Journal Homepage: - www.journalijar.com

INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)

ISSN NO. 2320-5407

Article DOI: 10.21474/IJAR01/2445
DOI URL: http://dx.doi.org/10.21474/IJAR01/2445

RESEARCH ARTICLE

VIRTUAL KEYBOARD.

THE INSPIRAL TIME AND THE ORBITAL DECAY OF THE PLANETS OF THE SOLAR SYSTEM DUE TO GRAVITATIONAL WAVES.

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Manuscript Info

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Manuscript History

Received: 20 October 2016
Final Accepted: 22 November 2016
Published: December 2016

Key words:-
General Relativity, Gravitational waves, Inspiral time

Abstract

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Einstein's general theory of relativity predicts that any binary orbiting massive objects will radiate gravitational waves, and consequently the separation between the binary systems would decrease and ultimately it will collapse to each other. In practice, the phenomenon has been observed though the theory was predicted hundred years ago by Albert Einstein. We apply here the theory of the general relativity to study the decay of the planetary orbits of the solar system due to the radiation of the gravitational waves and observe how stable the orbits are and how long it would take to collapse to the sun. The calculations of the inspiral time of the planets of the solar system have been compared to the inspiral time of the binary neutron stars and it is observed that the inspiral time of the planets are extremely high in comparison to the massive binary stars. It is interesting to observe that the inspiral time of the earth's orbit around the sun is 10^{13} times greater than the age of the universe.

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Introduction:-

According to the general theory of relativity of Albert Einstein, the space near a very massive objects are curved and the acceleration of a massive object comparable to the thousand times massive than our star sun creates gravitational waves. The gravitational waves are ripples created by massive body in the curved spacetime. When these gravitational waves cross a large astronomical distant, the energy of the waves diminishes to a large extent and it is difficult to detect them on earth. It took hundred of years to detect it after the prediction of Einstein's theory of relativity. Though many predictions of relativity have been verified, but the gravitational wave's existence is enormously difficult to prove directly.

In this context, the detection of gravitational waves[1] at the Laser Interferometer Gravitational-Wave Observatory is remarkable and historical so far as the gravitational waves are concerned. They are caused by the collapse of two spiral huge black holes 1.3 billion light-years from the Earth, colliding together in a vigorous and catastrophically violent way.

Majority of the scientists believe that there are some good reasons as to the existence of binary black holes and the observation of these gravitational waves undoubtedly proves the experimental verification of another predictions

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made by Einstein long before. The fate of these binary stars are that they ultimately merge[2] and are destroyed releasing a huge amount of energy through gravitational waves.

The short term effects of gravitational waves[3] on the orbits of the revolving planets around the sun in our solar system is expected to be small compared to the binary black holes or binary neutron stars. But in the long run, the general relativity predicts the shift of the periods of the revolving objects. The author investigates this long term effects due to gravitational waves in case of solar planets and compare the results with the binary massive neutron star.

If two black holes[1] having masses 36 and 29 times that of the sun merge, then after merging the single black hole has mass 62 times that of the sun. The missing mass 3 times that of the sun generates staggering amount of energy equivalent to $E = 3 \times 2.0 \times 10^{30} \times 9 \times 10^{10} = 5.4 \times 10^{41}$ joules. This single event releases energy which the sun can radiate in 10^{12} years.

According to the general theory of relativity space and time are the two facets of the same thing called space time continuum. In space time continuum, there are three coordinates and one time co-ordinates in which everything including us is embedded. In Newtonian mechanics gravity is thought of a force which attracts every material body but Albert Einstein saw it as warp of space time due to presence of material bodies. And due to this curvature of space time or the distortion of space time, a body moving along it gets accelerated. Therefore, it is said that matter tells space how to bend, and space tells matter how to move.

Another aspect of general relativity is that when a massive body is accelerated in space time it creates ripples just like a stone being dropped into water makes. These ripples move away from the object carrying energy with it called gravitational wave energy. The space time expands and contracts resulting in generation of gravitational waves like the ripples move out from a stone dropped into pond distorting the surface of the water of the pond.
The strength of the gravitational waves depends on the mass and acceleration of the object. The more massive of a object and the greater value of acceleration, the more is the strength of the waves. In the solar systems the planets move around the Sun and therefore it is natural to ask the question whether they emit any gravitational waves and how strong they are. It is also interesting to know the possibility of merging of the planets with the sun in near future. As we know that the motion of the planets is too slow and their masses are to low compared to the mass of the sun and therefore the strength of the gravitational waves are very feeble to detect. But in case of very massive binary neutron stars, the dense cores of stars generate waves that can be measured by sophisticated LIGO apparatus.

Theory: Formula for Orbital decay and Inspiral time of the Planets and binary neutron stars:

The formula for the rate of orbital decay[4,5] can be approximated by $\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{m_1 m_2(m_1+m_2)}{r^3}$.

The Earth Sun system has total energy (kinetic and potential energy) 1.14×10^{36} joules of which 200 watts are lost through gravitational radiation leading to an orbit decay of about 1×10^{-15} m per day. At this rate of decay the earth will take 10^{13} times the age of the universe to inspiral onto sun. The formula of the merger time is

$$T = \frac{5}{256}\frac{c^5}{G^3}\frac{r^4}{m_s m_2\,(m_s+m_2)} = 3.3 \times 10^{17}\, yrs.\left(\frac{a}{1AU}\right)^4 \frac{m_s^3}{m_s m_2 (m_s+m_2)},$$

where masses are measured in terms of the solar masses. For example, a pair of solar mass neutron stars in a circular orbit at a separation of $1.89 \times 10^8\ m\ (1{,}89{,}000\ km)$ has an orbital period of 1000 seconds and life time 4,14,000 years. The polarisations of the waves are given by

$$h_+ = -\frac{G^2}{Rc^4}\frac{2m_s m_2}{r}(1 + Cos^2\,\theta)Cos[2\omega\left(t - \frac{R}{c}\right)]$$

and

$$h_x = -\frac{G^2}{Rc^4}\frac{4m_s m_2}{r} Cos\theta\ Sin[2\omega\left(t - \frac{R}{c}\right)]$$

For the earth sun system in the X-Y plane, we have $\theta = \pi/2$, so

$h_x = 0$ and $h_+ = -\frac{G^2}{Rc^4}\frac{4m_s m_2}{r} = -\frac{1}{R}1.7 \times 10^{-10} m$ and frequencies of gravitational waves have ranges between $10^{-16} Hz < f < 10^4 Hz$. If we measure the merger time of the other planets of the solar system, we can write,

$$t_{rel}^{mars} = \frac{t_{decay}^{mars}}{t_{decay}^{earth}} = \frac{r^4}{r_E^4}\frac{m_E(m_E+m_s)}{m_M(m_M+m_s)} = \frac{r^4}{r_E^4}\frac{m_E}{m_M}\ (1 + \frac{m_E}{m_s})/(1 + \frac{m_M}{m_s}),$$

where t_{rel}^{mars} is the merger time of the planet mars with respect to the merger time of the earth. Here m_E and m_M are the masses of the earth and mars respectively. m_s is the mass of the sun. r_E is the distance between earth and the sun and r is the distance between the mars and the sun. The table1 gives the estimates of the merger times of the different planets of the solar system. From the table1 it is clear that the merger time is very large compared to the age of the universe 13.8×10^9 yrs.

In 1974, a binary neutron star orbiting about their common centre of mass was discovered by astronomers Joseph Taylor and Russel Hulse. As they move about their common centre of mass, they emit energy in the form of gravitational waves. As a consequence of this, the orbital energy of the stars decreases while emitting gravitational waves. Therefore, the orbit shrinks as it goes on radiating waves and the period of orbiting each other drops. These are orbital decay of the stars and they can be measured very accurately. There are experimental results of these decays which matched the predictions of general relativity very precisely.

The famous PSR 1913+16 binary star discovered by Hulse and Taylor[6] made the first proof of astrophysical evidence of binary star. The system parameters[5] of this star is a (separation between the stars) = 5 kpc, $m_1 = m_2 = 1.4\, m_s$, where m_s is the mass of the sun. The time period (P) of this star is 7 hours and 45 minutes. The merger time for this binary star can be calculated from the formula

$$T = \frac{5}{256}\frac{c^5}{G^3}\, r^4 \frac{1}{\mu m^2}$$

where μ is the reduced mass of the binary system ($\mu = \frac{m_1 m_2}{m_1 + m_2}$) and $m = m_1 + m_2$ is the total mass of the binary system. The expression for gravitational waves luminosity is

$$L_{GW} = \frac{32}{5}\frac{G^4}{c^5}\, m^3\, \mu^2 \frac{1}{r^5}$$

and we have already mentioned that the rate of decay is

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\, \mu \frac{m^2}{r^3}$$

The orbital frequency increases accordingly

$$\frac{1}{P}\frac{dT}{dt} = \frac{3}{2}\frac{1}{r}\frac{dr}{dt}$$

The merger time of this binary star can be calculated as T= 3.5×10^8 years.

Results:-

In Fig.1 we have shown the variation of the inspiral time (years) of the planets onto the sun of the solar system . It is clear from the figure that the inspiral time of the planets are enormously large compared to the binary system neutron star. The curves from the left of the figure are Jupiter, Saturn, Uranus and Neptune respectively.

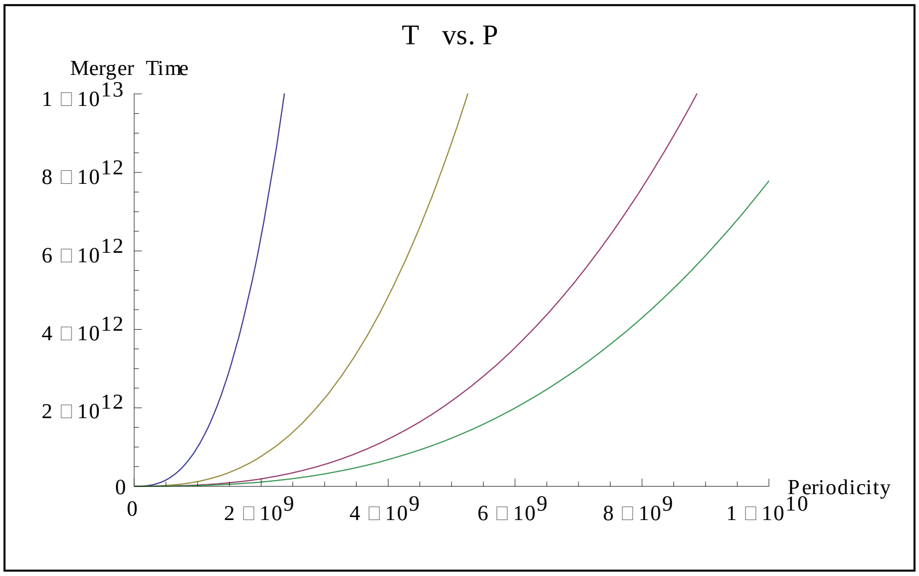

Fig.1:- The Plot of merger time (years) vs. Periodicity of the planets (in sec) in the solar system.

In the Table 1 given below, we have calculated and compared different merger time of the planetary orbits relative to the earth's merger time. We have further calculated the luminosity vs. distance of the circular orbits of the neutron stars. It is seen from the graphs that the luminosity of the binary stars increases enormously as the distances between them decrease. We have also shown luminosity distance plot in Fig.2. In Fig.3 the variation of inspiral time vs. distance between the binary stars is shown to compare it with the planets of our solar system.

Table 1:- The table shows the different parameters of the planets of the solar system.

Serial No.	Planets	Periods(P)	Mass in Kg	Distance from the sun in AU	Merger time with respect to earth
1.	Mercury	88 days.	3.30×10^{23}	.387	.4058
2.	Venus	225 days.	4.867×10^{24}	.923	.3353
3.	Earth	365.24 days.	5.972×10^{24}	1.0	1.000
4.	Mars	687 days.	6.417×10^{23}	1.524	50.2029
5.	Jupiter	11.9 yrs.	1.898×10^{27}	5.203	2.3037
6.	Saturn	29.5 yrs.	5.683×10^{26}	9.523	86.4003
7.	Uranus	84 yrs.	6.861×10^{25}	19.208	9363.99
8.	Neptune	165 yrs.	1.024×10^{26}	39.746	145537.0

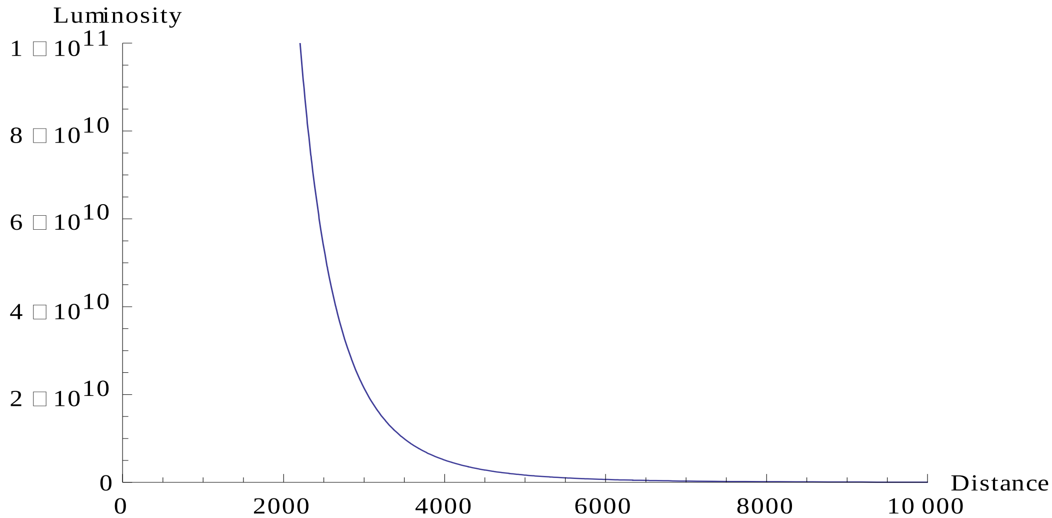

Fig.2:- The Luminosity in terms of sun's Luminosity vs. Distance (r in m) between the binary stars.

The origin of gravitational waves may be of different form and shapes and it actually distort the fabric of spacetime. It produces strain in the space around the heavy objects and these strain energy propagates at the speed of light in all directions from the source. As the waves proceed towards earth crossing of the order of megaparsec distances, naturally the strength of these distortion are very feeble and it is very difficult to detect them on earth.

This impossible task has been possible recently through the Laser Interferometer Gravitational Wave Observatory (LIGO). LIGO detectors are two observatories, one situated at Washington state and another in Lousiana (Operation done by Caltek and MIT).

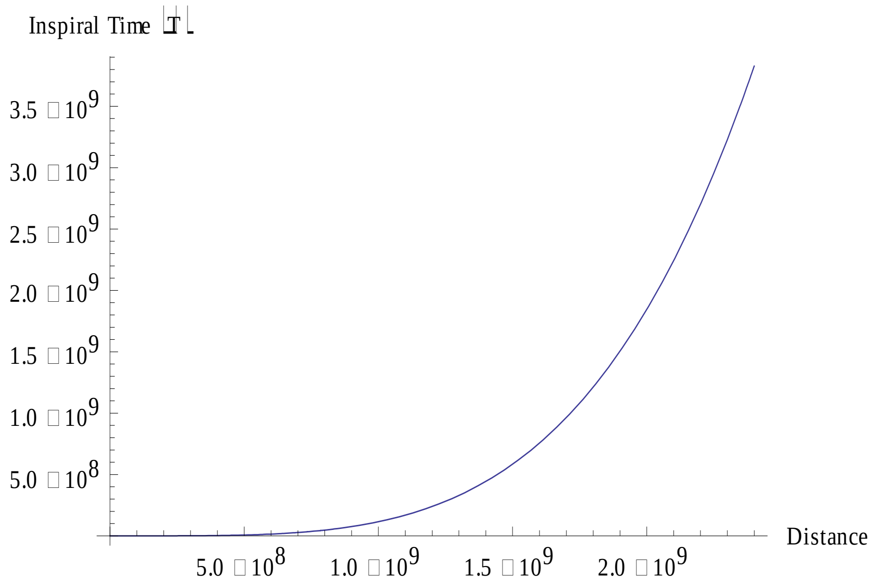

Fig.3:- The Inspiral time(T) vs. Separation distance between the binary stars.

If two very massive black holes orbit around each other with very high velocity nearly equal to a large fraction of the speed of light they lose their energy due to radiation of energy through gravitational waves. Both of them create ripples in the space time around them and therefore emit gravitational waves towards us expanding away at the speed of light. As they rotate with enormous speed they lose energy due to gravitational waves and come closer and closer gradually. The orbits of the black holes shrink and consequently they move faster and faster like the neutron stars that got Taylor and Hulse their Nobel.

This change of their orbital radius increases the frequency of the waves they emit. The frequency of the waves increases as the time period of the black holes decreases. The faster the orbits of the black holes shrink the faster the frequency of the waves increases. As a consequence it emit more gravitational waves. The black holes approach closer and closer until they merge and form a single black hole.

The authors[6] determined the theoretical change in orbit period P over time (due to orbital energy loss from emission of gravitational waves) from a formula (Peters & Matthews (1963)) of the form:-
The frequency of the orbit changes as $\frac{1}{P}\frac{dP}{dt} = \frac{3}{2}\frac{1}{r}\frac{dr}{dt}$, where $\frac{dP}{dt} = \frac{K}{P^{\frac{5}{3}}}$ (from Peters and Mathews), and K is an effectively a constant function of the masses of the two stars and the orbit's eccentricity. For the Hulse- Taylor binary, the period P is approximately 7.75 hours and the value of K is approximately -6×10^{-5} giving an approximate value of $\frac{dP}{dt} = 2.34 \times 10^{-12}\,\frac{s}{s}$ and for the solar system objects the value of K for earth sun system is $K = -\frac{64}{5}\frac{G^3}{c^5}\, m_E m_s (m_E + m_s) \times \frac{6\pi^{\frac{8}{3}} \times 4^{\frac{1}{3}}}{(Gm_s)^{\frac{4}{3}}} = -1.019 \times 10^{-11}$ and the expression for the periodicity of the earth around the sun is $P = P_0\left(1 - \frac{K}{P_0}\right)^{\frac{3}{8}}$. Therefore, the cumulative orbit decay of the earth can be written as $P - P_0 = P_0\left(1 - \frac{K}{P_0}\right)^{\frac{3}{8}} - P_0$. The plot of the function $P - P_0$ as a function of time (t) has been shown in Fig.4. Note that the graph resembles that of the plot by Weisberg and Taylor [6].

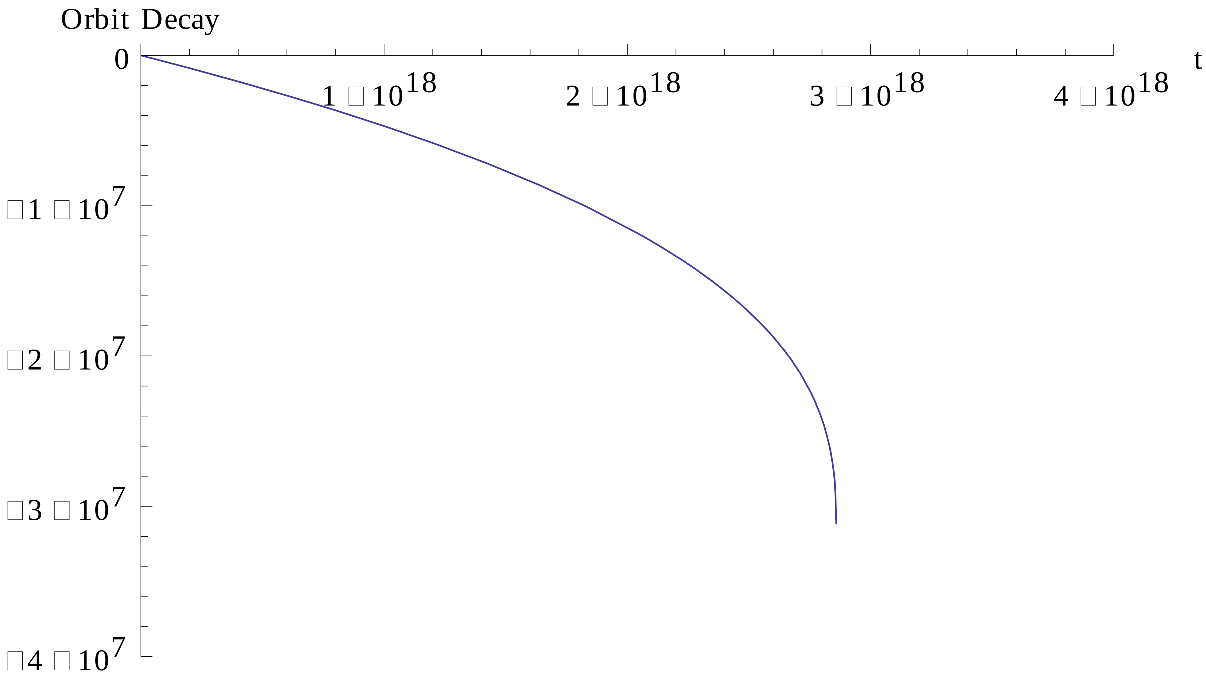

Fig.4:- The Plot of Orbital decay time (in second) vs. time (t in second).

If we estimate roughly the orbit decay of the earth in a year, we get the approximate value of change of period in one year= $365 \times 24 \times 3600 \times 1.019 \times 10^{-11}$ = 0.035 millisecond.

Conclusions:-

It is to be seen that the decay time of the orbits of the planets are extremely large and even sometimes it is greater than the age of the universe. The energy loss due to the gravitational waves in case of the massive neutron stars whose masses are several times greater than the mass of the sun is enormous than that of the planets of the solar system. The massive stars move harder and the radiation strength is large. The planets of the solar system are extremely stable than the binary neutron stars as far as the orbits are concerned. We see that the inspiral time of the binary neutron stars is of the order of 10^8 years, whereas that of the planets of the solar system is 10^{23} years for the earth and it is even greater for the planets like Jupiter, Saturn, Uranus and Neptune.

Acknowledgements: The author is grateful to Dr. Kuntal Biswas for encouragement and help to prepare the manuscript.

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