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RESEARCH ARTICLE

EFFECT OF PERMEABILITYAND HEAT GENERATION IN MHD FLOW ALONG A VERTICAL ISOTHERMAL POROUS PLATE UNDER VARIABLE ELECTRICAL CONDUCTIVITY.

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Abstract

MHD convective flow of a viscous incompressible fluid along a vertical isothermal porous plate is discussed with the action of variable electrical conductivity and heat generation in presence of external transverse magnetic field. It is supposed that there is an internal heat generation along the plate that decays exponentially while electrical conductivity of the fluid is a function of fluid temperature. The process of similarity transformation is used to transform the partial governing equations into ordinary. Considering fluid flow of low PrandltNumber Pr<< 1}, Numerical solutions and results are obtained for the fluid Prandlt Number = 0.025 (mercury 20^0 c) and 1.0 (electrolytic solution) using Runge-Kutta method, while Shooting method is used to find the missing initial conditions. The results are used to plot velocity and temperature profiles near the plate and skin-friction and heat transfer at the plate for various values of physical parameters used. The results show significant effects of fluid electrical conductivity and medium porosity on the flow and heat transfer in presence and absence of heat generation under the action of transverse magnetic field.

Nomenclature:-

x, y : Cartesian coordinates;
ρ: density of fluid;
Ψ: stream function;
f: dimensionless stream function;
T: Temperature of the fluid;
Q: volumetric rate of heat generation
S: heat generation parameter;
k: coefficient of thermal conductivity;
B_o: magnetic field intensity;
ε: electrical conductivity parameter
u, v: velocity components along x- and y- directions, respectively;
μ: coefficient of viscosity; υ:kinematic viscosity;
g: Acceleration due to gravity of the Earth.
η: similarity variable; τ: skin-friction coefficient;:
T_∞: Temperature of fluid far away from plate;
β: coefficient of thermal expansion; θ: dimensionless temperature;
Pr: Prandtl number; k_1= permeability of the medium
C_p: specific heat at constant pressure : Nu: Nusselt number;
M: Hartmann number; σ: electrical conductivity
σ_1: variable electrical conductivity; Da: Darcy number

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Introduction:-

Magntohydrodynamic (MHD) thermal boundary layer flow with variable electrical properties in the presence of a transverse magnetic field has been to a great deal of attention because of its wide ranging applications, for example in geophysics, thermal insulation engineering, industrial fields such as chemical engineering process, drying process etc., MHD generators, Pumps, Accelerators, Flow-meters, and many such applications. By selecting fluids of suitable electrolyte and its medium porosity (i.e Darcy Number) in presence of magnetic field, one can control many metallurgical processes, e.g. cooling of continuous strips etc. The effect of heat generation or absorption in MHD flows can be effectively dealt by taking into account the variation of fluid properties along with temperature field, Herwig, et al [1]. The effect of internal heat generation is especially pronounced for low Prandtl number fluid e.g. liquid metal like Mercury, Bismuth, KCl solution, NaCl solution etc. This is because of the fact that they have smaller Prandtl number but higher thermal conductivity which provides them ability to transport heat even if small temperature difference exists. The MHD flow with suitable electrically conducting fluid under magnetic field can control the rate of cooling while achieved desired results as well, Chakrabarti et al. [2]. Some liquid metals have smaller Prandtl number, of order 0.01 to 0.1; e.g. Bismuth=0.01, Mercury =0.023at 20^0 temperature etc. They are generally used as coolants because of higher thermal conductivity. Many authors have studied problems of natural convection flow along vertical isothermal plate with such fluids of low Prandtl number. Flow of such kind of fluids at stagnation point have been discussed by Pai et al.[3]. Kay [4] reported that thermal conductivity of liquids with low Prandtl number varies linearly with temperature in range of 0°F to 400°F. Arunachalam and Rajappa [5] considered forced convection in liquid metals with variable thermal conductivity and capacity in potential flow and derived explicit closed form of analytical solution. Chen [6] considered laminar mixed convection flow adjacent to vertical, continuously stretching sheet. Molla et al. [7] studied the natural convection flow along a horizontal cylinder in the presence of heat generation. Recently, Gorla et al [8], Alam M. S [9], Chain, T. C [10], Hazen A. Allia [11] and many others have studied MHD flow with heat generation problem with various geometries. Recently, Boracic et al. [12], has studied natural convection MHD flow with variable electrical conductivity and heat generation along an isothermal plate. More recently, Sharma et al. [13] have studied steady MHD natural convection flow with variable electrical conductivity and heat generation along an isothermal vertical plate.

In this paper, we are going to investigate a steady, fully developed MHD convective heat and mass transfer problem of an incompressible fluid flow along a porous plate in porous medium where fluid is sucked through vertical plate and maintained at constant suction velocity under the action of heat generation and variable electrical conductivity in presence of transverse magnetic field. Considering the energy dissipation due to porosity of the medium, the nature of flow parameters like fluid velocity, temperature, skin friction and heat transfer at the plate, are analyzed graphically and discussed to draw some conclusions.

Formulation of the problem:-

We consider steady laminar natural convection flow of a viscous incompressible fluid over a vertical non-conducting plate in a medium. It is considered that the plate is porous, and at constant temperature that generates an internal volumetric heat within the plate that affect fluid flow while the fluid electrical conductivity varies inversely with temperature [12]. The x-axis is taken along the plate and y-axis is normal to the plate. A uniform magnetic field of intensity Bo is applied normal to the plate. It is assumed that the electrical field due to polarization of charges and Hall Effect are negligibly small. Incorporating the Boussinesqs approximation within the boundary layer , the governing equations of continuity , momentum and energy, Schlichting [14], respectively are given as follows.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+g\beta\,(T_w-T_\infty)-\frac{\sigma_1 B_0^2}{\rho}\,u-\frac{\upsilon}{k_1}u=0 \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K\frac{\partial^2 T}{\partial y^2}+-\frac{\mu}{K_1}u^2+Q \tag{3}$$

$$\sigma_1=\frac{\sigma}{1+\varepsilon\theta} \tag{4}$$

The boundary conditions are

$$y=0: u=0\,,\ v=0\,,\ T=T_w$$
$$y\to\infty:\quad u\to 0,\quad T\to T_\infty \tag{5}$$

Method of Solution:-

Introducing the stream function Ψ(x, y) such that

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \Psi}{\partial x} \quad (6)$$

Where,

$$\Psi(x,y) = 4\upsilon f(\eta)\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \quad \text{and} \quad \eta = \frac{y}{x}\left(\frac{Gr}{4}\right)^{\frac{1}{4}} \quad (7)$$

Following Crepeau and Clarksean [15] ,the volumetric rate of heat generation is given as

$$Q=S\{k(\frac{T_w - T\infty}{x^2})(\frac{Gr}{4})^{1/2} e^{-n})\} \quad (8)$$

Since equation (1) is identically satisfied equation (6), using equations (6) , (7), and (8), equations (2) and (3), along with the equation (4) ,the resulted coupled non-linear ordinary differential equations, given are as follows

$$f''' -2 f'^2 + 3f f'' +\theta - [\frac{M}{1+\epsilon\theta} + \frac{1}{D_a}(\frac{4}{G_r})^{\frac{1}{2}}]f' =0 \quad (9)$$

and

$$\theta'' + \theta' +3 P_r \theta' f - [16\frac{E P_r}{D_a}(\frac{G_r}{4})^{\frac{1}{2}} (f')^2] + Se^{-n} = 0 \quad (10)$$

Where,

$\upsilon = \frac{\mu}{\rho}$; $Gr = (\frac{g\beta(T_w - T_x)x^3}{v^2})$; $M = \frac{\sigma B_0^2 x^2}{\mu}(\frac{Gr}{4})^{-1/2}$

$Pr = \frac{\mu C_p}{K}$; $Q= K(\frac{T_w - T_x}{x^2})(\frac{G_r}{4})^{\frac{1}{2}} e^{-n})$; $\theta = \frac{T - T_\infty}{T_w - T_\infty}$; $Da=\frac{K_1}{X^2}$

The boundary conditions are reduced to

$$f(0) = 0,\ f'(0) = 0,\ f'(\infty) = 0,\ \theta(0) = 1 \text{ and } \theta(\infty) = 0 \quad (11)$$

The governing boundary layer equations (9) and (10) with boundary conditions (11) are solved using Runge-Kutta fourth order technique along with double shooting technique.

Skin –Friction Coefficient:-

$$\tau = \frac{\tau}{\frac{1}{2}\rho u_o^2} = 2\left(\frac{Gr}{4}\right)^{\frac{1}{4}} f''(0) \quad (12)$$

Where,

$(\tau)_{y=0} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}$, shear stress at the plate

$u_0 = \sqrt{\{g\beta x(T_w - T\alpha)\}}$, convective fluid velocity near the plate.

Rate of Heat Transfer:-

The rate of heat transfer in terms of the Nusselt number at the plate is given by

$$Nu = \frac{qx}{K(T_w - T_\infty)} = -\left(\frac{Gr}{4}\right)^{\frac{1}{4}}\theta'(0)$$

Where, $q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$

Solutions of equations:-

Solution for the equations (9 & 10) subject to the boundary condition(11) are obtained using Shooting iteration technique along with fourth order Runge-Kutta method for different values of physical parameters. In calculating numerical results for physical quantities of our interest f, T, τ & Nu we have considered, S= 0.0 stands for absence of heat generation while S > 0 in presence of heat generation, Gr= 5.00 because it relates to various magnetohydrodynamic phenomena also in the problems of cooling in nuclear reactors; Pr=0.025 since it is connected to the popular liquids metal mercury at 20^0c ; n= 1.0 (choosen arbitarily). The physical parameters whose effects on flow motion are the objectives of this study, varied as Da = 1.0 to 2.5 ; ε = 0.01 (KCl solution ε = 1.05 at 15^oc), to 20.0 (NaCl solution □ = 20.14 at 15^oc); M = 0.2 to 1.5; S = 0.0 to 1.5. We suppose that the electrical conductivity of the liquid (electrolyte) stands ε = 0.01 as smaller, ε = 1.0 as larger. The various values of non-dimensional parameters fluid-velocity (f), fluid-temperature(T) , Skin-friction at the plate $(\tau)_{\eta=0}$, and the rate of heat transfer at the plate $(Nu)_{\eta=0}$, has ben obtained from the numerical solutions. Values are plotted for above mentioned values of Da, ε, M and S; the results are shown in the figures 1-8.

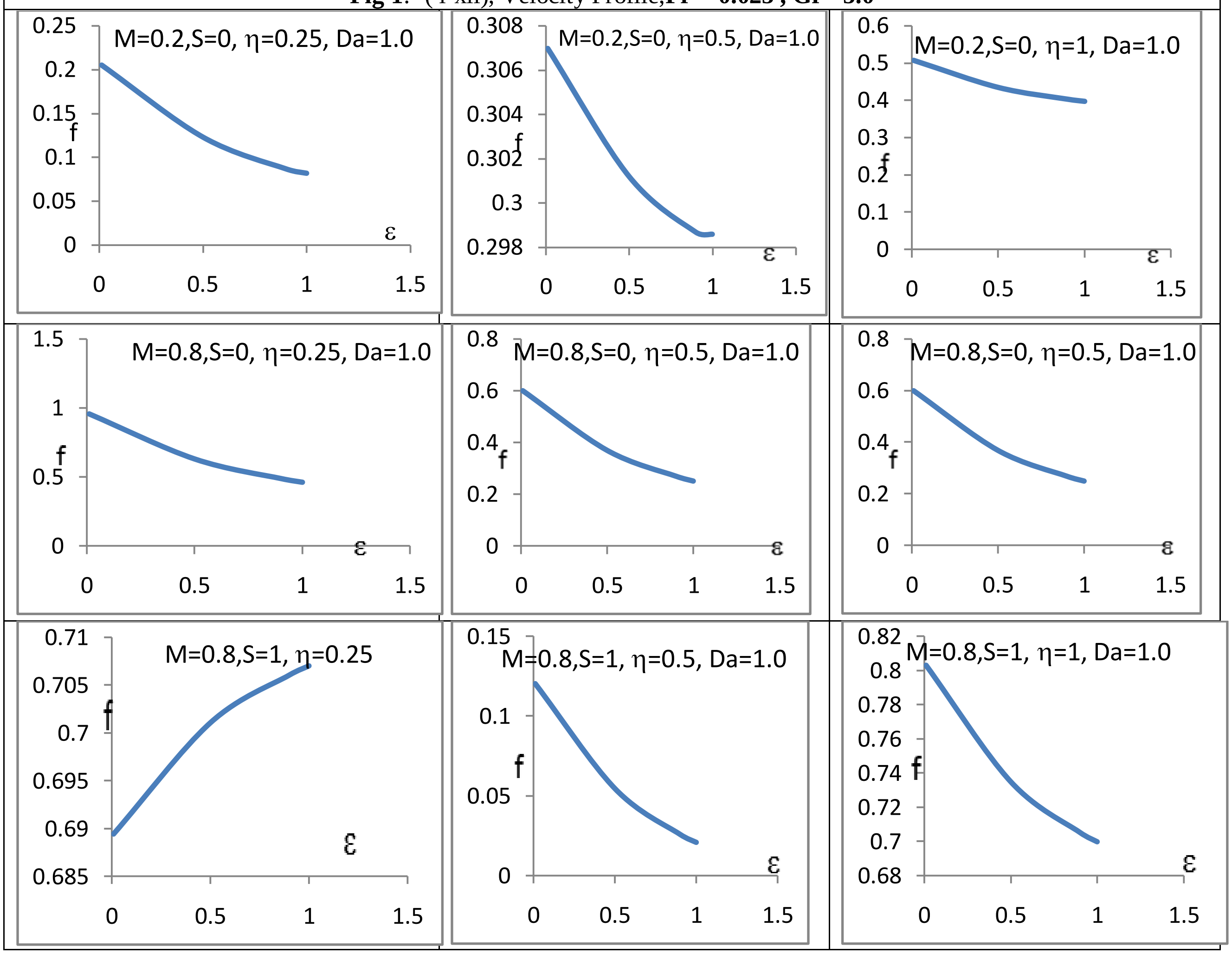
Technique For Numerical Solutions:-

The system of non-linear ordinary differential equations (9 - 10) together with the boundary conditions (11) are solved numerically using Nachtsheim-Swigert shooting iteration technique (guessing the missing values) along with fourth order Runge- Kutta initial value solver. Chakrabortyet. al.[16], Hazarikaet. al.[17], Alam et. al.[18] have also used same technique to solve their problems.

Result and Discussion:-

(i) Velocity Profile

Fig 1:- (i-xii), Velocity Profile,**Pr = 0.025 , Gr= 5.0**



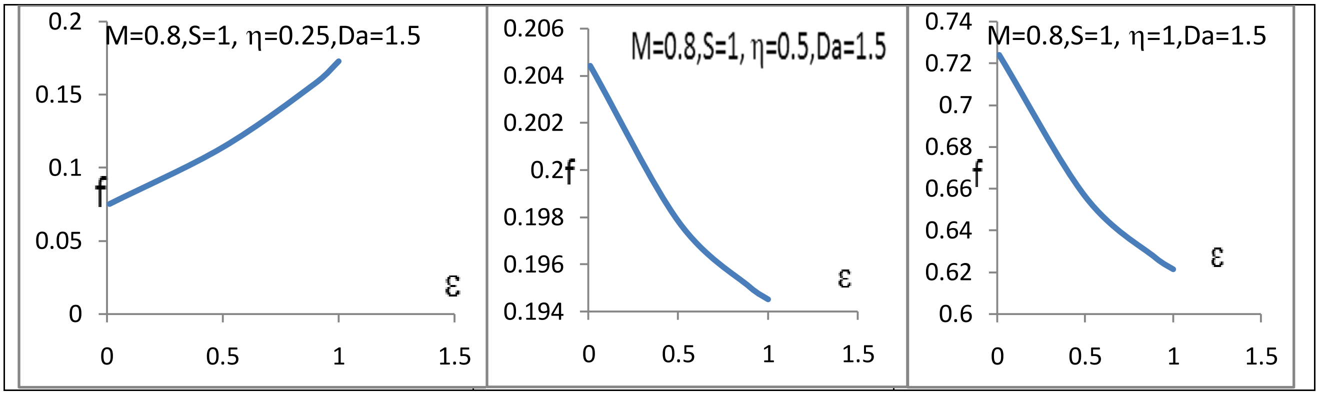
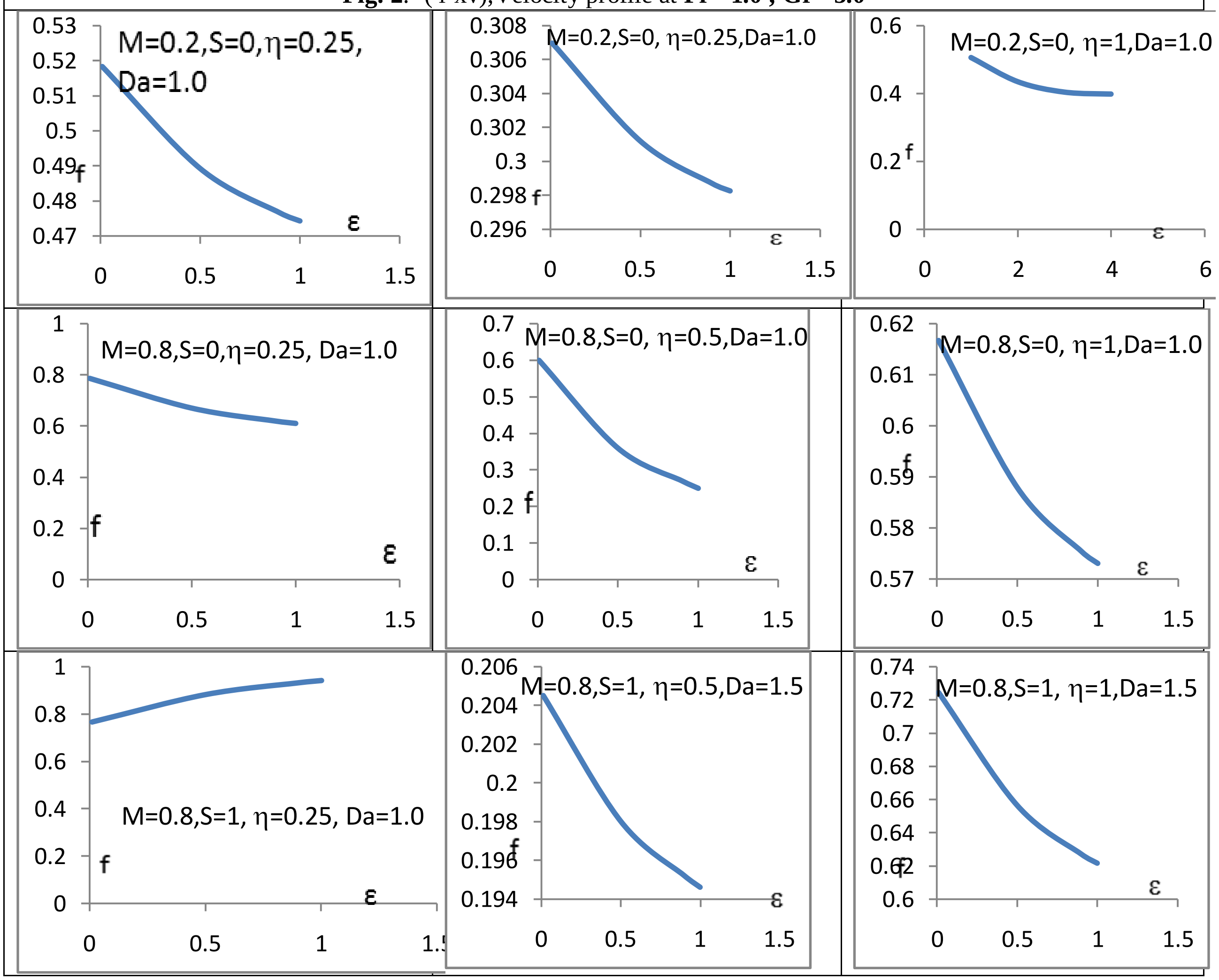


Fig. 2:- (i-xv),Velocity profile at **Pr =1.0 ; Gr= 5.0**



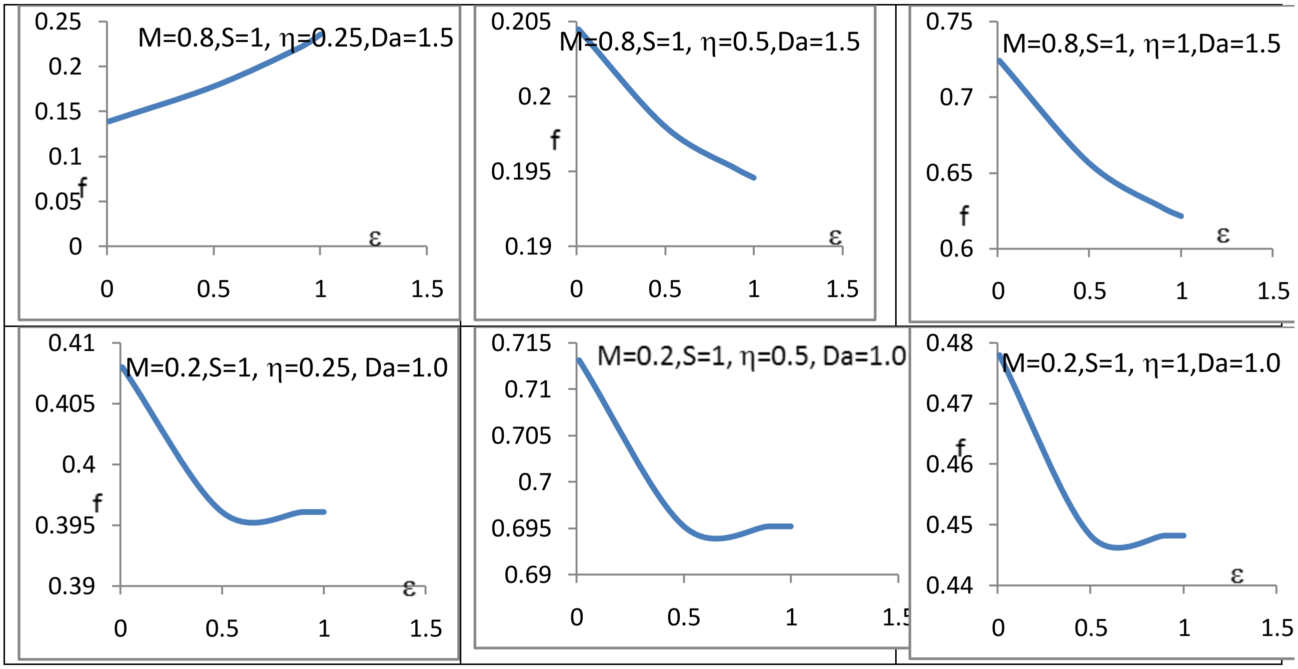
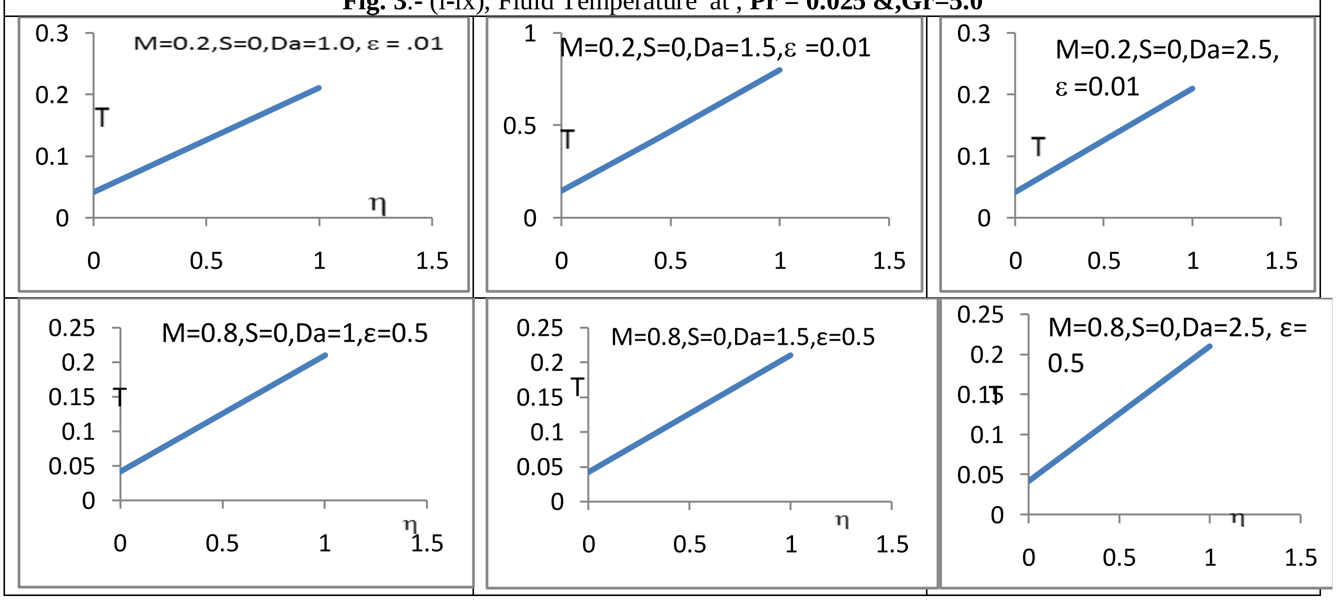


Fig. 1 & 2 (i-vi), away from the plate (η= 0.25 to 1.0), Fluid velocity (f) decreases with the rise of ε ; rate of decrease depends upon M, Da, S and Pr. Fluid velocity (f) increases with the increase of M,but decreases with the increase of heat generation parameter S fig. 1 & 2 (i & iv, ii & v, iii & vi) and fig. (iv& vii, v & viii, vi & ix) respectively. In presence of heat generation (S=1), f increases with the increase of ε near the plate (η= 0.25), fig. 1 &2 (vii & ix) while gradually decreases away from the plate. When Da increases from 1.0 to 1.5, the value of fluid velocity f also increases fig. (vii& x, viii & xi, ix & xii).With the increase of Pr (0.025 to 1.0) which means decrease of thermal diffusivity of the medium, f decreases; this is opposite in nature in presence of heat generation S=1 fig.(1 & 2).

Temperature Profile:-

Fig. 3:- (i-ix), Fluid Temperature at , **Pr = 0.025 &,Gr=5.0**



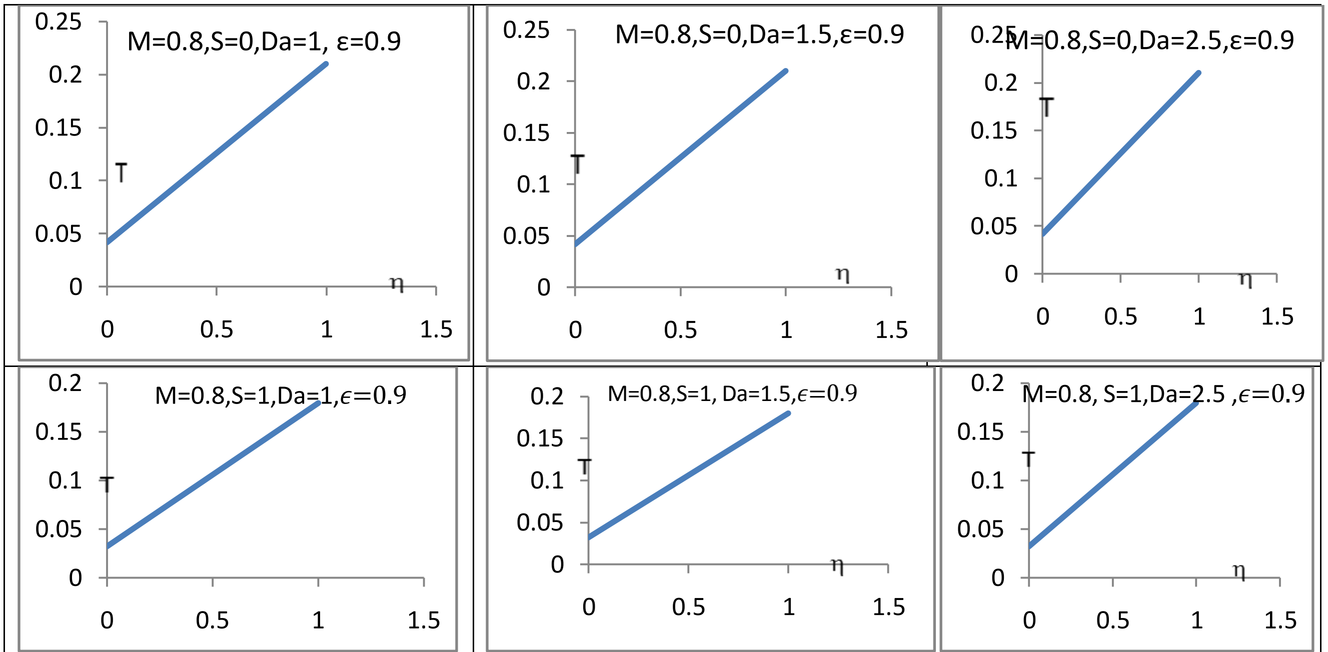
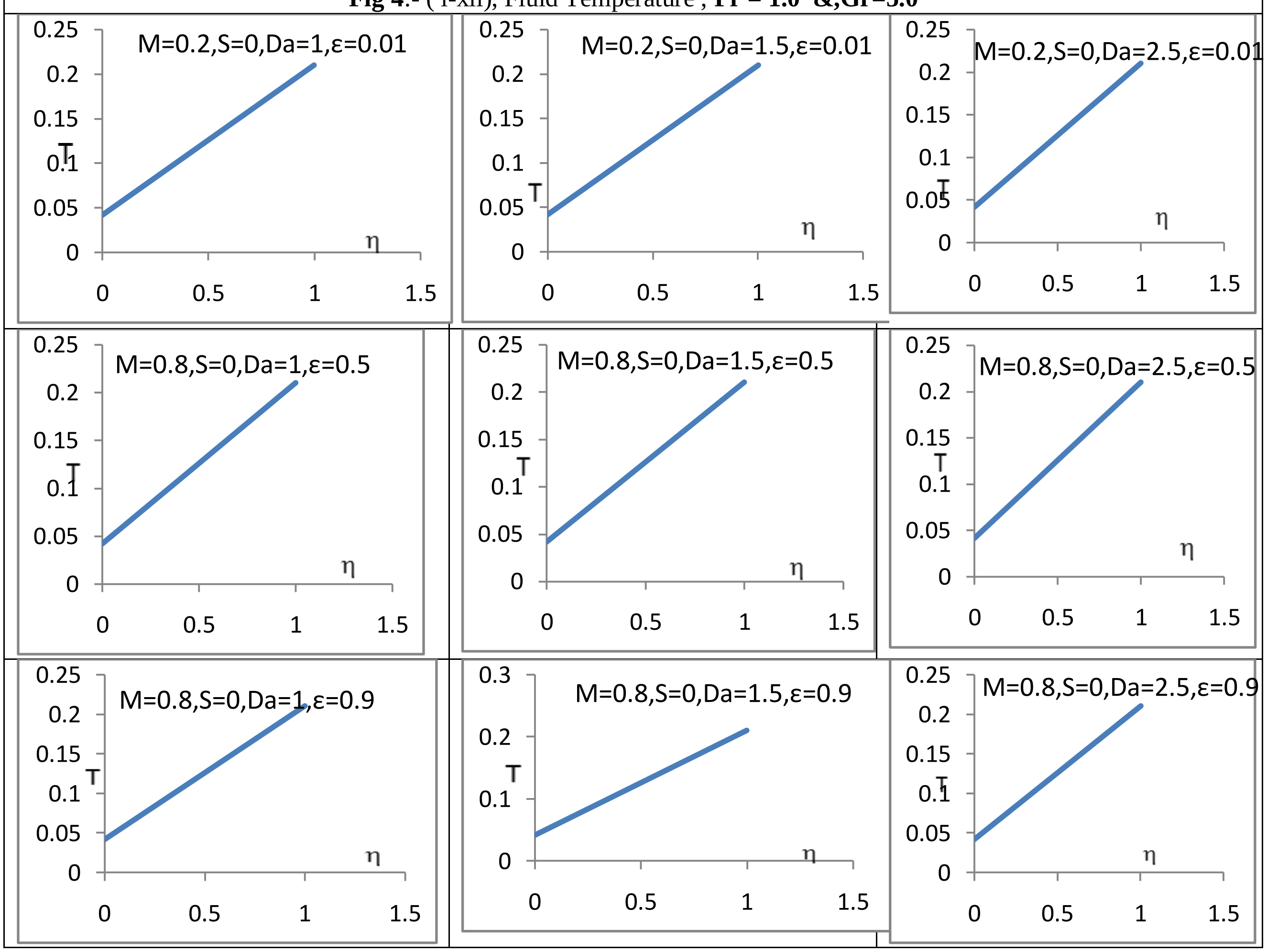
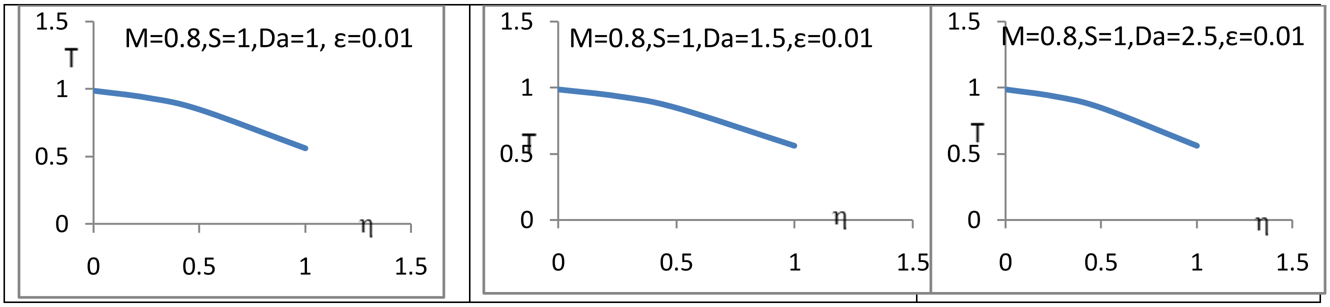


Fig 4:- (i-xii), Fluid Temperature , **Pr = 1.0 &,Gr=5.0**

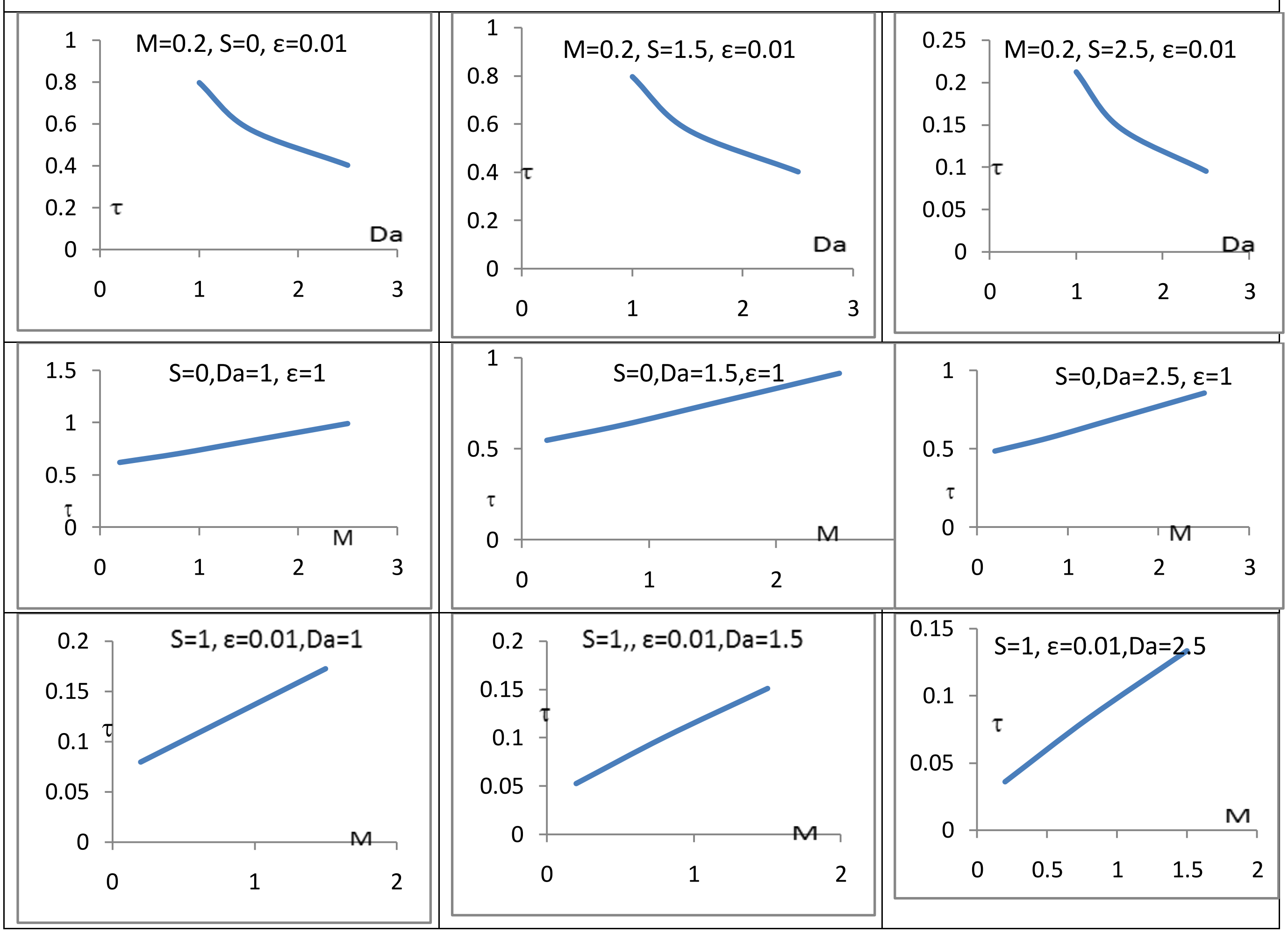




From fig. 3 & 4, the fluid temperature (T) within the medium away from the plate rises gradually forvarious values of S, M &ε. Fluid temperature (T)decreases slowly with the increase of Da , but rises slowly with the rise of M fig 3 & 4, (i-iii, iv-vi, vii- ix) and (i& iii, iii & vi) respectively. When Pr is increased from 0.025 to 1.0,in presence of heat generation S=1 and reasonably higher magnetic field M ≅ 0.8 the nature of variation of T is reciprocal with η , i.e T decreases away from the plate, fig 4, (vii& x, viii & xi, ix & xii). With the increase of ε, fluid temperature (T) alsoincreases, similarly for M, fig. (vii& x) & (ii &vi) respectively.

Skin Friction Profile:-

Fig 5: - (i-ix), Skin Friction (τ) profile at the plate (η=0.0), **Pr = 0.025 , Gr=5.0**



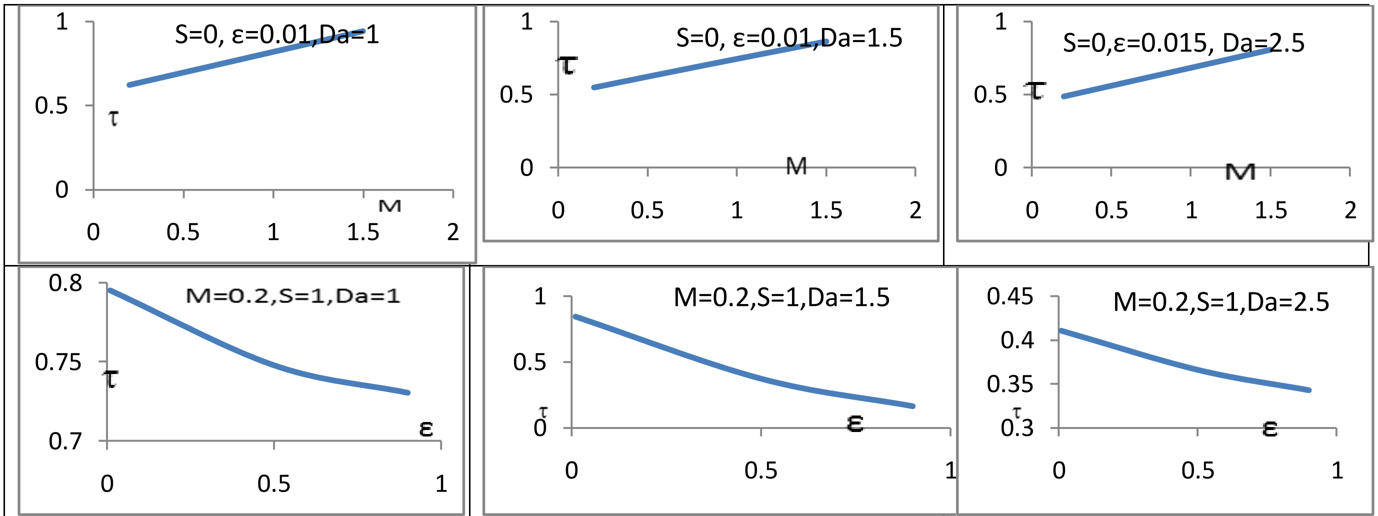


Fig 6: (i-ix), Skin Friction profile at the plateη=0.0, **Pr = 1.0 &Gr=5.0**

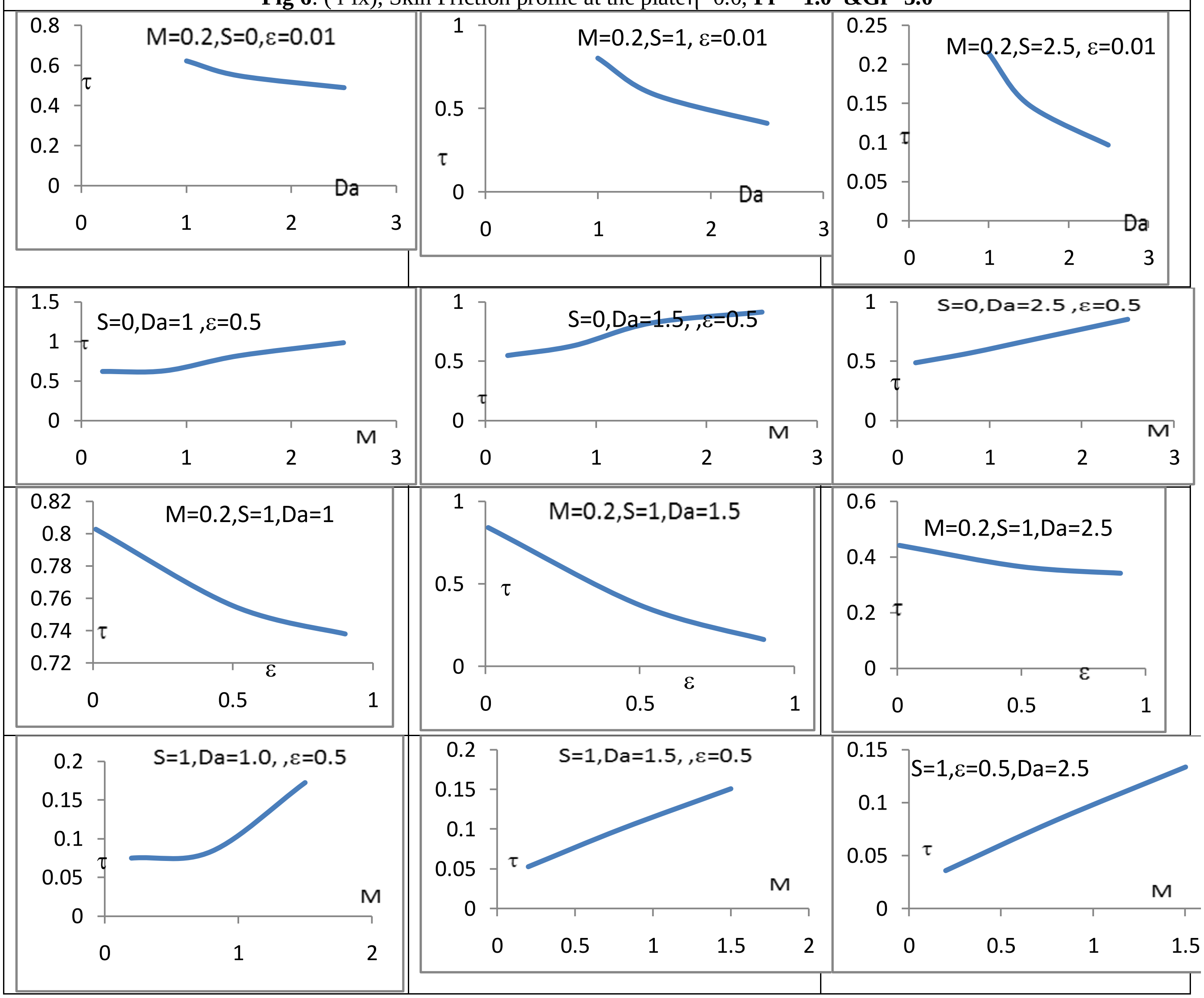


Fig. 3 & 4, the Skin Friction at the plate$(\tau)_{\eta=0}$, varies reciprocally with the rise of Da both in absence (S=0.0) and presence of heat generation ($S \cong 1$). At higher heat generation(S >1), $(\tau)_{\eta=0}$ is higher fig.5 &6.(i-iii). With the rise of M ,τ increases both in absence and presence of heat generation S = 0 & S=1 fig. 5 & 6.(ii i-ix)respectively . The value of τ decreases with the increase of Da (=1.0, 1.5, 2.5) fig. 5 & 6.(x-xii). With the increase of electrical conductivity ε, $(\tau)_{\eta=0}$ gradually decreases fig. 5 (xiii-xv).

Heat Transfer Profile:

Fig 7:- (i-ix), Heat Transfer (Nu) profile at the plate(η=0.0), **Pr = 0.025&Gr=5.0**

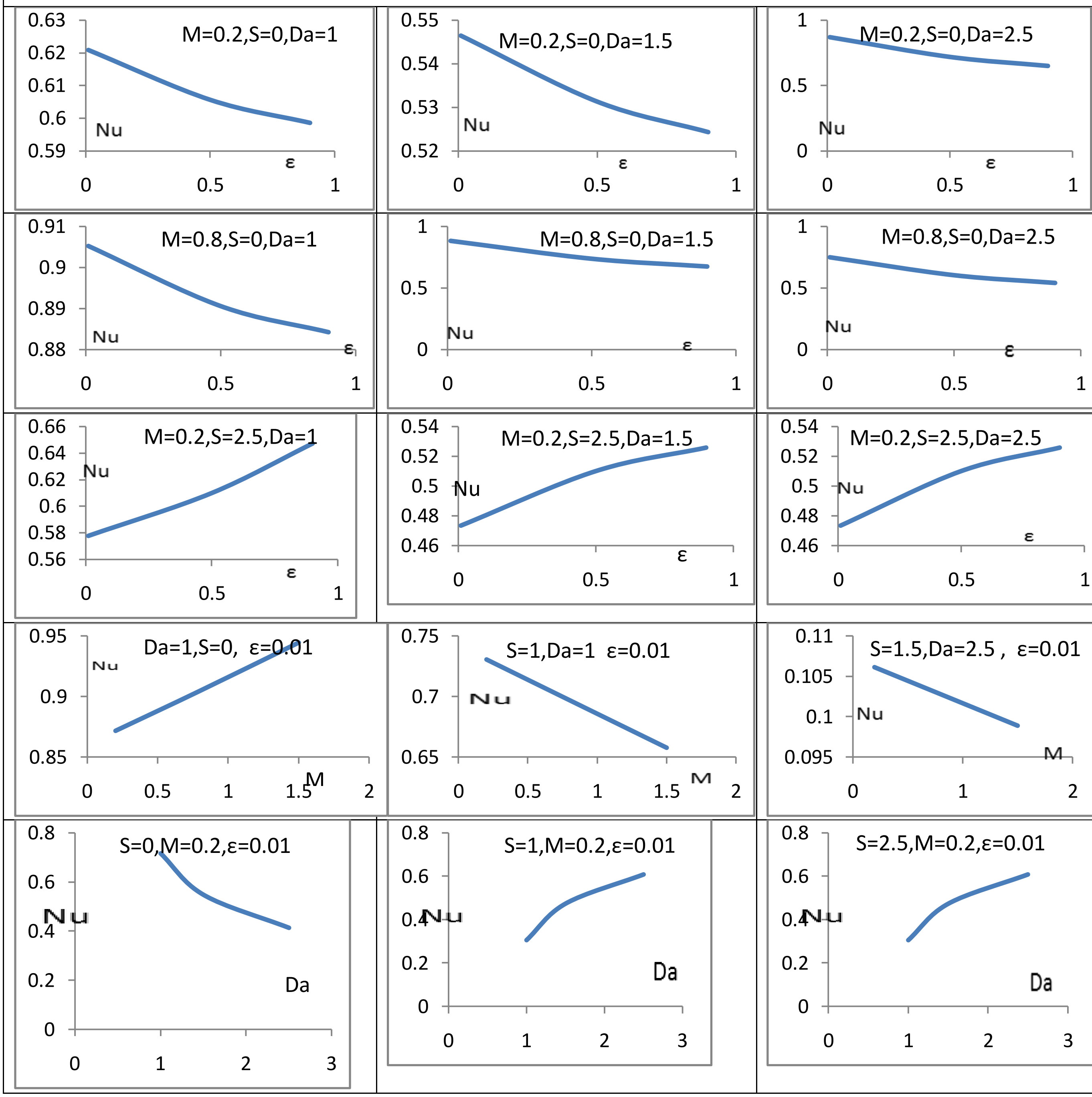
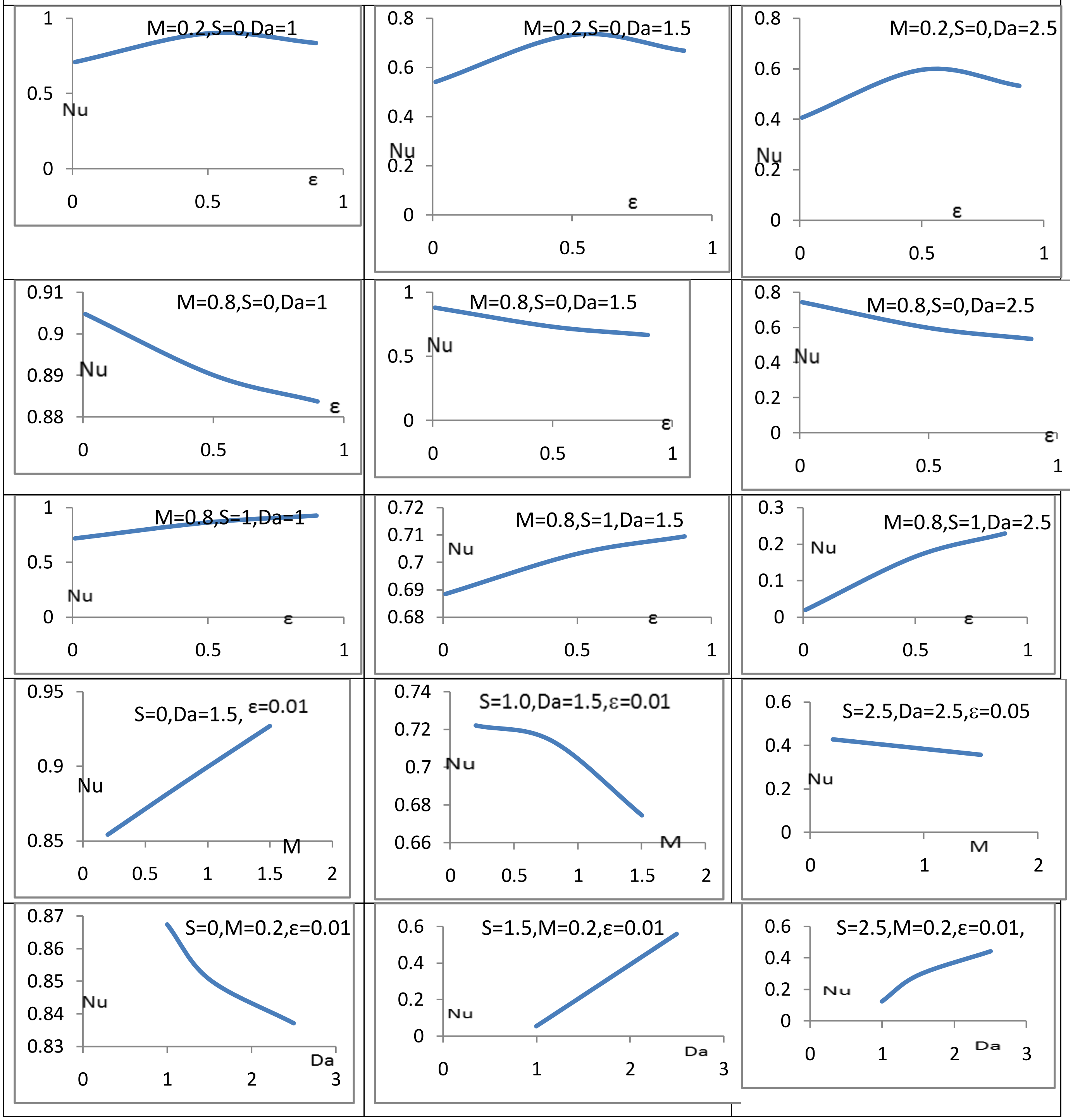


Fig 8:- (i-ix), Heat Transfer (Nu) profile at the plate(η=0.0), **Pr = 1.00 & Gr = 5.0**



The nature of variation of heat transfer coefficient Nu with electrical conductivity ε is almost opposite; Nu decreases with the increase of ε in absence of heat generation S=0.0 but increases gradually in presence of heat generation S=2.5, fig. 7 & 8. (i-ix). With the increase of M, heat transfer Nu decreases in absence of heat generation S=0 while increases in presence S =1.0, 2.5 fig. 7,8 (x – xv). Nu decreases with Da in absence of heat generation (S = 0) but increases gradually in presence of heat generation S >0, fig 7 & 8 (xiii to xv).M=0.8,S=1.5 ,Da=1.5

Conclusions:-

- Fluid velocity decreases with the rise of electrical conductivity (ε) in absence and presence of heat generation; exception is that at very near to the plate in presence of heat generation, fluid velocity increases with the increase of ε . The rate of decrease is higher in presence of heat generation. For higher applied magnetic field, the rate of decrease is higher; rate of decrease is higher in presence of heat generation in compare to the absence. Fluid velocity falls with the rise of permeability of the medium, rate of fall is more in absence of heat generation than in presence The nature of variation of fluid velocity at Pr=0.025 & 1.0 are similar but magnitude is higher at Pr=1.0.
- Near the plate, in absence and in presence of heat generation, fluid temperature increases but in case of higher magnetic field and higher heat generation, temperature falls near the plate and away from it.
- Skin-friction at the plate varies reciprocally with the rise of permeability of the medium both in absence and presence of heat generation. At higher heat generation (S >1), skin-friction is higher. When magnetic field increases, skin-friction increases in absence and presence of heat generation. Skin-friction decreases with the increase of electrical conductivity in presence and absence of heat generation.
- In absence of heat generation, the rate of heat transfer at the plate, decreases with the increase of electrical conductivity while in presence it increases. Heat transfer decreases with the increase ofmediumpermeabilityin absence of heat generation while increases in its presence of it. Heat transfer rises with the increase of magnetic field in absence of heat generation but decreases in presence of heat generation.

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