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RESEARCH ARTICLE

FITTING G-FAMILY PROBABILITY DISTRIBUTION AT THE RAINFALL DATA OF RAJSHAHI DISTRICT, BANGLADESH

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Abstract

Classical statistical distributions were recognized tool in case of different areas such as biological science, economics, finance, engineering and environmental science. But these types of distribution have some limitations for modeling real data. Researchers are trying to find out the best performing statistical distribution for modeling environmental data by adding extra parameters in these existing distributions which provide super flexibility in modeling. These extend to new families of these distributions generally defined as G-family distribution. Rainfall is one of the most important natural resources for agricultural production. The excess and deficit amount of rainfall sometimes be dangerous for agriculture production. So, the prior knowledge about the pattern of rainfall will help to solve food insecurity, water erosion and water pollution problems. In this study we compare the performance of Gamma uniform G, Log gamma G type-I, Log gamma G type-II , Marshall-Olkin G and Weibull G distribution where we consider lomax, f, chen and Weibull distribution as baseline distribution for modeling the rainfall data in case of Rajshahi district Bangladesh. The monthly rainfall data from January 1971 to December 2020 consider as study period. The empirical study shows that Marshall-Olkin Chen distribution gives better fitting results in case of Rajshahi district. This study will help policy makers to become aware of upcoming situations for solving water related problems.

Introduction:-

To fulfill the requirement of the scientific community, recent study need to analysis the data efficiently. But sometimes the standard statistical models cannot solve these problems accurately and effectively. For this reason, the creation of new flexible models, well adapted to the context, remains a passionate challenge for the statisticians. So, there have been growing interests in developing families of statistical distributions by adding one or more parameter with the existing statistical distributions to create new distribution family. This kind of new distribution is treated as G-family distribution. The target to generate more flexible distributions that will provide superior fitting performance than traditional statistical distributions in case of many real world problems.

Rainfall is one of the most important natural resources for agricultural production. As least some amount of water is essential for every kind of plants. Rainfall, usually meet up the demand of surface water becomes dangerous if its

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amount is more or less. So, the distribution of rainfall in time and space is necessary for the development of economy. Descriptive statistics give some idea about rainfall status but prior knowledge about rainfall status is enhanced by different statistical distributions. They are proven tool to describe many natural and social problem by providing suitable models and methods. The performances of different statistical distributions are investigated by Sen and Eljadid (1999) in case of Libyan monthly rainfall data and they found that the Gamma distribution performed better than other statistical distribution for 20 years data. Al-Mansory (2005) compared the performance different statistical distribution such as Normal, Log-Normal, Log-Normal type III, Pearson type III, Log-Person type III, and Gumbel and found that Person type III and Gumbel distributions performed better for describing maximum monthly rainfall in case of maximum monthly rainfall of Basrah, Iraq. Maliva and Missimer (2012) showed different probability distribution Normal, Gamma, Gumbel and Weibull give best fit of rainfall data in case of arid and semi-arid regions. Alghazali and Alawadi (2014) fitted three statistical distributions Normal, Gamma and Weibull in case of thirteen Iraqi weather stations and found that Gamma distribution was suitable for five stations. Afruz et.al. (2023) compared the performance of different statistical distribution with different estimation technique in case Pabna and Dinajpur district in Bangladesh. Different statistical distributions showing better performance in case of different geographic locations. But all of these distributions have some limitations. To overcome these restrictions and get more flexible condition, G-family distribution is introduced. Several authors proposed the G-family distribution and found that most of case these extended distribution perform better than classical distribution. Marshall-Olkin G distribution proposed by Marshall and Olkin (1997), Kumaraswamy G family by Cordeiro and Castro (2011), McDonald (Mc) G family by Alexander et. al. (2012), Weibull-G family by Bourguignon et. al. (2014). Alizadeh et. al. (2015) proposed Kumaraswamy Marshal-Olkin G family distribution, Yousof et. al. (2018) described Marshall-Olkin generalized-G family distribution.

From the above discussion it is clear that traditional distribution Normal, log-Normal, Gamma, Gumbel and Weibull were well fitted distribution for rainfall data. Recently researchers are trying to find more performing distribution by adding extra parameter of the existing classical distribution. The application of G-family distribution in environmental science especially rainfall data is rare and quite interesting. Rahman et al. (2022) compared the performance of Gamma uniform G family, Kumaraswamy G family, Marshall-Olkin G-family distribution and Weibull–G family distribution in case of rainfall data of Rajshahi division for the time period January 1971 to December 2015, Bangladesh. So, in this paper we use alternative G-family probability distributions for modeling rainfall data of Rajshahi district and also extend the time period up to December 2020. Therefore, the aim of this paper is to find out best performing distribution from a set of different G-family distributions such as Gamma uniform G family, Log gamma G type-I, Log gamma G family type-II, Marshall-Olkin G-family distribution and Weibull G family distribution with different base line distribution such as lomax, f, chen and weibull distribution in case of rainfall data of Rajshahi district, Bangladesh. Section 1 present the introduction of the study, section 2 present the methodology, section 3 present the result and discussion and finally section 5 present the conclusion and recommendation.

Methodology:-

Study Area

Rajshahi district is a district in mid-western part of Bangladesh under Rajshahi division. Geographically Rajshahi is situated within Barind Tract, 23 m (75 ft) above sea level, and lies at 24°22′26″N 88°36′04″E. It is bounded by Naogaon district to the north, Natore district to the east, Chapai Nababganj district to west and a small part of Kushtia district and the river Padma to the south. The district consists of alluvial plain. There are ten rivers in this district which cover 146 km in length. The main river is the Padma Ganges. Other rivers are Mahananda, Boral and Barnai. The climate of this district belongs to monsoons, high temperature, considerable humidity and moderate rainfall. The maximum mean temperature observed is about 32 to 36 °C (90 to 97 °F) during the months of April, May, June and July and the minimum temperature recorded in January is about 7 to 16 °C (45 to 61 °F). The highest rainfall is observed during the months of monsoon. The annual rainfall in the district is about 1,448 millimetres (57.0 in) (Wikipedia, 2024)

Source: http://www.banglapedia.org**.**

Data Sources

The data was collected from the Bangladesh Meteorological Department (BMD). The daily rainfall data of Rajshahi station from the period January, 1971 to December, 2015 considered for analysis. The daily data was converted to monthly data by Microsoft Excel. Missing data is common in environmental data. The missing values were random, however continuous missing data for one month to several months also evident in some year. We estimate missing data taking from the average value of two nearest stations, then it was finally ready to apply our methodology.

G-family distribution

Rainfall data analysis data depends on its different pattern of distribution. It is always interesting for researchers to find out best performing distribution for modeling rainfall data of certain areas. In this methodology we present mathematical form of different G-family distribution and also present some measure of goodness of fit test. Teimouri and Nadarajah (2019) developed Maximum Product Spacing (MPS) package for computing probability density function, cumulative distribution function, parameter estimation and drawing q-q plot from different G-family distribution. In our study we use MPS package of Teimouri and Nadarajah (2019) for empirical study.

Gamma Uniform G distribution

General form for the probability density function of gamma uniform G distribution due to Torabi and Montazeri (2012) is given by

$$f(x,\Theta)=\frac{h(x-\mu,\theta)}{\Gamma(\alpha)\left(1-H(x-\mu,\theta)\right)^{2}}\left(\frac{H(x-\mu,\theta)}{1-H(x-\mu,\theta}\right)^{\alpha-1}\exp\left(-\frac{H(x-\mu,\theta)}{1-H(x-\mu,\theta)}\right)$$

Where θ is the baseline family parameter vector. $\alpha > 0$ and μ are extra parameters induced to the baseline cumulative distribution function H whose probability density function is h. The general form for the cumulative distribution function of the gamma uniform G distribution is given by

$$F(x,\Theta) = \int_0^{\frac{H(x-\mu,\theta)}{1-H(x-\mu,\theta)}} \frac{y^{\alpha-1}\exp(-y)}{\Gamma(\alpha)} dy$$

The baseline H refers to the cumulative distribution function of different families such as Lomax, F, Chen, and Weibull distribution. The parameter vector is $\Theta = (\alpha, \theta, \mu)$ where θ is the baseline G family parameters which contain shape and scale parameter. The parameter μ is the location parameter.

Log Gamma G type-I distribution

Amini et. al. (2013) proposed the probability density function (pdf) of Log Gamma G type-I distribution as

$$f(x,\Theta) = \frac{\beta^\alpha h(x-\mu,\theta)}{\Gamma(\alpha)} [-\log(1 - H(x-\mu,\theta))]^{\alpha-1} (1 - H(x-\mu,\theta))^{\beta-1}$$

Where θ is the baseline family parameter vector. $\alpha > 0$, $\beta > 0$ and μ are extra parameters induced to the baseline cumulative distribution function H whose probability density function is h. The general form for the cumulative distribution function of the Log gamma G type-I distribution is given by

$$F(x,\Theta) = \int_0^{-\beta\log(1-H(x-\mu,\theta))} \frac{y^{\alpha-1}\exp(-y)}{\Gamma(\alpha)} dy$$

The baseline H refers to the cumulative distribution function of different families such as Lomax, F, Chen, and Weibull distribution. The parameter vector is $\Theta = (\alpha, \theta, \mu)$ where θ is the baseline G family parameters which contain shape and scale parameter. The parameter μ is the location parameter.

Log Gamma G type-II distribution

Amini et. al. (2013) proposed the probability density function (pdf) of Log Gamma G type-II distribution as

$$f(x,\Theta) = \frac{\beta^\alpha h(x-\mu,\theta)}{\Gamma(\alpha)} [-\log(H(x-\mu,\theta))]^{\alpha-1} (H(x-\mu,\theta))^{\beta-1}$$

Where θ is the baseline family parameter vector. $\alpha > 0$, $\beta > 0$ and μ are extra parameters induced to the baseline cumulative distribution function H whose probability density function is h. The general form for the cumulative distribution function of the Log gamma G type-II distribution is given by

$$F(x,\Theta) = \int_0^{-\beta\log(H(x-\mu,\theta))} \frac{y^{\alpha-1}\exp(-y)}{\Gamma(\alpha)} dy$$

The baseline H refers to the cumulative distribution function of different families such as Lomax, F, Chen, and Weibull distribution. The parameter vector is $\Theta = (\alpha, \theta, \mu)$ where θ is the baseline G family parameters which contain shape and scale parameter. The parameter μ is the location parameter.

Marshall-Olkin G distribution

Marshall and Olkin (1997) proposed one G family distribution which is known as Marshall and Olkin G distribution. The general form for the probability density function of this distribution is given below:

$$f(x,\Theta) = \frac{\alpha h(x-\mu,\theta)}{[1-(1-\alpha)(1-H(x-\mu,\theta))]^2}$$

Where θ is the baseline family parameter vector. $\alpha > 0$ and μ are extra parameters induced to the baseline cumulative distribution function H whose probability density function is h. The general form for the cumulative distribution function of the Marshall-Olkin G distribution is given by

$$F(x,\Theta) = 1 - \frac{\alpha\left(1 - G(x-\mu,\theta)\right)}{[1-(1-\alpha)(1-G(x-\mu,\theta))]}$$

The baseline H refers to the cumulative distribution function of different families such as Chen, Frechet, Long-Normal and Weibull distribution. The parameter vector is $\Theta = (\alpha, \theta, \mu)$ where θ is the baseline G family parameters which contain shape and scale parameter and μ is the location parameter.

Weibull G distribution

Weibull G distribution was proposed by Alzaatreh et. al. (2013). The general form for the probability density function of this distribution is shown below:

$$f(x,\Theta)=\frac{\alpha}{\beta^{\alpha}}\frac{h(x-\mu,\theta)}{1-H(x-\mu,\theta)}[-\log(1-H(x-\mu,\theta))]^{\alpha-1}\exp\left(-\left(\frac{-\log(1-H(x-\mu,\theta))}{\beta}\right)^{\alpha}\right)$$

Where θ is the baseline family parameter vector $\alpha>0$, $\beta>0$ and μ are extra parameters induced to the baseline cumulative distribution function H whose probability density function is h. The general form for the cumulative distribution function of the Weibull G distribution is given by

$$F(x,\Theta)=1-\exp\left(-\left(\frac{-\log(1-H(x-\mu,\theta)}{\beta}\right)^{\alpha}\right)$$

The weibull G distribution is the special case of Weibull-X family distribution of the Alzaatreh et. al. (2013). The baseline H refers to the cumulative distribution function of different families such as Chen, Frechet, Long-Normal and Weibull distribution. The parameter vector is $\Theta=(\alpha,\beta,\theta,\mu)$ where θ is the baseline G family parameters which contain shape and scale parameter and μ is the location parameter.

Model Evaluation Statistics

Choosing the suitable model is always challenging task for researchers. Smirnov (1948) introduced the table for estimating the goodness of fit for different empirical distributions. In this study we used different well known model evaluation criteria and goodness-of-fit statistics such as: Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) (Brownlee, 2019), Anderson Darling statistic (AD) (Anderson and Darling, 1952), log-likelihood statistic (log). The Kolmogorov-Smirnov (KS) (Smirnov, 1948) test statistic and corresponding p-value.

Result and Discussion:-

In this study we consider monthly rainfall data of Rajshahi district in Bangladesh. Papalexious (2012) proposed three steps for choosing probability distribution in case of rainfall data. These are: choose a priori some parametric families of distributions, estimate the parameters with appropriate fitting method, find the best performing model based on some model evaluation criteria. Recent proposed G-family distributions are performing better than traditional probability distribution in many cases. So, in this study Gamma Uniform G, Log gamma G type-I, Log gamma G type-II, Marshall-Olkin G and Weibull G distribution with Lomax, F, Chen and Weibull distribution consider for analysis. The summary statistics of rainfall data of Rajshahi district is given below in Table 1.

Table 1:- Summary statistics for monthly rainfall data.

	Min	First Qu.	Median	Mean	Third Qu.	Max.	Skewness	Kurtosis
Rajshahi	0.30	2.29	6.98	12.71	20.45	76.23	1.3346	1.8765

Table 1 showed that the average monthly rainfall for Rajshahi district is 12.71 mm where maximum rainfall amount in Rajshahi is 76.23 mm and minimum rainfall amount is 0.30. The data present the positive skewness and kurtosis is less than 3. The parameter estimation result with standard error is given in Table 2.

Table 2:- Parameter estimation result with standard error.

Model	**Parameter estimation**			
	$\hat{\alpha}$ (SE)	**$\hat{\beta}$ (SE)**	**$\hat{\theta}$ (SE)**	**$\hat{\mu}$(SE)**
	Lomax			
Gamma Uniform G	2.880 (2.890)		0.694 (2.871)	0.333 (1.890)
Log gamma G Type-I	0.922 (2.787)	1.467 (2.123)	0.003 (2.765)	2.131 (1.912)
Log gamma G Type-II	3.664 (2.547)	14.493 (1.987)	0.607 (2.121)	0.424 (1.867)
Marshall-Olkin G	0.714 (2.168)		0.032 (1.982)	3.398 (1.776)
Weibull G	2.930 (2.987)	1.832 (2.667)	0.421 (2.998)	0.104 (1.883)
	F			
Gamma Uniform G	11.739 (2.001)		0.055 (1.901)	0.393 (1.877)
Log gamma G Type-I	6.062 (1.876)	1.001 (0.978)	0.128 (1.887)	4.402 (1.865)
Log gamma G Type-II	7.278 (1.654)	1.957 (1.548)	0.405 (1.789)	2.324 (1.675)
Marshall-Olkin G	8.576 (1.332)		1.099 (1.673)	2.355 (1.343)
Weibull G	4.565 (2.110)	1.032 (1.976)	6.287 (2.112)	0.347 (1.901)
	Chen			
Gamma Uniform G	4.339 (1.341)		0.0923 (0.528)	0.653 (0.631)
Log gamma G Type-I	2.686 (1.212)	0.284 (1.550)	0.185 (0.153)	1.011 (0.233)
Log gamma G Type-II	0.495 (1.132)	1.735 (1.432)	0.213 (0.065)	0.182 (0.142)

Marshall-Olkin G	1.769 (1.023)		0.051 (0.045)	0.789 (0.121)
Weibull G	2.450 (1.563)	0.559 (1.876)	0.149 (0.675)	0.0822 (0.781)
	Weibull			
Gamma Uniform G	1.648 (1.564)		0.398 (0.557)	0.956 (0.876)
Log gamma G Type-I	1.001 (1.561)	1.024 (1.791)	0.798 (0.954)	5.124 (1.543)
Log gamma G Type-II	0.895 (1.230)	0.785 (1.763)	0.884 (0.862)	6.764 (1.120)
Marshall-Olkin G	0.969 (1.076)		0.866 (0.887)	5.180 (0.896)
Weibull G	0.901 (1.775)	0.735 (1.986)	0.900 (1.112)	15.543 (2.329)

The estimated result from Table 2 indicates that all of the parameters from gamma uniform lomax, gamma uniform f, gamma uniform chen, gamma uniform weibull, log gamma G type-I lomax, log gamma G type-I f, log gamma G type-I chen, log gamma G type-I weibull, log gamma G type-II lomax, log gamma G type-II f, log gamma G type-II chen, log gamma G type-II weibull, Marshall-Olkin G lomax, Marshall-Olkin G f, Marshall-Olkin G chen, Marshall-Olkin G weibull, Weibull G lomax, Weibull G f, Weibull G chen and Weibull G weibull distribution meet the criteria for parameter significant. The standard error from Marshall-Olkin chen produce lowest standard error whereas Weibull G weibull distribution produce higher standard error for all parameters except the parameter μ. The model evaluation criteria such as AIC, BIC, log likelihood and some test of goodness of fit test statistics such as AD and KS is given in Table 3.

From Table 3 we found that the Marshall-Olkin G chen distribution produce the lowest AIC and BIC value whereas this distribution produce the highest log likelihood value which indicate that the Marshall-Olkin G chen distribution provide better performance and the Weibull G lomax distribution showed highest value of AIC and BIC value. The Anderson Darling statistic (AD) and Kolmogorov-Smirnov (KS) test statistic shows that all of these models are significant. For comparing model the Marshall-Olkin chen model produce lowest value of Anderson and darling test statistic and Weibull G lomax distribution produce highest value. So, if we compare the performance of all of the distributions which are used here found that the first performing distribution is Marshall-Olkin G, second performing distribution is log gamma G type-II, third performing distribution is Gamma uniform G and last performing distribution is Weibull G. And if we compare the baseline distribution then found that base line distribution Chen with all of the distributions showed better performance whereas Lomax with all of the distributions showed the worst result.

Table 3:- Model evaluation statistics for different distributions.

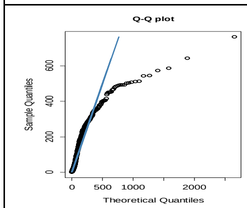
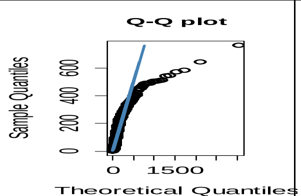
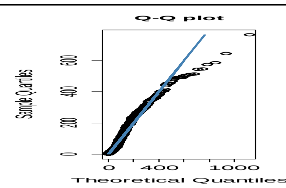
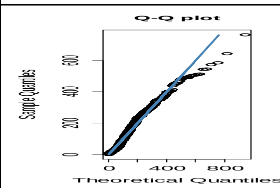
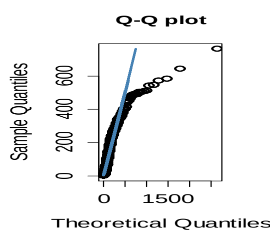
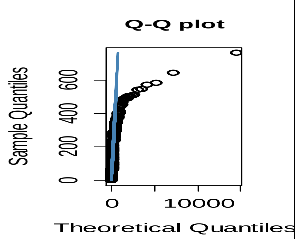
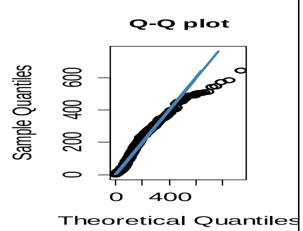
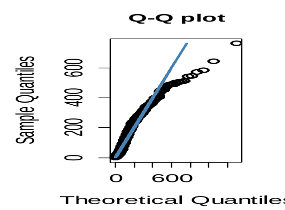
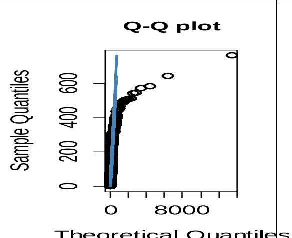
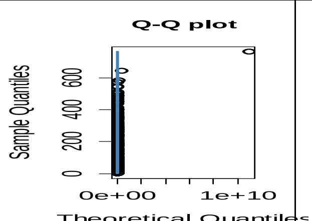
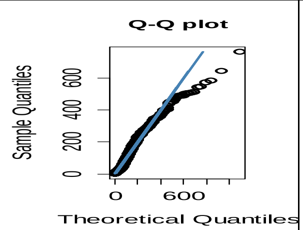
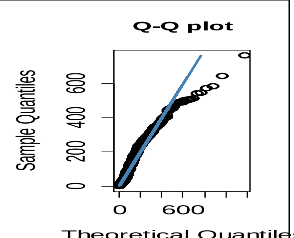
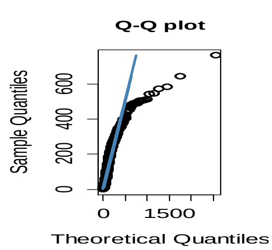
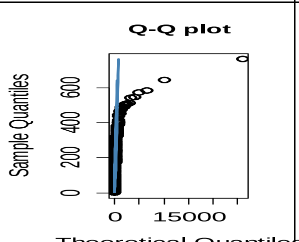
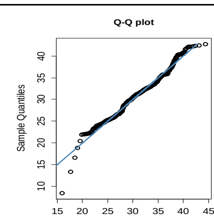
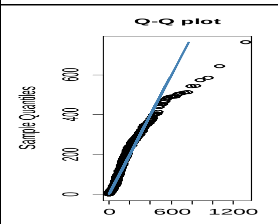
Model	**Model evaluation statistics**				
	AIC	**BIC**	**-LL**	**AD**	**KS (p-value)**
	Lomax				
Gamma Uniform G	7018	7038	3500	10.01	0.126 (0.000)
Log gamma G Type-I	7013	7033	3496	9.52	0.099 (0.002)
Log gamma G Type-II	7009	7021	3479	8.24	0.085 (0.000)
Marshall-Olkin G	7001	7018	3436	7.45	0.076 (0.001)
Weibull G	7028	7040	3504	10.83	0.879 (0.000)
	F				
Gamma Uniform G	7010	7021	3486	7.01	0.087 (0.001)
Log gamma G Type-I	6999	6990	3479	6.92	0.079 (0.000)
Log gamma G Type-II	6968	6984	3466	6.75	0.073 (0.000)
Marshall-Olkin G	6946	6969	3421	6.05	0.070 (0.001)
Weibull G	7021	7038	3493	7.21	0.094 (0.002)
	Chen				
Gamma Uniform G	6975	6993	3412	5.35	0.084 (0.001)
Log gamma G Type-I	6765	6888	3377	5.18	0.077 (0.001)
Log gamma G Type-II	6753	6775	3372	5.08	0.772 (0.000)
Marshall-Olkin G	6739	6756	3365	3.86	0.012 (0.001)
Weibull G	6903	6915	3483	5.92	0.087 (0.000)
	Weibull				
Gamma Uniform G	6892	6838	3441	6.07	0.082 (0.000)
Log gamma G Type-I	6821	6914	3418	6.01	0.073 (0.016)

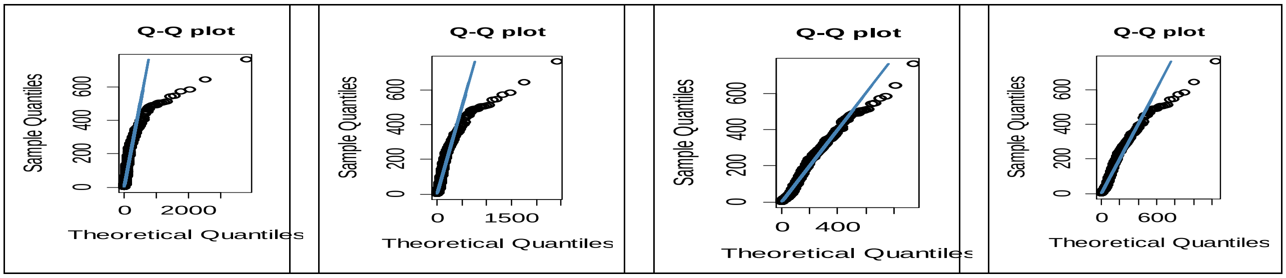
Log gamma G type-II	6815	6807	3406	5.98	0.064 (0.137)
Marshall-Olkin G	6803	6800	3401	5.87	0.061 (0.000)
Weibull G	6909	6931	3450	6.14	0.095 (0.009)

Therefore, from the parameter estimation result and model evaluation criteria and goodness of fit test statistics we conclude that Marshall-Olkin G chen distribution is the most suitable distribution for fitting monthly rainfall data in case of Rajshahi district which also confirm the result of Rahman et al (2022).

The Q-Q plot of these models for Rajshahi district is shown in Figure 2. The Q-Q plot also confirm that the Marshall-Olkin chen distribution provide better result for fitting monthly rainfall data of Rajshahi district.

Figure 2:- Q-Q plot of the fitted distributions.

Lomax	**F**	**Chen**	**Weibull**
Gamma Uniform G	Gamma Uniform G	Gamma Uniform G	Gamma Uniform G
			
Log gamma G Type-I	Log gamma G Type-I	Log gamma G Type-I	Log gamma G Type-I
			
Log gamma G Type-II	Log gamma G Type-II	Log gamma G Type-II	Log gamma G Type-II
			
Marshall-Olkin G	Marshall-Olkin G	Marshall-Olkin G	Marshall-Olkin G
			
Weibull G	Weibull G	Weibull G	Weibull G



Conclusion:-

Agriculture production highly depends on the certain amount of water and rainfall is the main source of surface water. The preceding knowledge about the rainfall pattern help policy maker to take necessary action in advance to solve some water related problems. Statistical distribution is one of the main tools to get idea about the pattern of rainfall and G family probability distribution is the advance and contains more flexible condition than classical statistical distribution. In this study we compare the performance of Gamma uniform G, Log gamma G type-I, Log gamma G type-II, Marshall-Olkin G and Weibull G distribution where we consider lomax, f, chen and Weibull distribution as baseline distribution for modeling the rainfall data in case of Rajshahi district Bangladesh.

For modeling monthly rainfall data from the time period January, 1971 to December, 2020 in case of Rajshahi district is considered for analysis. The model evaluation criteria indicate that Marshall-Olkin Chen distribution give best fitting results in case of Rajshahi district and Weibull G Lomax distribution give worst fitting performance. As a base line distribution Chen and F distribution provide better results than Weibull and Lomax distribution for all of the cases. Although this paper is an extension of Rahman et al (2022) work's but still we need to check new G family distributions and also base line distribution. This is for future work.

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