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RESEARCH ARTICLE

ON SαRW CLOSED AND SαRW OPEN MAPS IN TOPOLOGICAL SPACES.

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Abstract

In this paper, we introduce and studies a new class of closed and open maps is called sαrw-closed and sαrw-open maps in topological space. Also some of their properties have been investigated. We also introduce sαrw*-closed and sαrw*-open maps in topological spaces and study of their properties.

Introduction:-

In 1982, the concept of generalized closed maps are introduced and studied by S. R, Malghan [22], wg-closed maps and rwg-closed maps were introduced and studied by Nagaveni [25],Later regular closed maps, rw-closed maps and αrw-closed maps have been introduced and studied by Long [19], Benchalli [8] and R S Wali [34] respectively. The aim of this paper is to introduce and study sαrw-closed and sαrw-open, sαrw*-closed and sαrw*-open maps in topological spaces. Also some of their properties have been investigated.

Preliminaries:-

In this paper X or (X,τ) and Y or (Y,σ) denote topological spaces on which no separation axioms are assumed. For a subset A of a topological space X, cl(A), int(A), X-A or A^c represent closure of A, interior of A and complement of A in X respectively.

Definition 2.1:-A subset A of a topological space (X, τ) is called

1. Regular α-open set [32] (briefly, rα-open) if there is a regular open set U such that $U \subseteq A \subseteq \alpha cl(U)$.
2. Regular semi open set [11] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.
3. Regular open set [29] if A = int (cl A)) and a regular closed set if A = cl (int (A)).
4. Semi-preopen set [1] (β-open [10] if $A \subseteq cl(int(cl(A)))$ and a semi-pre closed set (β- closed) if $int(cl(int(A))) \subseteq A$.
5. α-open set [15] if $A \subseteq int(cl(int(A)))$ and α -closed set if $cl(int(cl(A))) \subseteq A$.
6. Pre-open set [23] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
7. Semi-open set [18] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.

Definition 2.2:-A map $f: (X, \tau) \to (Y, \sigma)$ is said to be

1. Regular-continuous(r-continuous) [2] if $f^{-1}(V)$ is r-closed in X for every closed subset V of Y.
2. Completely-continuous [2] if $f^{-1}(V)$ is regular closed in X for every closed subset V of Y.

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3. g-Continuous [5] if $f^{-1}(V)$ is g–closed in X for every closed subset V of Y.
4. sαrw-continuous [7] if $f^{-1}(V)$ is sαrw closed in X for every closed subset V of Y.
5. Strongly sαrw-continuous [7] if $f^{-1}(V)$ is closed set in X for every sαrw closed set V in Y.
6. αrw-continuous [34] if $f^{-1}(V)$ is αrw–closed in X for every closed subset V of Y.

Definition 2.3:-A map f: (X, τ) → (Y, σ) is said to be
1. Contra continuous [12] if $f^{-1}(V)$ is open in X for every closed subset V of Y.
2. Contra irresolute [23] if $f^{-1}(V)$ is semi-open in X for every semi-closed subset V of Y.
3. Contra r–irresolute [22] if $f^{-1}(V)$ is regular-open in X for every regular-closed subset V of Y
4. Irresolute [19] if $f^{-1}(V)$ is semi-closed in X for every semi-closed subset V of Y.
5. rω*-open(resp rω*-closed) [30] map if f(U) is rω-open (resp rω-closed) in Y for every rω-open (resp rω-closed) subset U of X.
6. sαrw– irresolute [7] if $f^{-1}(V)$ is sαrw closed in X for every sαrw-closed subset V of Y.
7. α*-quotient map[31] if f is α-irresolute and $f^{1}(V)$ is an α- open set in (X,τ) implies V is an open set in (Y,σ).
8. αg– irresolute [25] if $f^{-1}(V)$ is αg-closed in X for every αg-closed subset V of Y.
9. α– irresolute [23] if $f^{-1}(V)$ isα-closed in X for every α- closed subset V of Y.

Definition 2.4:-A map f: (X, τ) → (Y, σ) is said to be
1. α–closed map [15] if f (F) is α–closed in Y for every closed subset F of X.
2. gspr–closed map [27] if f (V) is gspr–closed in Y for every closed subset V of X
3. g–closed map [5] if f (V) is g–closed in Y for every closed subset V of X
4. ω–closed map [30] if f (V) is ω–closed in Y for every closed subset V of X
5. rgα–closed map [32] if f (V) is rgα–closed in Y for every closed subset V of X
6. gr–closed map [10] if f (V) is gr–closed in Y for every closed subset V of X
7. g*p–closed map [21] if f(V) is g*p–closed in Y for every closed subset V of X
8. rps–closed map [28] if f (V) is rps–closed in Y for every closed subset V of X
9. R*–closed map [14] if f (V) is R*–closed in Y for every closed subset V of X
10. gprw–closed map [17] if f (V) is gprw–closed in Y for every closed subset V of X.
11. wgrα–closed map [16] if f (V) is wgrα–closed in Y for every closed subset V of X.
12. αg–closed map [20] if f (F) is αg–closed in Y for every closed subset F of X.
13. swg–closed map [25] if f (V) is swg–closed in Y for every closed subset V of X.
14. rω–closed map [9] if f (V) is rw–closed in Y for every closed subset V of X.
15. rgw–closed map [24] if f (V) is rgw–closed in Y for every closed subset V of X.
16. regular closed map[29] if f (F) is closed in Y for every regular closed set F of X
17. Contra closed map [4] if f (F) is closed in Y for every open set F of X.
18. Contra regular closed map [29] if f (F) is r-closed in Y for every open set F of X.
19. Contra semi-closed map [26] if f (F) is s-closed in Y for every open set F of X.
20. wg–closed map [25] if f (V) is wg–closed in Y for every closed subset V of X
21. rwg–closed map [25] if f (V) is rwg–closed in Y for every closed subset V of X
22. gs–closed map [3] if f (V) is gs–closed in Y for every closed subset V of X
23. gp–closed map [21] if f (V) is gp–closed in Y for every closed subset V of X
24. gpr–closed map [13] if f (V) is gpr–closed in Y for every closed subset V of X
25. αgr–closed map [33] if f (V) is αgr–closed in Y for every closed subset V of X
26. ωα–closed map [9] if f (V) is ωα–closed in Y for every closed subset V of X

Definition 2.5:-A map f: (X, τ) → (Y, σ) is said to be
1. g-open map [5] if f (U) is g-open in (Y, σ) for every open set U of (X, τ),
2. gpr-open map [13] if f (U) is gpr-open in (Y, σ) for every open set U of (X, τ),
3. Regular open map [29] if f (U) is open in (Y, σ) for every regular open set U of (X, τ).
4. rwg-open map [25] if f (U) is rwg-open in (Y, σ) for every open set U of (X, τ),
5. wg-open map [25] if f (U) is wg-open in (Y, σ) for every open set U of (X, τ),
6. w- open map [30] if f (U) is w-open in (Y, σ) for every open set U of (X, τ).

Results 2.6:- [6]
1. Every closed (resp regular-closed, α-closed) set is sαrw-closed set in X.
2. Every sαrw-closed set is sg-closed set

3. Every sαrw-closed set is gsp-closed (resp rps-closed, gs-closed, gspr-closed) set in X .

Results 2.7:-[6] If a subset A of a topological space X, and
1. If A is regular open and sαrw-closed then A is α-closed set in X
2. If A is open and αg-closed then A is sαrw-closed set in X
3. If A is open and gp-closed then A is sαrw-closed set in X
4. If A is regular open and gpr-closed then A is sαrw-closed set in X.
5. If A is open and wg-closed then A is sαrw-closed set in X.
6. If A is regular open and rwg-closed then A is sαrw-closed set in X.
7. If A is regular open and αgr-closed then A is sαrw-closed set in X.
8. If A is ω-open and ωα-closed then A is sαrw-closed set in X.

Results 2.8:-[6] If a subset A of a topological space X, and
1. If A is semi-open and sg-closed then it is sαrw-closed.
2. If A is semi-open and ω-closed then it is sαrw-closed.
3. A is sαrw-open iff U ⊆ αint (A), whenever U is sαrw- closed and U ⊆A.

Definition 2.9:-A topological space (X, τ) is called
1. $T_{½}$ space [22] if every g-closed set is closed.
2. $T_{s\alpha rw}$ space [7] if every sαrw-closed set is closed.

3. sαrw-closed maps and sαrw-open maps:-

Definition 3.1:-A map f: (X, τ) → (Y, σ) is said to be semi α-regular weakly closed (briefly sαrw-closed) if the image of every closed set in (X, τ) is sαrw-closed in (Y, σ).

Theorem 3.2:-Every closed map is sαrw-closed map, but not conversely.
Proof:-Let f: (X,τ)→(Y, σ) be closed map and V be any closed set in X. Then f (V) is closed set in Y, since every closed set is sαrw-closed set. Hence f (V) is sαrw-closed set in Y. Therefore f is sαrw-closed.

Example 3.3:-Let X = Y = {a, b, c} τ ={X, ϕ, {a}, {b}, {a, b}} be a topology on X. τ ={Y, ϕ, {a}, {b, c}} be a topology on Y. Let f: (X, τ) → (Y, σ) ,defined by f (a) = b, f (b) = c, f(c) = a, then f is sαrw-closed map but not closed as image of closed set {b, c} in X is {a, c}, which is not closed set in Y.

Theorem 3.4:-
1. Every α-closed map is sαrw-closed map, but not conversely.
2. Every regular closed map is sαrw-closed map, but not conversely

Proof:-
1. The proof follows from the fact that every α-closed set is sαrw-closed set.
2. The proof follows from the fact that every regular closed set is sαrw-closed set.

Example 3.5:-
1. In example 3.3, f is sαrw-closed map but not closed as image of closed set {b, c} in X is {a, c}, which is not α-closed set in Y.
2. In example 3.3, f is sαrw-closed map but not closed as image of closed set {b, c} in X is {a, c}, which is not regular closed set in Y.

Theorem 3.6:-Every sαrw-closed map is sg closed map, but not conversely.

Proof:-The proof follows from the fact that every sαrw-closed set is sg-closed set.

Example 3.7:-Let X = Y = {a, b, c} τ ={X, ϕ, {a}, {b, c}} be a topology on X. τ ={Y, ϕ, {a}, {b}, {a, b}, {a, c}} be a topology on Y. Let f: (X, τ) → (Y, σ) defined by f (a) = c, f (b) = a, f(c) = b, then f is sg-closed map but not sαrw-closed as image of closed set {b, c} in X is {a, c}, which is not sαrw-closed set in Y.

Theorem 3.8:-
1. Every sαrw-closed map is gs closed map, but not conversely.
2. Every sαrw-closed map is rps closed map, but not conversely.
3. Every sαrw-closed map is gsp closed map, but not conversely.
4. Every sαrw-closed map is gspr closed map, but not conversely.

Proof:-
1. The proof follows from the fact that every sαrw-closed set is gs closed set.
2. The proof follows from the fact that every sαrw-closed set is rps closed set.
3. The proof follows from the fact that every sαrw-closed set is gsp closed set.
4. The proof follows from the fact that every sαrw-closed set is gspr closed set.

Example 3.9:-
1. In example 3.7, f is gs closed map but not sαrw-closed as image of closed set {b, c} in X is {a, c}, which is not sαrw-closed set in Y.
2. In example 3.7, f is rps closed map but not sαrw-closed as image of closed set {b, c} in X is {a, c}, which is not sαrw closed set in Y
3. In example 3.7, f is gsp closed map but not sαrw-closed as image of closed set {b, c} in X is {a, c}, which is not sαrw closed set in Y.
4. In example 3.7, f is gspr closed map but not sαrw-closed as image of closed set {b, c} in X is {a, c}, which is not sαrw closed set in Y.

Remark 3.10:-The following examples show that sαrw-closed maps are independent of pre-closed, β-closed, gp-closed, gpr-closed, swg-closed, rwg-closed, wg-closed, rgw-closed, gprw-closed, pgpr-closed maps.

Example 3.11:-Let X={a,b,c}and Y={a,b,c,d}, τ ={X, ϕ, {a}, {b} , {a , b} , {a, c}} be a topology on X. τ ={Y, ϕ, {a}, {b, c}, {a,b,c}} be a topology on Y. Let f: (X, τ) → (Y, σ) defined by f (a) = c, f (b) = a, f(c) = b then f is of pre-closed, β-closed, gp-closed, gpr-closed , swg-closed, rwg-closed, wg-closed, rgw-closed, gprw-closed, pgpr-closed maps. But f is not sαrw-closed map, as closed set {c} in X is {b}, which is not sαrw-closed set in Y.

Example 3.12:-Let X={a,b,c}and Y={a,b,c,d}, τ ={X, ϕ, {a}, {b} , {a , b} , {a, c}} be a topology on X. τ ={Y, ϕ, {a}, {b, c}, {a,b,c}} be a topology on Y. Let f: (X, τ) → (Y, σ) defined by f(a) = c, f(b) = a, f(c) = b then f is sαrw-closed, but not a pre-closed, β-closed, gp-closed, gpr-closed , swg-closed, rwg-closed, wg-closed, rgw-closed, gprw-closed, pgpr-closed maps ,as closed set {a , c} in X is {b , c}, which is not pre-closed(respectively β-closed, gp-closed, gpr-closed , swg-closed, rwg-closed, wg-closed, rgw-closed, gprw-closed, pgpr-closed) set in Y.

Remark 3.13;-From the above discussions and known facts, the relation between sαrw-closed map and some existing closed maps in topological space is shown in the following figure.

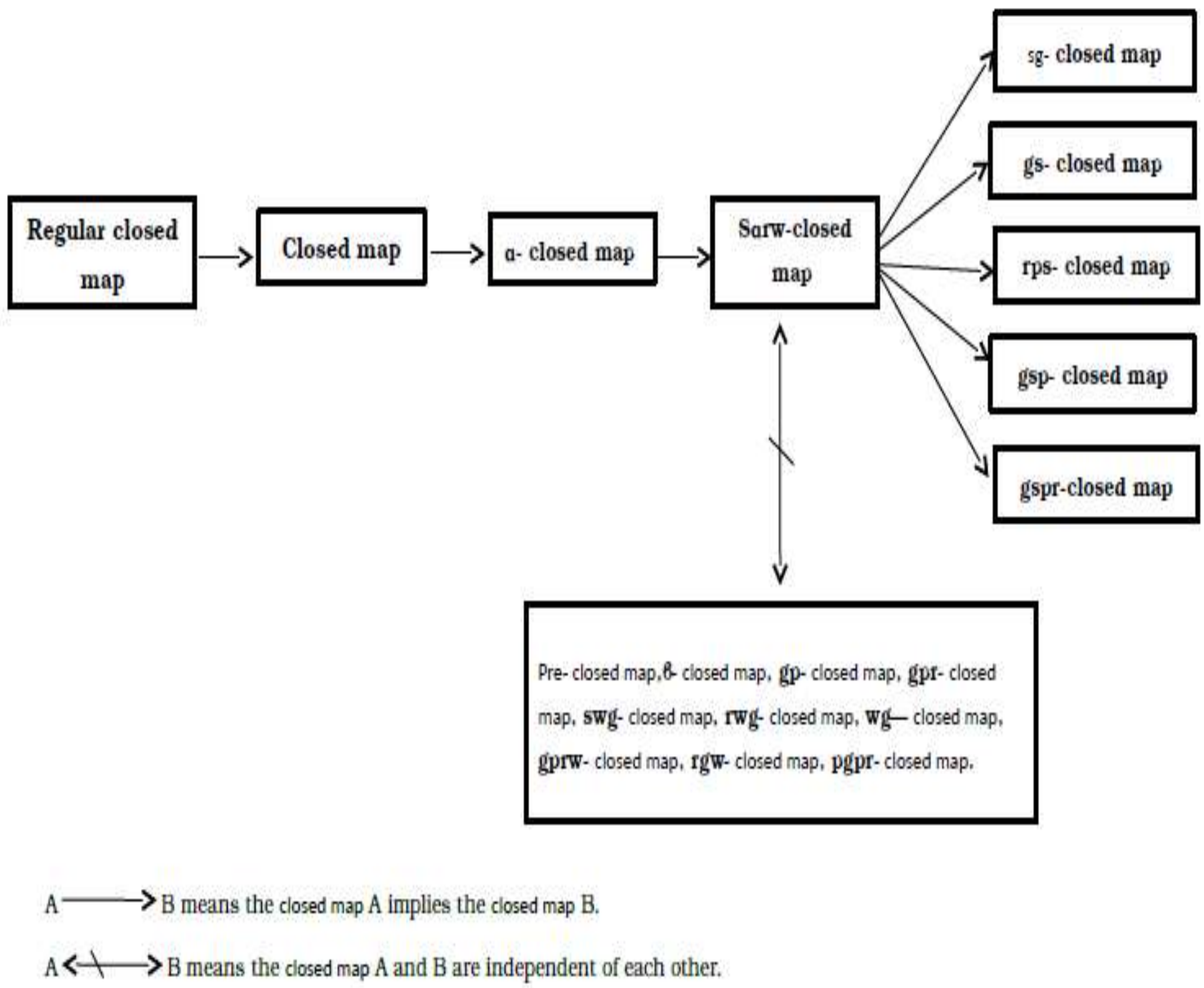

Figure-1

Theorem 3.14:-
1. If a map f: (X, τ) → (Y, σ) is contra regular closed and αgr–closed map then f is sαrw closed map.
2. If a map f: (X, τ) → (Y, σ) is contra w- closed and wα–closed map then f is sαrw closed map.
3. If a map f: (X, τ) → (Y, σ) is contra closed and αg–closed map then f is sαrw closed map.
4. If a map f: (X, τ) → (Y, σ) is contra regular closed and rwg–closed map then f is sαrw closed map.
5. If a map f: (X, τ) → (Y, σ) is contra closed and wg–closed map then f is sαrw closed map.
6. If a map f: (X, τ) → (Y, σ) is contra regular closed and gpr–closed map then f is sαrw closed map.
7. If a map f: (X, τ) → (Y, σ) is contra closed and gp–closed map then f is sαrw closed map

Proof:-
1. Let V be any closed set in (X, τ). Then f (V) is open and αgr–closed. By results 2.7 f (V) is sαrw closed in (Y, σ). Therefore f is sαrw closed map.
2. Let V be any closed set in (X, τ). Then f (V) is open and wα–closed. By results 2.7 f (V) is sαrw closed in (Y, σ). Therefore f is sαrw closed map.
3. Let V be any closed set in (X, τ). Then f (V) is open and αg–closed. By results 2.7 f (V) is sαrw closed in (Y, σ). Therefore f is sαrw closed map.
4. Let V be any closed set in (X, τ). Then f (V) is open and rwg–closed. By results 2.7 f (V) is sαrw closed in (Y, σ). Therefore f is sαrw closed map.
5. Let V be any closed set in (X, τ). Then f (V) is open and wg–closed. By results 2.7 f (V) is sαrw closed in

(Y, σ). Therefore f is sαrw closed map.
6. Let V be any closed set in (X, τ). Then f (V) is open and gpr–closed. By results 2.7 f (V) is sαrw closed in (Y, σ). Therefore f is sαrw closed map.
7. Let V be any closed set in (X, τ). Then f (V) is open and gp–closed. By results 2.7 f (V) is sαrw closed in (Y, σ). Therefore f is sαrw closed map.

Theorem 3.15 If a mapping f: (X, τ) → (Y, σ) is sαrw closed, then sαrw-cl (f (A)) ⊆ f (cl (A)) for every subset A of (X, τ).

Proof:- Suppose that f is sαrw-closed and A ⊆X. Then cl (A) is closed in X and so f (cl (A)) is sαrw-closed in (Y, σ). We have f (A) ⊆ f (cl (A)), sαrw-cl(f(A)) ⊆ sαrw-cl(f(cl(A))) → (i). Since f (cl (A)) is sαrw-closed in (Y, σ), sαrw-cl (f (cl (A))) = f (cl (A)) → (ii), from (i) and (ii), we have sαrw-cl (f (A)) ⊆ f (cl (A)) for every subset A of (X, τ).

Corollary 3.16:-
If a mapping f: (X, τ) → (Y, σ) is sαrw-closed, then the image f (A) of closed set A in (X, τ) is $\tau_{s\alpha rw}$-closed in (Y, σ).

Proof:-Let A be a closed set in (X, τ). Since f is sαrw-closed, sαrw-cl (f (A)) ⊆ f (cl (A)) → (i). Also cl (A) = A, as A is a closed set and so f (cl(A)) = f(A) →(ii). From (i) and (ii), we have sαrw-cl (f (A)) ⊆ f (A). We know that f (A) ⊆ sαrw-cl (f (A)) and so sαrw-cl (f (A)) =f (A). Therefore f (A) is $\tau_{s\alpha rw}$ -closed in (Y, σ).

Theorem 3.17:-Let (X, τ) be any topological spaces and (Y, σ) be a topological space where" sαrw-cl (A) = α-cl (A) for every subset A of Y" and f: (X, τ) →(Y, σ) be a map, and then the following are equivalent.
1. f is sαrw-closed map.
2. sαrw-cl (f (A)) ⊆ f (cl (A)) for every subset A of (X, τ).

Proof:- (1) ⇒ (2) Follows from the Theorem 3.15.
(2) ⇒ (1) let A be any closed set of (X, τ). Then A=cl (A) and so f (A) = f (cl (A)) ⊇ sαrw-cl (f (A)) by hypothesis. We have f (A) ⊆ sαrw-cl (f (A)), Therefore f (A) = sαrw-cl (f (A)). Also f (A) = sαrw-cl (f (A)) = α-cl (f (A)), by hypothesis. That is f (A) = α-cl (f (A)) and f (A) is α-closed in (Y, σ). Thus f (A) is sαrw-closed set in (Y, σ) and hence f is sαrw-closed map.

Theorem 3.18:- A map f : (X, τ) → (Y, σ) is sαrw-closed if and only if for each subset S of (Y, σ) and each open set U containing $f^{-1}(S)$ ⊆U, there is a sαrw-open set V of (Y, σ) such that S ⊆V and $f^{-1}(V)$ ⊆ U.

Proof;- Suppose f is sαrw-closed. Let S ⊆ Y and U be an open set of (X, τ) such that $f^{-1}(S)$ ⊆U. Now X − U is closed set in (X, τ). Since f is sαrw-closed, f(X − U) is sαrw closed set in (Y, σ). Then V = Y – f (X − U) is a sαrw-open set in (Y, σ). Note that $f^{-1}(S)$ ⊆ U implies S ⊆ V and $f^{-1}(V)$ =X − f^{1} (f (X − U)) ⊆ X − (X − U) = U.
That is $f^{-1}(V)$⊆ U.

For the converse, let F be a closed set of (X, τ). Then $f^{-1}((f(F))^c) \subseteq F^c$ and F^c is an open in (X, τ). By hypothesis, there exists sαrw-open set V in (Y, σ) such that $f(F)^c \subseteq V$ and $f^{-1}(V) \subseteq F^c$ and so $F \subseteq (f^{-1}(V))^c$. Hence $V^c \subseteq f(F) \subseteq f(((f^{-1}(V))^c) \subseteq V^c$ which implies $f(F) = V^c$. Since V^c is sαrw-closed, f (F) is sαrw-closed. Thus f (F) is sαrw-closed in (Y, σ) and therefore f is sαrw-closed map.

Remark 3.19:- The composition of two sαrw-closed maps need not be sαrw-closed map in general and this is shown by the following example.

Example 3.20:- Let X = Y = Z = {a, b, c}, τ={X, ϕ, {a}, {b}, {a, b}, {a, c}}, σ = {Y, ϕ, {a}, {b, c}} and η = {Z, ϕ, {a}, {b}, {a, b}}. Define f: (X, τ) → (Y, σ) defined by f (a) =b, f (b) =c, f(c) =a and g: (Y, σ) → (Z, η) are the identity maps. Then f and g are sαrw-closed maps, but their composition (g ∘ f): (X, τ) → (Z, η) is not sαrw-closed map, because F = {a,c} is closed in (X, τ), but (g ∘ f) (F) = (g ∘ f) ({a,c}) = g[f({a,c})] = g[{a,b}] = {a,b} which is not sαrw-closed in (Z, η).

Theorem 3.21:- If f: (X, τ) → (Y, σ) is closed map and g: (Y, σ) → (Z, η) is sαrw-closed map, then the composition (g ∘ f): (X, τ) → (Z, η) is sαrw-closed map.

Proof:-Let F be any closed set in (X, τ). Since f is closed map, f (F) is closed set in (Y, σ). Since g is sαrw-closed map, g [f (F)] is sαrw-closed set in (Z, η). That is (g ∘ f) (F) = g [f (F)] is sαrw-closed and hence (g ∘ f) is sαrw-closed map.

Remark 3.22:- If f: (X, τ) → (Y, σ) is sαrw-closed map and g: (Y, σ) → (Z, η) is closed map, then the composition need not be sαrw-closed map as seen from the following example.

Example 3.23:-Let X = Y = Z = {a, b, c}, τ={X, ϕ, {a}, {b}, {a, b}}, σ = {Y, ϕ, {a}, {b, c}} and η = {Z, ϕ, {a}, {b}, {a, b},{a, c}}. Define f: (X, τ) → (Y, σ) defined by f(a)=b, f(b)=c, f(c)=b and g: (Y, σ) → (Z, η) defined by g(a)=c, g(b)=a, g(c)=c. Then f is sαrw-closed map and g is closed map, but their composition (g ∘ f): (X, τ) → (Z, η) is not sαrw-closed map, because F = {c} is closed in (X, τ), but (g ∘ f)(F) = (g ∘ f) ({c}) = g[f({c})] = g[{b}] = {a} which is not sαrw-closed in (Z, η).

Theorem 3.24:-If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) is sαrw-closed maps and (Y, σ) be a $T_{s\alpha rw}$-space then (g ∘ f): (X, τ) → (Z, η) is sαrw-closed map.

Proof. Let A be a closed set of (X, τ). Since f is sαrw-closed, f (A) is sαrw-closed in (Y, σ). Then by hypothesis, f (A) is closed. Since g is sαrw-closed, g (f (A)) is sαrw-closed in (Z, η) and g [f (A)] = (g ∘ f) (A). Therefore (g ∘ f) is sαrw-closed map.

Theorem 3.25:-If f: (X, τ) → (Y, σ) is g-closed, g: (Y, σ) → (Z, η) be sαrw-closed and (Y, σ) is $T_{1/2}$-space then their composition (g ∘ f): (X, τ) → (Z, η) is sαrw-closed map.

Proof:-Let A be a closed set of (X, τ). Since f is g-closed, f (A) is g-closed in (Y, σ). Since (Y, σ) is $T_{1/2}$-space, f (A) is closed in (Y, σ). Since g is sαrw-closed, g [f (A)] is sαrw-closed in (Z, η) and g (f (A)) = (g ∘ f) (A).
Therefore (g ∘ f) is sαrw-closed map.

Definition:-3.26: A map f: (X, τ) → (Y, σ) is called sαrw-open map if the image f (A) is sαrw-open in (Y, σ) for each open set A in (X, τ).
From the definitions we have the following results.

Theorem 3.27:-
1. Every open map is sαrw–open but not conversely.
2. Every α-open map is sαrw–open but not conversely.
3. Every sαrw–open map is sg-open but not conversely.
4. Every sαrw–open map is gs-open but not conversely.
5. Every sαrw–open map is gsp-open but not conversely.
6. Every sαrw–open map is rps-open but not conversely.
7. Every sαrw–open map is gspr-open but not conversely.

Proof:-proof is straight forward since the compliments of closed sets are open sets.

Theorem 3.28:-For any bijection map f: (X, τ) → (Y, σ), the following statements are equivalent:
1. f^{-1}: (Y, σ) → (X, τ) is sαrw-continuous.
2. f is sαrw-open map
3. f is sαrw-closed map.

Proof:-

(1)⇒ (2) Let U is an open set of (X, τ). By assumption, $(f^{-1})^{-1}$ (U) = f (U) is sαrw-open in (Y, σ) and so f is sαrw-open.

(2)⇒ (3) Let F is a closed set of (X, τ). Then F^c is open set in (X, τ). By assumption, f (F^c) is sαrw-open in (Y, σ). That is f (F^c) = f $(F)^c$ is sαrw-open in (Y, σ) and therefore f(F) is sαrw-closed in (Y, σ). Hence f is sαrw-closed.

(3)⇒ (1) Let F is a closed set of (X, τ). By assumption, f (F) is sαrw-closed in (Y, σ). But f (F) = $(f^{-1})^{-1}$(F) and therefore f^{-1} is sαrw-continuous.

Theorem 3.29:-If a map f: (X, τ) → (Y, σ) is sαrw-open, then f (int (A)) ⊆ sαrw-int (f(A)) for every subset A of (X, τ).

Proof:-Let f: (X, τ) → (Y, σ) be an open map and A be any subset of (X, τ). Then int (A) is open in (X, τ) and so f (int (A)) is sαrw-open in (Y, σ). We have f (int (A)) ⊆ f(A). Therefore f (int (A)) ⊆ sαrw-int (f (A)).

Theorem 3.30:-If a map f: (X, τ) → (Y, σ) is sαrw-open, then for each neighbourhood U of x in (X, τ), there exists a sαrw-neighbourhood W and f(x) in (Y, σ) such that W ⊆ f (U).

Proof:-Let f: (X, τ) → (Y, σ) be an sαrw-open map. Let x ∈X and U be an arbitrary neighbourhood of x in (X, τ). Then there exists an open set G in (X, τ) such that x ∈ G ⊆ U. Now f(x) ∈f (G) ⊆ f (U) and f (G) is sαrw-open set in (Y, σ), as f is an sαrw-open map. f (G) is sαrw– neighbourhood of each of its points. Taking f (G) = W, W is an sαrw–neighbourhood of f(x) in (Y, σ) such that W ⊆ f (U).

Definition 3.31:-A map f: (X, τ) → (Y, σ) is said to be sαrw*-closed map if the image f(A) is sαrw-closed in (Y, σ) for every sαrw-closed set A in (X, τ).

Theorem 3.32:-Every sαrw*-closed map is sαrw-closed map but not conversely.

Proof:-The proof follows from the definitions and fact that every closed set is sαrw-closed.

Example 3.33:-Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ ={Y, ϕ, {a}, {b}, {a, b}} and f: (X, τ) → (Y, σ) be the identity map. Then f is sαrw-closed map but not sαrw*-closed map. Since {a, b} is sαrw-closed set in (X, τ), but its image under f is {a, b}, which is not sαrw-closed in (Y, σ).

Theorem 3.34:-If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are sαrw*-closed maps, then their composition (g ∘ f): (X, τ) → (Z, η) is also sαrw∗-closed.

Proof:-Let F be an sαrw-closed set in (X, τ). Since f is sαrw*-closed map, f (F) is sαrw- closed set in (Y, σ). Since g is sαrw∗-closed map, g (f (F)) is sαrw closed set in (Z, η). Therefore (g ∘ f) is sαrw*-closed map.

Analogous to sαrw*-closed map, we define another new class of maps called sαrw*-open maps which are stronger than sαrw-open maps.

Definition 3.35:-A map f: (X, τ) → (Y, σ) is said to be sαrw∗-open map if the image f (A) is sαrw-open set in (Y, σ) for every sαrw-open set A in (X, τ).

Remark 3.36:-Since every open set is a sαrw-open set, we have every sαrw∗-open map is sαrw-open map. The converse is not true in general as seen from the following example.

Example 3.37:-Let X = Y = {a, b, c}, τ = {X, ϕ, {a}, {b, c}} and σ ={Y, ϕ, {a}, {b}, {a, b}} and f: (X, τ) → (Y, σ) be the identity map. Then f is sαrw-open map but not sαrw*-open map. Since {c} is sαrw-open set in (X, τ), but its image under f is {c}, which is not sαrw-open in (Y, σ).

Theorem 3.38:-If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) are sαrw∗-open maps, then their composition (g ∘ f): (X, τ) → (Z, η) is also sαrw∗-open.

Proof:-Proof is similar to the Theorem 3.34.

Theorem 3.39:-For any bijection map f: (X, τ) →(Y, σ), the following statements are equivalent:
1. f^{-1}: (Y, σ) → (X, τ) is sαrw irresolute.
2. f is sαrw∗–open map
3. f is sαrw∗–closed map.

Proof:-Proof is similar to that of Theorem 3.28

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