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RESEARCH ARTICLE

OPTIMIZATION OF AN ECONOMIC GREEN MANUFACTURING INVENTORY MODEL USING FUZZY GEOMETRIC PROGRAMMING.

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Abstract

The alarm of globalization is ringing in every corner of the world, the caution of environmental conservation sounds high in these days, and all these initiatives are the steps towards creating a pollution free society. It is not a day work rather it is a day by day work. The prime origin place of pollutants of all the forms is the industrial sectors who take the sole responsibility of product production to please their customers of wide range. These sectors cannot be blamed for disturbing the environmental conservation as because they are hurdled with many commitments towards their business administrators, clients and the government. In this commotional state the production process is bound to financial constraints which rank first among the rest. For a soothe functioning of the production sectors they have to set a balance between cost minimization, profit maximization and environmental sustainability for which an inventory model if formulated will result in optimal solution to their problem. In this paper the conventional aspect of Economic Manufacturing Quantity (EMQ) model is modified as Economic Green Manufacturing Quantity (EGMQ) Inventory Model and also fuzzy approach is applied to handle parameters of uncertainty. To determine the optimal order quantity Fuzzy Geometric Programming(FGP)is used for cost minimization and profit maximization with the inclusion of environmental conservation cost parameters, also using Zadeh's extension principle , two main programs are transformed to a pair of two-level of mathematical programs.

Introduction:-

The consumers in these days are attracted towards natural products as health is concerned. As a result of it the market place of these products is getting widened which has influenced the manufacturing sectors to implement the green mechanism of producing these products. Setting green machinery costs high as it an incarnation of several technologies of mitigating waste generation and pollutants emission, but still the business people are working for the installation of green manufacturing tactics to satisfy the laws of the government and to gain the confidence of the customers. Inventory models are formulated to benefit the production sectors, to minimize the total costs of product production. These models are transformed in accordance to the needs and constraints of these sectors by considering shortages, backlogs, trade credit, transportation, reverse logistics and its related issues, but these models are now have started to focus on environmental aspects. The inventory models highlighting the environmental concerns are only deterministic in nature, but in this paper fuzzy geometric tool is employed to determine the optimal order quantity so as to face the challenges of the manufacturing sectors.

The economic centered manufacturing inventory models get transformed to economic centered green manufacturing inventory models with the inclusion of the environmental costs. In the earlier works of Maurice,(Maurice,2011) the

pioneer of formulating environmental oriented inventory models have used deterministic input costs parameters in all his works, but in this work fuzzy input costs parameters are used to handle the problems of uncertainty.
In this paper the demand and the costs parameters are taken as fuzzy variables. There are two main factors for developing these two models. The first factor is the use of FGP to derive the optimal solution. The FGP approach can be effectively applied to this model, a pair of two–level mathematical program is formulated to calculate the upper and lower bounds of the objective value at possibility level. The membership function of the fuzzy objective value is derived numerically by enumerating different values.

The remainder of the paper is organized as follows: section 2 gives a brief overview of the basic definitions, section 3 and section 4 develops the fuzzy model with the total cost minimization and profit maximization, section 5 provides a detailed numerical illustration along with a discussion of key insights and section 6 concludes the paper.

Preliminaries:-

Fuzzy Set:-

A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \in [0, 1]\}$. In the pair $\{(x, \mu_{\tilde{A}}(x))\}$, the first element x belong to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belong to the interval [0, 1], called membership function or grade of membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

α – Cut:-

The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α level set or α - cut.
$A(\alpha) = \{x \in X : \mu_{\tilde{A}}(x) \ge \alpha$

Generalized Fuzzy Number:-

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalized fuzzy number (Bellman 1970)

(i) $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].

(ii) $\mu_{\tilde{A}}(x) = 0, -\infty < x \le a_1$,

(iii) $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1, a_2]$,

(iv) $\mu_{\tilde{A}}(x) = 1, a_2 \le x \le a_3$,

(v) $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3, a_4]$,

(vi) $\mu_{\tilde{A}}(x) = 0, a_4 \le x < \infty$,

where a_1, a_2, a_3 and a_4 are real numbers.

Trapezoidal Fuzzy Number:-

The fuzzy number $\tilde{A}(a_1, a_2, a_3, a_4)$ where $a_1<a_2<a_3<a_4$ and defined on R is called the trapezoidal fuzzy number, its membership function $\tilde{A}$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1 \text{ or } x > a_4 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x < a_2 \\ 1, & a_2 \le x < a_3 \\ \dfrac{x - a_4}{a_3 - a_4}, & a_3 \le x \le a_4 \end{cases}$$

Model formulation:-

Notations:-

The following notations are used throughout the paper

Q : order quantity
D : demand (units/unit time, decision variable for maximization model) that is fuzzy
C : manufacturing cost per unit(dollar/unit)

A : set up cost (dollar / batch)that is fuzzy
h : inventory carrying cost rate(% unit time)
T : Green Costs per unit
F : Environmental sustainability costs per unit
a : scaling constant for P
b : scaling constant for C
α : price elasticity with respect to demand
β : degree of economies of scale

The Green costs T includes the costs of green inputs usage, green energy consumption, green processing and maintenance of machinery and F includes the costs of cleaner technologies, waste treatment using bio processes engineering techniques and costs of particle air filtration technology (Nivetha 2015, Paulo 2009)

Minimizing Model:-
We have the following Mathematical formulation for the total cost per unit time (TC) with order quantity (Q) becomes the following FGP

Min $\tilde{TC}(Q)$ = Setup cost per unit time + production cost per unit time + inventory holding cost unit time + Green cost per unit time + Environmental sustainability cost per unit time.

$$= \frac{\tilde{A}\tilde{D}}{Q} + C(D)\tilde{D} + ic(D)\frac{Q}{2} + \frac{\tilde{T}\tilde{D}}{Q} + \frac{\tilde{F}\tilde{D}}{Q}$$

$$= \tilde{A}\tilde{D}Q^{-1} + b\tilde{D}^{1-\beta} + 0.5ib\tilde{D}^{-\beta}Q + \tilde{T}\tilde{D}Q^{-1} + \tilde{F}\tilde{D}Q^{-1}$$

where Q is the decision variable and cost per unit is expressed as a power function of the demand per unit time displaying economics of scale (ie) $C(D) = bD^{-\beta}$.

Optimal Solution Procedure:-
According to Fuzzy Geometric Programming the objective function in this model is an unconstrained polynomial with (m – n – 1) degree of difficulty [m – number of terms, n – number of variables](Rajgopal 2002, Shiang et al 2006)

So, by above definition and assumption, we are facing by this mathematical formulation for the total fuzzy cost per unit time (= $\tilde{TC}$) from the FGP perspective

$$\text{Min } \tilde{TC}(Q) = \tilde{A}\tilde{D}Q^{-1} + b\tilde{D}^{1-\beta} + 0.5ib\tilde{D}^{-\beta}Q + \tilde{T}\tilde{D}Q^{-1} + \tilde{F}\tilde{D}Q^{-1}$$

The objective function in this model is a fuzzy unconstrained polynomial (m – n – 1). Now in this part for easily solving, we use the duality FGP technique.

Suppose that $\tilde{A}, \tilde{T}, \tilde{F}$ and $\tilde{D}$ are trapezoidal fuzzy number, where

$$\text{Let } S = \frac{\left(\tilde{A}, \tilde{T}, \tilde{F}, \tilde{D}\right)}{A_a^L \le \tilde{A} \le A_a^U,\ T_a^L \le \tilde{T} \le T_a^U,\ F_a^L \le \tilde{F} \le F_a^U,\ D_a^L \le \tilde{D} \le D_a^U}$$

We denote $TC\left(\tilde{A}, \tilde{T}, \tilde{F}, \tilde{D}\right)$ to be the objective value of this model. Let TC^L and TC^U be the minimum and maximum of $TC\left(\tilde{A}, \tilde{T}, \tilde{F}, \tilde{D}\right)$ on S respectively namely

$$Z^L = \text{Min}\left\{ TC\left(\tilde{A}, \tilde{D}, \tilde{T}, \tilde{F}\right) \big/ \left(\tilde{A}, \tilde{D}, \tilde{T}, \tilde{F}\right) \in S\right\}$$

$$Z^U = \text{Max}\left\{ TC\left(\tilde{A}, \tilde{D}, \tilde{F}, \tilde{T}\right) \big/ \left(\tilde{A}, \tilde{D}, \tilde{T}, \tilde{F}\right) \in S \right\}$$

which can be reformulated as the following pair of two level mathematical program

$$TC^L = \text{Min}\,\underset{Q}{\text{Min}}\left\{ \begin{aligned} &\tilde{A}\tilde{D}Q^{-1} + b\tilde{D}^{1-\beta} + 0.5ib\tilde{D}^{-\beta}Q \\ &+ \tilde{T}\tilde{D}Q^{-1} + \tilde{F}\tilde{D}Q^{-1} \end{aligned} \right\}$$

$$\left(\tilde{A}, \tilde{T}, \tilde{F}, \tilde{D}\right) \in S \qquad \ldots (1)$$

$$TC^U = \text{Max}\,\underset{Q}{\text{Max}}\left\{ \begin{aligned} &\tilde{A}\tilde{D}Q^{-1} + b\tilde{D}^{1-\beta} + 0.5ib\tilde{D}^{-\beta}Q \\ &+ \tilde{T}\tilde{D}Q^{-1} + \tilde{F}\tilde{D}Q^{-1} \end{aligned} \right\}$$

$$\left(\tilde{A}, \tilde{T}, \tilde{F}, \tilde{D}\right) \in S \qquad \ldots (2)$$

So the model (1) is transformed as

$$TC^L = \underset{Q}{\text{Min}}\left(A_a\right)^L\left(D_a^L\right)Q^{-1} + b\left[\left(D_a^L\right)\right]^{1-\beta}$$
$$+ 0.5ib\left[\left(D_\alpha\right)^U\right]^{-\beta} u + \left(T_a^L\right)\left(D_a^L\right)Q^{-1} + \left(F_a^L\right)\left(D_a^L\right)Q^{-1}$$

For solving we use dual based algorithm so one can transform model (1) & (2) to the corresponding dual geometric program as follow

$$TC^L = \underset{w}{\text{Max}}\left[\frac{\left(A_a\right)^L\left(D_a\right)^L}{w_1}\right]^{w_1}\left[\frac{0.5ib\left\{\left(D_a\right)\right\}^{-\beta}}{w_2}\right]^{w_2}$$
$$+\left[\frac{\left(T_a\right)^L\left(D_a\right)^L}{w_3}\right]^{w_3} + \left[\frac{\left(F_a\right)^L\left(D_a\right)^L}{w_4}\right]^{w_4}$$

s.t $w_1 + w_2 + w_3 + w_4 = 1$

$-w_1 + w_2 - w_3 - w_4 = 0$

$w_i > 0 \qquad i = 1$

$$TC^L = \underset{(\tilde{A},\tilde{D},\tilde{F},\tilde{T})c\text{-}S}{\text{Max}}\ \text{Max}\left[\frac{\tilde{A}\tilde{D}}{w_1}\right]^{w_1}\left[\frac{0.5ib\tilde{D}^{-\beta}}{w_2}\right]^{w_2}\left[\frac{\tilde{T}\tilde{D}}{w_3}\right]^{w_3}\left[\frac{\tilde{F}\tilde{D}}{w_4}\right]^{w_4} + \left[\frac{\left(T_a\right)^L\left(D_a\right)^L}{w_3}\right]^{w_3} + \left[\frac{\left(F_a\right)^L\left(D_a\right)^L}{w_4}\right]^{w_4}$$

s.t $w_1 + w_2 + w_3 + w_4 = 1$

$-w_1 + w_2 - w_3 - w_4 = 0 \qquad w_i > 0.$

If we define the i^{th} term of the optimal primal objective function as U_i^0 we have

$$U_i^0 = \left(A_a\right)^L\left(D_a^L\right)\left(Q^L\right)^{-1}$$

$$U_3^0 = \left(A_a\right)^L\left(D_a^L\right)\left(Q^L\right)^{-1}$$

$$U_2^0 = \left(A_a\right)^L\left(D_a^L\right)\left(Q^L\right)^{-1}\left[0.51ib\left(D_a\right)^U\right]^{-\beta_Q^L}$$

$$U_4^0 = \left(F_a\right)^L\left(D_a^L\right)\left(Q^L\right)^{-1}$$

$$Q^L = \sqrt{\frac{2\left[A_a^L + T_a^L + F_a^L\right]D_a^L}{ib\left[\left(D_a^U\right)\right]^{-\beta}}} \qquad \text{and}$$

$$Q^U = \sqrt{\frac{2\left[A_a^L + T_a^L + F_a^L\right]D_a^L}{ib\left[\left(D_a^U\right)\right]^{-\beta}}}$$

Maximization Model:-

We are facing to formulation for the profit per unit time

Max $\pi(Q, D)$ = revenue per unit time – (set up cost per unit time + production cost per unit time + inventory holding cost per unit time + Green cost per unit time + Environmental sustainability cost per unit time.

$$= P(D)D - \left[\frac{AD}{Q} + C(D)D + \frac{iC(D)Q}{2} + \frac{FD}{Q} + \frac{TD}{Q}\right]$$

$P(D) = aD^{-\alpha}$, $C(D) = bD^{-\beta}$.

$= aD^{1-\alpha} - \tilde{A}DQ^{-1} - bD^{1-\beta} - 0.5ibD^{-\beta}Q - TDQ^{-1} - FDQ^{-1}$.

Let $S = \left\{A,T,P \middle| A_a^L \le \tilde{A} \le A_a^U, T_a^L \le \tilde{T} \le T_a^U, F_a^L \le \tilde{F} \le F_a^U\right\}$

for each $\tilde{A}, \tilde{T}, \tilde{P} \in S$. We denote $\pi\left(\tilde{A}, \tilde{F}, \tilde{T}\right)$ to be the objective value of this model. So

$$\pi^L = \operatorname{Min}\left\{\pi\left(\tilde{A}, \tilde{T}, \tilde{F}\right) \middle| \tilde{A}, \tilde{T}, \tilde{F} \in S\right\}$$

$$\pi^U = \operatorname{Max}\left\{\pi\left(\tilde{A}, \tilde{T}, \tilde{F}\right) \middle| \tilde{A}, \tilde{T}, \tilde{F} \in S\right\}$$

which can be reformulated as the following pair of two level mathematical program.

$$\pi^L = \min_{\tilde{A}, \tilde{T}, \tilde{F} \in S} \min_{D, Q} \{a^{-1}D^{\alpha-1} + Aa^{-1}Q^{-1}D^{\alpha} + ba^{-1}D^{\alpha-\beta}$$
$$+ 0.5iba^{-1}QD^{\alpha-\beta-1} + Ta^{-1}Q^{-1}D^{\alpha} + Fa^{-1}a^{-1}D^{\alpha}\}$$

$$\pi^U = \max_{\tilde{A}, \tilde{T}, \tilde{F} \in S} \max_{D, Q} \{a^{-1}D^{\alpha-1} + Aa^{-1}Q^{-1}D^{\alpha} + ba^{-1}D^{\alpha-\beta}$$
$$+ 0.5iba^{-1}QD^{\alpha-\beta-1} + Ta^{-1}Q^{-1}D^{\alpha} + Fa^{-1}a^{-1}D^{\alpha}\}$$

For 2 models π^L and π^U, we use the dual problem which is usually easier to solve for π^L

$$d(u) = \left[\frac{1}{w_0}\right]^{w_0} \left[\frac{\alpha^{-1}\lambda}{w_1}\right]^{w_1} \left[\frac{a^{-1}(Aa)^L\lambda}{w_2}\right]^{w_2} \left[\frac{a^{-1}b\lambda}{w_3}\right]^{w_3}$$
$$\left[\frac{0.5a^{-1}ib\lambda}{w_4}\right]^{w_4} \left[\frac{a^{-1}(Ta)^L\lambda}{w_5}\right]^{w_5} \left[\frac{a^{-1}(Fa)^L\lambda}{w_6}\right]^{w_6}$$

s.t. $w_0 = 1$; $-w_0 + w_1 = 0$

$(\alpha-1)w_1 + \alpha w_2 + (\alpha-\beta)w_3 + (\alpha-\beta-1)w_4 + \alpha w_5 + \alpha w_6 = 0$

$-w_2 + w_4 - w_5 - w_6 = 0$ $\quad w_i > 0 \quad i = 1, 2, 3, 4$

where $\lambda = w_1 + w_2 + w_3 + w_4$ and for π^U.

the same as π^L.

So according to duality techniques we have

$$Q = \left[\frac{A+T+F}{a\left(V_2+V_5+V_6\right)}\right]\left[\frac{\alpha V_3}{b}\right]\left[\frac{\alpha}{\alpha - \beta}\right]$$

$$Q = \left[\frac{A+T+F}{a\left(V_2+V_5+V_6\right)}\right]\left[\frac{\alpha V_3}{b}\right]\left[\frac{\alpha}{\alpha - \beta}\right]$$

$$D = \left[\frac{\alpha V_3}{b}\right]^{\frac{1}{\alpha - \beta}}$$

such that $V_2 = a^{-1}\tilde{A}D^{\alpha}Q^{-1}$; $\quad V_3 = a^{-1}bD^{\alpha-\beta}$

$V_5 = a^{-1}D^{\alpha}TQ^{-1}$; $\quad V_6 = a^{-1}D^{\alpha}FQ^{-1}$

IV. NUMERICAL EXAMPLE

Consider an inventory system with following data.

Let demand per unit be a trapezoidal number

$\tilde{D}$ (2900, 3000, 3200, 3400)

$\tilde{s}$ = (600, 700, 800, 900)

$\tilde{T}$ = (40, 50, 70, 80)

$\tilde{P}$ = (5, 10, 20, 25)

$\tilde{D}$ = $[2900 + 100\alpha, 3400 - 200\alpha]$

$\tilde{A}$ = $[600 + 100\alpha, 900 - 100\alpha]$

$\tilde{T}$ = $[40 + 10\alpha, 80 - 10\alpha]$

$\tilde{F}$ = $[5 + 5\alpha, 25 - 5\alpha]$

$$Q^L = \sqrt{\frac{2[(600 + 100\alpha)+(40 + 10\alpha)+(2900 + 100\alpha)}{iQ2[2900 + 100\alpha] - \beta}}$$

$$Q^U = \sqrt{\frac{2[(900 - 100\alpha)+(80 - 10\alpha)+(25 - 5\alpha)(3400 - 200\alpha)}{iQ[2900 + 100\alpha] - \beta}}$$

For α = 0, /1, ./2, ... namely different α-cut we can reach to an interval that Q satisfies it.

Conclusion:-

In this paper the significance and the practical implementation of green manufacturing model is discussed. This paper is indeed an integration of economic and environmental aspects which benefits the manufacturing sectors and the environment mutually. This model is highly reliable as it has been validated with numerical example. It is also a highly responsible model as it focuses on green aspects. It has a wide scope as it uses fuzzy mathematical approach.

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