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RESEARCH ARTICLE

FLUID MECHANICS: A NEW DISCRETE SOLUTION TO THE UNIDIMENSIONAL CONTINUITY DIFFERENTIAL EQUATION DERIVED FROM THE LAW OF AMP'ERE-MAXWELL.

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Abstract

It is well-known that we are able to obtain, using the Ampère-Maxwell Law, a continuity equation that shows the conservation of electric charge. Nowadays, there are already generalized solutions to any quantity that is involved in a continuity equation. However, these solutions use to be supported by non-elementary mathematical formalism. The author will show that it is possible to build an alternative solution to the unidimensional continuity differential equation related to electric charge, where a discrete point of view of electrodynamics will lead us to a unidimensional Navier-Stokes equation for continuity. It turns out that the product between Area and Speed being a constant is successfully obtained by the solution proposed by the author, accordingly with the objectives.

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Introduction:-

Fluid mechanics is a fundamental field of the known classical mechanics. Several engineering problems, for instance, were solved through the study of this field. Its principal applications were related to the flow of liquids and gases [1], hydraulic industry, besides several applications in the area of aerodynamics and aerospace engineering [2]. The continuity equation is one of the topics studied in fluid mechanics that allows the statement of conservation of determined quantity [3]. There were found some generalized solutions to continuity equations of any quantity such as mass, energy, electric charge, momentum, number of molecules, and others. However, approaches such as primitive-variable solutions to the Navier- Stokes equations, which describe the motion of fluids, although lead one to solve the Navier-Stokes equation for continuity, are supported by an extensive mathematical formalism and non-elementary [4].

The study of fluid mechanics is able to be applied in many others fields of theoretical physics. On the present study, the topic that will be related to fluid mechanics is Electromagnetism. Electromagnetism is a field of physics that started to get more notoriety after the studies of James Clerk Maxwell, in 1862, when he published his four equations that unified electricity and magnetism, giving them reasonable meaning and, through the language of divergence and rotational, a physical interpretation [5]. From then on, it has been developed several studies and scientific papers that uses Maxwell studies as theoretical basis [6]. Nowadays, it is well known that one may obtain a continuity equation that guarantees the conservation of electric charge in determined electric fluid [7].

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Making use of differential, integral, and multivariable calculus on this study will be indispensable, and having contributions on these areas from great scientists such as Sir Isaac Newton, Gottfried Leibniz, J. Willard Gibbs, and Oliver Heaviside is really pleasant. Therefore, the present study has as objective the construction of a more elementary alternative solution to the continuity differential equation related to electric charge, making use of a discrete point of view of electrodynamics.

Methodology:-

It is an inductive applied research expressed through the point of view of the author based in the theory of fluid mechanics, electromagnetism and infinitesimal calculus. In order to make this approach effectively, it was done a previous study exploring the theoretical basis of authors such as Çengel [3], Griffiths [5], Apostol, Lang, Boyce [8-10]. The analysis and literature were done through the analysis of texts, books and theories, making parallels that could generate either new technologies or methods with handful results.

Development

Differential Form of the Continuity Equation

Let q be an arbitrary quantity such as mass, energy, electric charge, etc. Let J be the rate of change of q per unit time per unit area. Consider S as an imaginary closed surface that encloses a volume V . Assuming there's neither sources (creators) nor sinks (destroyers) of the quantity, the integral form of the continuity equation for q is defined as [3]:

$$(1) \quad \frac{d}{dV} \left(\frac{dq}{dt} \right) + \frac{d}{dV} \left(\iiint_V (\nabla \cdot \mathbf{J}) dV \right) \implies$$

$$\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

$$\nabla \cdot (\nabla \times B) = \nabla \cdot (\mu_0 J) + \nabla \cdot \left(\mu_0 \epsilon_0 \frac{\partial E}{\partial t} \right)$$

The next steps will be useful to transform the continuity equation from its integral form to the differential form. Utilizing the theorem of **Ostrogradski-Gauss**, it's possible to rewrite the second term of the left-hand side of equation (1):

$$(2) \quad \frac{dq}{dt} + \iint_S \mathbf{J} \cdot d\mathbf{S} = 0$$

Also known as divergence theorem, shows the equivalence between a flux integral of J and a triple volume integral of the divergence of J [9].

$$\iint_S \mathbf{J} \cdot d\mathbf{S} = \iiint_V (\nabla \cdot \mathbf{J}) dV$$

Deriving with respect to the volume on both sides of equation (1) will make the triple integral to vanish. Utilizing the properties of the derivatives to change the order of derivation that involve the first term of the left-hand side of the equation, and considering ρ as the volume density of q , it may be obtained the continuity equation on its differential form for any arbitrary quantity q :

$$(3) \quad \frac{\partial J}{\partial x} + \frac{\partial \rho}{\partial t} = 0$$

Deriving a Continuity Equation from the Law of Ampère-Maxwell

It is possible to find a continuity equation related do electric charge as quantity. In order to do that, consider the Law of Ampère-Maxwell on its differential form [5]:

$$(\nabla \cdot J) + \frac{\partial \rho}{\partial t} = 0 \quad (4)$$

Now, take the divergence from both sides of the equation:

One should be familiar there's a property of vector calculus that the divergence of any rotational is always zero. Therefore, the left-hand side of the equation above is zero. Writing in evidence the constants of the right-hand side of the equation and making use of the properties of the derivatives, we shall rewrite the equation:

Note that $(\nabla \cdot E)$ is another Maxwell Equation, also known as Gauss's Law [5], which says:

Hence:

Simplifying, we have the continuity equation related to the conservation of electric charge:

$$J = \frac{\partial i}{\partial A} = \frac{\partial}{\partial A}(\eta A \nu e) \quad (5)$$

$$\eta \nu e \frac{\partial A}{\partial A} = \eta \nu e = \frac{N \nu e}{V} = \frac{Q}{V} \nu \implies$$

$$J = \rho \nu$$

Where J is the current density and ρ is the volume charge density.

Obtaining the Navier-Stokes unidimensional equation for continuity

Now that equation (5) was presented, one may solve it to obtain explicitly a meaning of continuity. For the purposes of this study, one may consider that the current flow is unidimensional. This way, one may account for the x component of the divergence operator only, rewriting equation (5):

$$\frac{\partial(\rho \nu)}{\partial x} + \frac{\partial \rho}{\partial t} = 0 \quad (6)$$

One may know that J is the current density per unit area ($J = \frac{\partial i}{\partial A}$) In a discrete electrodynamic point of view and imagining a collection of electric charges flowing through an imaginary surface, there will be an electric current such that $i = \eta A \nu e$, where $\eta = N/V$, where N = number of electric charge carriers, V = volume, A = area, ν = speed of the electric charge carriers and e = elementary charge [5].

Substituting this value of electric current in the current density expression:

$$(7) \quad 0 = \mu_0(\nabla \cdot J) + \mu_0 \epsilon_0 \frac{\partial(\nabla \cdot E)}{\partial t}$$

Making use of equations (7) and (6):

$$\nu \frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial t} = 0 \quad (8)$$

One may identify equation (8) as the unidimensional Navier-Stokes continuity equation [3].

$$(\nabla \cdot E) = \frac{\rho}{\epsilon_0} \quad 0 = \nu \frac{\partial}{\partial x} \left(\frac{Q}{V} \right) = - \frac{\partial}{\partial t} \left(\frac{Q}{V} \right)$$

Solving the Equation

Considering constant the speed of the electric charge carriers, ν doesn't change with x, and hence one is able to remove the speed of the partial derivative:

One may know that $\rho = Q/V$, being $Q = \sum q_i$ where q_i is the charge of each electric charge carrier and V = volume past by the electric charge carriers.

Since Q does not change with x, one may remove it from the partial derivative.

$$2A_1X_1\nu_1 = 2A_2X_2\nu_2$$

$$\frac{\partial V}{\partial x} = Q\left(\frac{1}{V^2}\right)\frac{\partial V}{\partial x} = Q\left(\frac{1}{V^2}\right)\frac{\partial V}{\partial t}$$

$$-\nu A = \frac{\partial V}{\partial t} \tag{9}$$

However,

$$(10)$$

Using equation (10) in equation (9), we have:

$$-\nu A = \frac{\partial}{\partial t}(Ax) \implies -\nu A = A\frac{\partial x}{\partial t} + \frac{\partial A}{\partial t}x$$

Substituting V = Ax and developing, one may have:

Where $\frac{\partial}{\partial t}(Ax)$ was accounted using the product rule of the derivatives, since both A and x change with time.

Continuing the development:

$$-\nu A = \nu A + \frac{\partial A}{\partial t}x \implies -2\nu A = \frac{\partial A}{\partial t}x$$

$$\frac{-2\nu\partial t}{x} = \frac{\partial A}{A} \implies -2\frac{\partial x}{x} = \frac{\partial A}{A}$$

One may recognize that in the last step $\nu\partial t = \partial x$ since $\nu = \frac{\partial x}{\partial t}$. Integrating from both sides of the last equation:

$$-2 \int_{X_1}^{X_2} \frac{\partial x}{x} = \int_{A_1}^{A_2} \frac{\partial A}{A}$$

$$2\ln\left(\frac{X_1}{X_2}\right) = \ln\left(\frac{A_2}{A_1}\right) \implies \ln\left(\frac{A_2}{A_1}\right) = \ln\left(\frac{X_1}{X_2}\right)^2$$

In the last step it was used a property of logarithms:

$$n \cdot \log_a b = \log_a b^n$$

Continuing the development of the logarithm:

$$\frac{A_2}{A_1} = e^{\ln\left(\frac{X_1}{X_2}\right)^2} \implies \frac{A_2}{A_1} = \left(\frac{X_1}{X_2}\right)^2$$

Where in the last step it was used another property of logarithms:

$$a^{\log_a b} = b$$

Without loss of generality, the terms A₁ and A₂ are locally constant with time (see Figure 1 below).

$$\nu Q \frac{\partial}{\partial x} \left(\frac{1}{V}\right) = -Q \frac{\partial}{\partial t} \left(\frac{1}{V}\right)$$

Therefore, rewriting and deriving with respect to time both sides of the last equation, we have:

This equation allows one to state that if we analyze two imaginary closed surfaces with equal **length** ($X_1 = X_2$) along the space that the flow is submitted, then the product Av is a constant:

$$(11) \quad A_1 v_1 = A_2 v_2$$

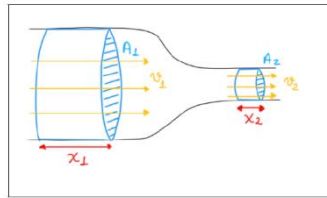


Figure 1: Tube with flow of electric charge carriers in unidimensional movement. (Author).

Conclusion:-

The study presents an alternative solution and relatively more elementary than the generalized solutions to continuity equations that there are nowadays. There is an immediate physical interpretation from equation (11) that may be useful to hydroelectric issues that involve tubulations: equation (11) says that the product Av is a constant. Hence, the **product** between the transversal section of determined tube, past by electric charge carriers, and the speed of these electric charge carriers, is **constant with time**. It implies that if in any moment the transversal section gets reduced, the speed of electric charge carriers is automatically increased and vice-versa. This way, the product Av maintains constant and it respects the result obtained in equation (11).

Figure 1 allow one to visualize this phenomenon. Initially, one has determined number of electric charge carriers of speed v_1 passing through a transversal section A_1 . However, in determined interval of time, the transversal section of the tube is reduced to A_2 . That makes the speed of the electric charge carriers to be increased to v_2 . In order to work accordingly to the result found in the equation (11), one is able to state that $v_2 > v_1$.

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