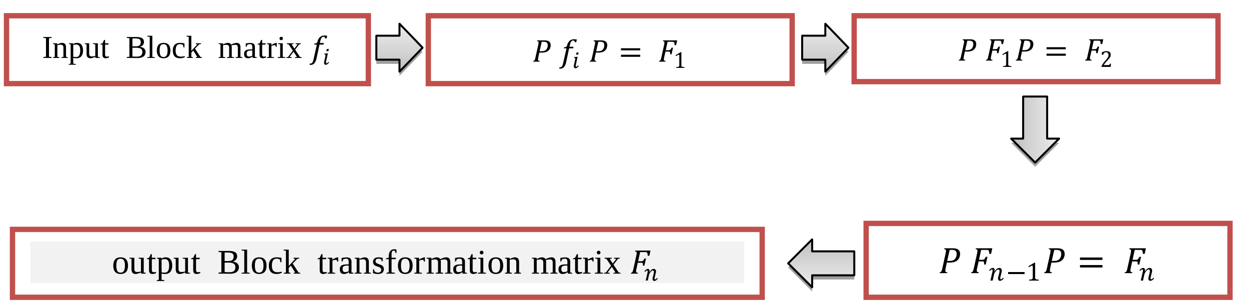
2- $P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} =\equiv C_2$, when P of size 2x2

3- $P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} =\equiv (C_2 v)$, when P of size 4x4

4- Transformation f_i matrix when P be of size 2×2:



5- Transformation f_i matrix P be of size 4×4:



So we get the final matrix encoded F_n And for the purpose of obtaining the original matrix and without the use of inverse matrix, so the answer will be in the following main theorems

3- Main result

In this section we present important theorems on open cipher , where its cipher an n loop.

Lemma(3.1):

If $P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ then $P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

Lemma (3.2):

If $P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ then $P^{-1} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$.

Theorem (3.3):
Let P_1 and P_2 are matrices of size 2×2 , then :

$$P_1 \times P_2 = 2I_2 \text{ , where } I_2 \text{ identity matrix of size } 2 \times 2$$

Proof:

$$P_1 \times P_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 2I_2$$

Theorem (3.4):
Let P_1 , P_2 , … P_{n-1} , P_n are matrices n-time of size 2×2 and n even then :

$$P_1 \times P_2 \times \ldots \times P_{n-1} \times P_n = 2^{\frac{n}{2}} I_2 \text{ , where } I_2 \text{ identity matrix}$$

Proof:

$$\begin{aligned} P_1 \times P_2 \times \ldots \times P_{n-1} \times P_n &= \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \ldots \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= 2I_2 \times 2I_2 \times \ldots \times 2I_2 \\ &= 2^{\frac{n}{2}} I_2. \end{aligned}$$

Theorem (3.5):
Let P_1 and P_2 are matrices of size 4×4 , then :

$$P_1 \times P_2 = 4I_4 \text{ , where } I_4 \text{ identity matrix of size } 4 \times 4.$$

Proof:

$$P_1 \times P_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = 4I_4$$

Theorem (3.6):
Let P_1 , P_2 , … P_{n-1} , P_n are matrices n-time of size 4×4 and n even then :

$$P_1 \times P_2 \times \ldots \times P_{n-1} \times P_n = 4^{\frac{n}{2}} I_4 \text{ , where } I_4 \text{ identity matrix}$$

Proof:

$$\begin{aligned} P_1 \times P_2 \times \ldots \times P_{n-1} \times P_n &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \times \ldots \times \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \\ &= 4I_4 \times 4I_4 \times \cdots \times 4I_4 \\ &= 4^{\frac{n}{2}} I_4 \end{aligned}$$

Theorem (3.7):
Let P be matrix of size 2×2, and F_i is transformation for matrix of size 2×2 , for level nthen:

1- Keys when n even :

$$f = \frac{1}{2^n}([F_i])$$

1- Keys when nodd :

$$f = \frac{1}{2^n}\left(P^{-1}\left[[F_n]P^{-1}\right]\right)$$

Proof:
If n is even level:
Let F_n be transformationin level n (n even), then we need n-time inverse matric for P . i.e :

$$f = [P^{-1}.P^{-1} \ldots P^{-1} F_n P^{-1} \ldots P^{-1}.P^{-1}]$$

$$f = \{\frac{1}{2}P.\frac{1}{2}P \ldots \frac{1}{2}P\, F_n \frac{1}{2}P.\ldots\frac{1}{2}P.\frac{1}{2}P\ \}$$

$$f = \{\frac{1}{2^n}(P.P \ldots P)F_n \frac{1}{2^n}P.P \ldots P\}$$

$$f = \{\frac{1}{2^n}\left(2^{\frac{n}{2}}I_2\right)F_n \frac{1}{2^n}\left(2^{\frac{n}{2}}I_2\right)\}$$

$$f = \{\frac{2^n}{4^n}\ (I_2 F_n I_2)\}$$

$$f = \{\frac{1}{2^n}\ (F_n)\}$$

If n is odd level:

Let F_n be transformation in level n (n odd), then we need n-time inverse matric for P . i.e :

$$f = [P^{-1}.P^{-1} \dots P^{-1}F_n P^{-1}.P^{-1} \dots P^{-1}]$$
$$f = \{\frac{1}{2}P.\frac{1}{2}P \dots \frac{1}{2}P\ (\ P^{-1}F_n P^{-1}\ )\ \frac{1}{2}P.\frac{1}{2}P \dots \frac{1}{2}P\ \}$$
$$f = \{\frac{1}{2^n}(P.P \dots P)(\ A^{-1}F_n A^{-1}\ )\ \frac{1}{2^n}(P.P \dots P)\}$$
$$f = \{\frac{1}{2^n}\left(2^{\frac{n}{2}}I_2\right)(P^{-1}F_n P^{-1}\ )\ \frac{1}{2^n}\left(2^{\frac{n}{2}}I_2\right)\}$$
$$f = \{\frac{2^n}{4^n}\ (\ I_2\ (P^{-1}F_n P^{-1}\ )\ \ I_2)\}$$
$$f = \{\frac{1}{2^n}\ (\ P^{-1}F_n P^{-1}\ )\}$$
$$f = \left\{\frac{1}{2^n}(P^{-1}F_n P^{-1})\right\}$$

Theorem(3.8):

Let P be matrix of size 4×4, and F_i is transformation matrix, for level n then:

1- Keys when n is even number:

$$f = \frac{1}{4^n}([F_n])$$

2- Keys when n isodd number :

$$f = \frac{1}{4^n}\left(P^{-1}\left[[F_n]P^{-1}\right]\right)$$

Proof:

If n is even level:

Let F_n be betransformation for n-time , then we need n-time inverse matric for P . i.e:

$$f = [P^{-1}.P^{-1} \dots P^{-1}F_n P^{-1}.P^{-1} \dots P^{-1}]$$
$$f = \{\frac{1}{4}P.\frac{1}{4}P \dots \frac{1}{4}P\ F_n\ \frac{1}{4}P.\dots.\frac{1}{4}P.\frac{1}{4}P\ \}$$
$$f = \{\frac{1}{4^n}(P.P \dots P)F_n\ \frac{1}{4^n}(P.P \dots P)\}$$
$$f = \{\frac{1}{4^n}\left(4^{\frac{n}{2}}I_4\right)F_n\ \frac{1}{4^n}\left(4^{\frac{n}{2}}I_4\right)\}$$
$$f = \{\frac{4^n}{16^n}\ (\ I_4\ (F_n\ )I_4\ )\}$$
$$f = \{\frac{1}{4^n}\ (F_n)\}$$

If n is odd level:

Let F_n be betransformation for n-time , then we need n-time inverse matric for P . i.e:

$$f = [P^{-1}.P^{-1} \dots P^{-1}F_n P^{-1} \dots\ P^{-1}.P^{-1}]$$
$$f = \{\frac{1}{4}P.\frac{1}{4}P \dots \frac{1}{4}P\ (P^{-1}F_n P^{-1})\frac{1}{4}P.\dots.\frac{1}{4}P.\frac{1}{4}P\ \}$$
$$f = \{\frac{1}{4^n}(P.P \dots P)(P^{-1}F_n P^{-1})\frac{1}{4^n}(P.P \dots P)\}$$
$$f = \{\frac{1}{4^n}\left(4^{\frac{n}{2}}I_4\right)(P^{-1}F_n P^{-1})\frac{1}{4^n}\left(4^{\frac{n}{2}}I_4\right)\}$$
$$f = \{\frac{4^n}{16^n}\ (I_4(P^{-1}F_n P^{-1})I_4\ )\}$$
$$f = \{\frac{1}{4^n}(P^{-1}F_n P^{-1})\}$$

4- Application :

Now we can applied the transformation

$$f = \begin{bmatrix} f_1 & \cdots & f_n \\ \vdots & \ddots & \vdots \\ f_m & \cdots & f_r \end{bmatrix} = \begin{bmatrix} \begin{matrix} 12 & 213 & 123 & 244 \\ 110 & 211 & 9 & 234 \\ 11 & 122 & 243 & 134 \\ 10 & 221 & 233 & 30 \end{matrix} & \cdots & f_n \\ \vdots & \ddots & \vdots \\ f_m & \cdots & f_r \end{bmatrix}$$

Then the block matrix f_1 of matrix f be

$$f_1 = \begin{bmatrix} 12 & 213 & 123 & 244 \\ 110 & 211 & 9 & 234 \\ 11 & 122 & 243 & 134 \\ 10 & 221 & 233 & 30 \end{bmatrix}.$$

Now the transformation be : $F = P\ f\ Q$, where $P = Q = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$ Then

$$F_1 = \begin{bmatrix} 2160 & -658 & -340 & -590 \\ 44 & 10 & -432 & -10 \\ 152 & -638 & 212 & 678 \\ 12 & -2 & -8 & -398 \end{bmatrix} F_2 = \begin{bmatrix} 192 & 3408 & 1968 & 3904 \\ 1760 & 3376 & 144 & 3744 \\ 176 & 1952 & 3888 & 2144 \\ 160 & 3536 & 3728 & 480 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} 34560 & -10528 & -5440 & -9440 \\ 704 & 160 & -6912 & -160 \\ 2432 & -10208 & 3392 & 10848 \\ 192 & -32 & -128 & -6368 \end{bmatrix} F_4 = \begin{bmatrix} 3072 & 54528 & 31488 & 62464 \\ 28160 & 54016 & 2304 & 59904 \\ 2816 & 31232 & 62208 & 34304 \\ 2560 & 56576 & 59648 & 7680 \end{bmatrix}$$

Here n=4 and degree of partition 4x4 then :

$$f = \frac{1}{4^n}([F_n])$$

$$f = \frac{1}{4^4}\begin{bmatrix} 3072 & 54528 & 31488 & 62464 \\ 28160 & 54016 & 2304 & 59904 \\ 2816 & 31232 & 62208 & 34304 \\ 2560 & 56576 & 59648 & 7680 \end{bmatrix} = \begin{bmatrix} 12 & 213 & 123 & 244 \\ 110 & 211 & 9 & 234 \\ 11 & 122 & 243 & 134 \\ 10 & 221 & 233 & 30 \end{bmatrix} = f$$

Conclusion :

Transferring messages secretly between participants has interested people for long time, and it's been really important and needed in our modern world, especiallywith the advent of electronic messaging and the internet. This paper is a suggestion of a transformation that transform the pixels which are correlated into a representation where they are decor related. The new values are usually smaller on an average than the original values. The net effect is to reduce the redundancy of representation. For lossy compression, the transform coefficients can now be quantized according to their statistical properties, producing a much compressed representation of the original the image data. This idea of this transformation can be applied in encrypting and decrypting important data, and that would be my future modification of this paper

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