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## RESEARCH ARTICLE

## ON SPECTRA AND BOUNDS OF FUZZY GRAPH.

S. Vimala<sup>1</sup> and P. Jayalakshmi<sup>2,3</sup>.

1. Assistant Professor, Department of Mathematics, Mother Teresa Women's University, Kodaikanal.
2. Research Scholar in Mathematics, Mother Teresa Women's University, Kodaikanal.
3. Assistant Professor in Mathematics, Sri G.V.G Visalakshi College For Women (Autonomous), Udumalpet.

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**Abstract**

Energy of graph / fuzzy graph is the sum of absolute values of eigen values for adjacent/Fuzzy adjacent matrix [1,3]. In this paper, upper bound and lower bound in terms of its degree and membership values of fuzzy graph is calculated. The number of closed walks of a given length is spectrum of fuzzy graph. The  $k^{th}$  spectral moment of the fuzzy graph  $M_k(FG)$  is defined, where  $M_k$  is the number of closed walks of length  $k$  in  $FG$ . The relation between the energy and spectral moments of fuzzy graph eigen values are derived.

**\*Corresponding Author**

S.Vimala.

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**Introduction:-**

Let  $G=(V,E)$  be a graph comprising a set  $V$  of vertices and a set  $E$  of edges such that an edge is related with two vertices, and the relation is represented as an unordered pair of the vertices with respect to the particular edge. This type of graph may be described precisely as undirected and simple. A graph is a symbolic representation of a network and of its connectivity. Graph theory is a branch of mathematics concerned about how networks can be encoded and their properties measured[11].

Fuzzy graph was developed by L.A.Zadeh in 1965. The first definition of fuzzy graphs was proposed by Kauffmann [3] in 1975, from the Zadeh's fuzzy relations. In 1975, Rosenfeld defined fuzzy relation and developed fuzzy graph theoretical concepts[22]. Later on Fuzzy graph was developed for regular graph, complete graph, bigraph, irregular graph, complement graph etc by many authors[21,22]. Fuzzy graphs has many successful applications. Fuzzy matrix was developed by Chandrashekar Adiga and M. Smitha in 2009[7].

Balakrishnan R [3] introduced energy of graph in 2004. Anjali .N and Sunil Mathew[1] introduced energy, upper and lower bounds of fuzzy graph. A fuzzy graph is completely determined by specifying either its adjacency structure or its incidence structure. As computers are more adept at manipulating numbers than at recognising pictures, it is standard practice to communicate the specification of a fuzzy graph to a computer in matrix form.

Meenakshi A.R[19] described about fuzzy matrix theory and applications in 2008. Matrices with entries  $[0,1]$  and matrix operation defined by fuzzy logical operations are called fuzzy matrices. The fuzzy adjacency matrix is a  $[0,1]$ -matrix with zeros on its diagonal. There are several observations made about fuzzy adjacency matrix [13]. The diagonal is zero since there are no loops. Many kinds of fuzzy matrices are associated with fuzzy graphs. The spectrum of one such fuzzy matrix, fuzzy adjacency matrix is called the spectrum of fuzzy graph[6]. They fuzzy adjacency matrix is symmetric and so that its spectrum is real. Fuzzy graph spectrum appears in problems in physics and in Mathematics. Fuzzy graph spectrum plays an important role in pattern recognition, computer networks and in securing personal data in databases.

In 1997, Halpern J developed the set adjacency measures in Fuzzy graphs [12]. The emergence of fuzzy adjacent matrix on fuzzy graph has initiated the new research views in fuzzy graph theory. A concept related to the spectrum of fuzzy graph is that of energy of fuzzy graph. In the last half century, considerable progress has been made in the energy of graphs [8]. Certain bounds on energy is studied. The physical meaning and application of energy fuzzy graph may not be known exactly at present but the properties it is found to possess are of area of interest to a Mathematician.

In this paper section 2, the definition of fuzzy adjacent matrix is given. The product of two fuzzy adjacent matrices, product of fuzzy adjacent matrix with crisp number and its power are discussed. Section 3 deals with the bounds of fuzzy graph. Section 4 characterizes the relation between the spectra and the energy of fuzzy graph as well as the spectral moments and the energy of fuzzy graph.

Throughout this paper a simple, undirected fuzzy graph is considered.

### Preliminaries:-

#### Definition:-

A fuzzy subset of a nonempty set  $S$  is a mapping  $\sigma: S \rightarrow [0,1]$ . A fuzzy relation on  $S$  is a fuzzy subset of  $S \times S$ . If  $\mu$  and  $\nu$  are fuzzy relations, then  $\mu \vee \nu(u, w) = \sup\{\mu(u, v) \wedge \nu(v, w) : v \in S\}$  and  $\mu^k(u, v) = \sup\{\mu(u, u_1) \wedge \nu(u_1, u_2) \wedge \mu(u_2, u_3) \wedge \dots \wedge \mu(u_{k-1}, v) : u_1, u_2, \dots, u_{k-1} \in S\}$ , where ' $\wedge$ ' stands for minimum.

#### Definition:-

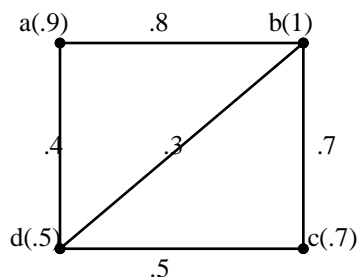
A fuzzy graph is a pair of functions  $G: (\sigma, \mu)$  where  $\sigma: V \rightarrow [0,1]$  is a fuzzy subset of non-empty set  $V$  and  $\mu: V \times V \rightarrow [0,1]$  is symmetric fuzzy relation on  $\sigma$  such that for all  $x, y$  in  $V$  the condition  $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$  is satisfied for all  $(x, y)$  in  $E$ .

#### Definition:-

Let  $G = (\sigma, \mu)$  be a fuzzy graph. The degree of a vertex  $u$  is  $d(u) = \sum_{u \neq v} \mu(u, v)$ .

#### Definition:-

The adjacency fuzzy matrix of FG is defined as  $A_{FG} = \begin{cases} \mu(u, v) & \text{if } i, j \text{ in the neighbourhood} \\ 0 & \text{otherwise} \end{cases}$



Fuzzy Graph 1.1

The fuzzy adjacent matrix of the above fuzzy graph is given by

$$A_{FG} = \begin{bmatrix} 0 & .8 & 0 & .4 \\ .8 & 0 & .7 & .3 \\ 0 & .7 & 0 & .5 \\ .4 & .3 & .5 & 0 \end{bmatrix}$$

Let us consider one more fuzzy adjacent matrices of same fuzzy graph 1.1

$$B_{FG} = \begin{bmatrix} 0 & .9 & 0 & .8 \\ .9 & 0 & .6 & .3 \\ 0 & .6 & 0 & .6 \\ .8 & .3 & .6 & 0 \end{bmatrix}$$

**Definition:-**

Let  $G: (\sigma, \mu)$  be a finite, undirected and simple fuzzy graph. The energy of a fuzzy graph,  $E(FG)$  is defined as the sum of the absolute values of eigen values  $A_{FG}$ .

(i.e)  $E(FG) = \sum_{i=1}^n |\lambda_i|$ . The set  $\{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n\}$  is the spectrum of  $(FG)$ .

The eigen values of  $A_{FG}$  is -1.1227, -0.2922, 0.0317 and 1.3832.

Energy of  $A_{FG}$  is  $E(A_{FG}) = 2.8298$ .

Spectrum of  $A_{FG} = \{-1.1227, -0.2922, 0.0317, 1.3832\}$

**Definition:-**

The  $k^{th}$  spectral moment of the fuzzy graph  $G: (\sigma, \mu)$  is defined as  $M_k = M_k(FG) = \sum_{i=1}^n (\lambda_i)^k$  where  $M_k$  is the number of closed walks of length  $k$  in  $G$ .

**Definition:-**

The product of two fuzzy adjacent matrix is a new adjacent matrix  $A_{FG} \cdot B_{FG}$  whose membership function is defined as  $\mu_{A_{FG} \cdot B_{FG}}(a_{ij} \cdot b_{ij}) = \mu_{A_{FG}}(a_{ij}) \cdot \mu_{B_{FG}}(b_{ij})$ .

$$\mu_{A_{FG} \cdot B_{FG}}(a_{ij} \cdot b_{ij}) = \begin{bmatrix} 0 & .72 & 0 & .32 \\ .72 & 0 & .42 & .09 \\ 0 & .42 & 0 & .3 \\ .32 & .09 & .3 & 0 \end{bmatrix}$$

**Product of fuzzy adjacent matrix with a crisp number:-**

Multiplying a fuzzy adjacent matrix  $A_{FG}$  by a crisp number 'a' results in a new fuzzy matrix product

$$\mu_{a \cdot A_{FG}}(a_{ij}) = a \cdot \mu_{A_{FG}}(a_{ij}).$$

$$\text{If } a = 0.5 \text{ then } \mu_{a \cdot A_{FG}}(a_{ij}) = \begin{bmatrix} 0 & .4 & 0 & .2 \\ .4 & 0 & .35 & .15 \\ 0 & .35 & 0 & .25 \\ .2 & .15 & .25 & 0 \end{bmatrix}$$

**Power of a fuzzy adjacent matrix:-**

The  $\alpha$  power of a fuzzy adjacent matrix  $A_{FG}$  is a new fuzzy adjacent matrix  $A^\alpha$  whose membership function is given by  $\mu_{A_{FG}}^\alpha(a_{ij}) = [\mu_{A_{FG}}(a_{ij})]^\alpha$ .

$$\mu_{A_{FG}}^2(a_{ij}) = \begin{bmatrix} 0 & .64 & 0 & .16 \\ .64 & 0 & .49 & .09 \\ 0 & .49 & 0 & .25 \\ .16 & .09 & .25 & 0 \end{bmatrix}$$

Raising a fuzzy adjacent matrix to its second power is called Concentration(CON) and taking the square root is called Dilation(DIL).

**Bounds for fuzzy adjacency matrix:-**

Several bounds for fuzzy adjacency eigen values are obtained:

- $E_{FG} \leq \frac{\sum_{i,j=1}^n \mu(u_i, v_j)}{2} [1 + \sqrt{\sum_{i,j=1}^n \mu(u_i, v_j)}]$
- $E_{FG} \leq \sqrt{2 \sum_{i,j=1}^n d[\sigma(u_i)]}$
- $E_{FG} \leq \sqrt{4 \sum_{i,j=1}^n \mu(u_i, v_j)}$

$$\begin{aligned}
\bullet \quad E_{FG} &\geq \sqrt{2 \sum_{i,j=1}^n \mu(u_i, v_j)} \\
\bullet \quad E_{FG} &\geq \sqrt{\sum_{i,j=1}^n d[\sigma(u_i)]} \\
\bullet \quad \lambda_1 &\geq \sqrt{\frac{\sum_{i,j=1}^n \mu^2(u_i, v_j)}{2}}
\end{aligned}$$

**Theorem:-**

Let  $G: (V, \sigma, \mu)$  be a fuzzy graph with  $|V|=n$  and  $\mu = (e_1, e_2, e_3, \dots, e_m)$  then  $\lambda_{max} \leq \sum_{i,j=1}^n \mu(u_i, v_j)$

**Proof**

Let  $x$  be an eigen vector for the eigen value  $\lambda$  and  $x_j = \max_i \{x_i\}$  be the largest co ordinate value.

$$\begin{aligned}
\text{Now } \lambda x_j &= (Ax)_j \\
&= \sum_{u_i \in N[u_j]} x_i \\
\lambda x_i &\leq \sum_{i,j=1}^n \mu(u_i, v_j) x_j \\
\lambda_{max} &\leq \sum_{i,j=1}^n \mu(u_i, v_j)
\end{aligned}$$

**Result:-**

The above inequality does not holds for  $\lambda_{min}$

**Theorem:-**

Let  $G: (V, \sigma, \mu)$  be a fuzzy graph with  $|V|=n$  and  $\mu = (e_1, e_2, e_3, \dots, e_m)$ . Let  $A_{FG} = (a_{ij})$  be the fuzzy adjacent matrix of  $G(\sigma, \mu)$  and  $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$  be the eigen values of  $A_{FG}$  with  $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_n$ . Then

$$\begin{aligned}
\text{(i)} \quad \sum_{i=1}^n \lambda_i &= 0. \\
\text{(ii)} \quad \sum_{i=1}^n \lambda_i^2 &= 2 \sum_{i,j=1}^n \mu^2(u_i, v_j)
\end{aligned}$$

**Proof**

(i) The diagonal is zero since there is no loops. Since  $A_{FG}$  is a symmetric matrix with trace zero, the eigen values are real with sum equal to zero.

(ii) Using trace properties of matrix,

$$\begin{aligned}
\text{tra}(A_{FG}^2) &= [0 + \mu^2(u_1, u_2) + \dots + \mu^2(u_1, u_n)] + [\mu^2(u_2, u_1) + 0 + \dots + \mu^2(u_2, u_n)] \\
&\quad + \dots + [\mu^2(u_n, u_1) + \mu^2(u_n, u_2) \dots + 0] \\
&= 2 \sum_{i,j=1}^n \mu^2(u_i, v_j)
\end{aligned}$$

**Theorem:-**

Let  $G$  and  $G_1$  be the fuzzy graphs with  $n$  vertices. If  $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_n$  are the adjacent eigen values of  $G$  and  $\eta_1 \geq \eta_2 \geq \eta_3 \geq \dots \geq \eta_n$  are the adjacent eigen values of  $G_1$  then

$$\sum_{i=1}^n \lambda_i \bullet \eta_i \leq 2 \sqrt{d[\sigma(u_i)] \bullet d[\sigma(u_j)]} \text{ where } d[\sigma(u_i)] \text{ and } d[\sigma(u_j)] \text{ are the degree of vertices of } G$$

and  $G_1$  respectively and  $\bullet$  is the max- min composition.

**Proof:-**

By Cauchy-Schwartz inequality,

$$\sum_{i,j=1}^n (\lambda_i \bullet \eta_i)^2 \leq (\sum_{i,j=1}^n \lambda_i^2) \bullet (\sum_{i,j=1}^n \eta_i^2)$$

$$\begin{aligned}
&= [2 \sum_{i,j=1}^n \mu^2(u_i, u_j)] \bullet [2 \sum_{i,j=1}^n \mu^2(v_i, v_j)] \\
&= 2^2 [d[\sigma(u_i)] \bullet d[\sigma(u_j)]] \\
\sum_{i=1}^p \lambda_i \bullet \eta_i &\leq 2 \sqrt{d[\sigma(u_i)] \bullet d[\sigma(u_j)]}
\end{aligned}$$

**Corollary:-**

Let  $G$  and  $G_1$  be the fuzzy graphs with  $n$  vertices. If  $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_n$  are the adjacent eigen values of  $G$  and  $\eta_1 \geq \eta_2 \geq \eta_3 \geq \dots \geq \eta_n$  are the adjacent eigen values of  $G_1$

then  $\sum_{i=1}^n \lambda_i \bullet \eta_i \leq 4 \sqrt{\sum_{i,j=1}^n \mu(u_i, u_j) \bullet \sum_{i,j=1}^n \mu(v_i, v_j)}$  where  $\sum_{i,j=1}^n \mu(u_i, u_j)$  and  $\sum_{i,j=1}^n \mu(v_i, v_j)$  are the vertices of  $G$  and  $G_1$  respectively and  $\bullet$  is the max- min composition.

**Theorem 3.4**

Let  $G: (V, \sigma, \mu)$  be a fuzzy graph with  $|V|=n$  and  $\mu = (e_1, e_2, e_3, \dots, e_m)$

then  $E_{FG} \geq \sqrt{\sum_{i,j=1}^n \mu^2(u_i, v_j)}$

**Proof:-**

$$\begin{aligned}
E_{FG} &= \sum |\lambda_i| \\
&\geq \sqrt{\frac{\sum_{i=1}^n \lambda_i^2}{2}} \\
&\geq \sqrt{\frac{\text{Traces } A_{FG}^2}{2}} \\
&\geq \sqrt{\frac{2 \sum_{i,j=1}^n \mu^2(u_i, v_j)}{2}} \\
&\geq \sqrt{\sum_{i,j=1}^n \mu^2(u_i, v_j)}
\end{aligned}$$

**Theorem:-**

Let  $G: (V, \sigma, \mu)$  be a fuzzy graph with  $|V|=n$  and  $\mu = (e_1, e_2, e_3, \dots, e_m)$  then

$$\sum_{i=1}^n d[\sigma(u_i)] = \sqrt{2 \sum_{i,j=1}^n \mu(u_i, v_j) + \left( \sum_{i,j=1}^n d[\sigma(u_i)] - 1 \right) \left( \sum_{i,j=1}^n d[\sigma(u_i)] \right)}$$

**Proof:-**

For any fuzzy graph  $G$ ,  $\sum_{i=1}^n d[\sigma(u_i)] = 2 \sum_{i,j=1}^n \mu(u_i, v_j)$

Consider

$$(\sum_{i=1}^n d[\sigma(u_i)] - 1)(\sum_{i=1}^n d[\sigma(u_i)]) = (\sum_{i=1}^n d^2[\sigma(u_i)] - (\sum_{i=1}^n d[\sigma(u_i)]))$$

$$\begin{aligned}
&= (\sum_{i=1}^n d^2[\sigma(u_i)]) - 2 \sum_{i,j=1}^n \mu(u_i, v_j) \\
&(\sum_{i=1}^n d^2[\sigma(u_i)]) = 2 \sum_{i,j=1}^n \mu(u_i, v_j) (\sum_{i=1}^n d[\sigma(u_i)] - 1) (\sum_{i=1}^n d[\sigma(u_i)]) \\
\sum_{i=1}^n d[\sigma(u_i)] &= \sqrt{2 \sum_{i,j=1}^n \mu(u_i, v_j) + (\sum_{i,j=1}^n d[\sigma(u_i)] - 1) (\sum_{i,j=1}^n d[\sigma(u_i)])}
\end{aligned}$$

**Theorem 3.6**

Let  $G: (V, \sigma, \mu)$  be a fuzzy graph with  $|V| = n$  and  $\mu = (e_1, e_2, e_3, \dots, e_m)$  then

$$E_{FG} \leq \sqrt{\frac{\sum_{i,j=1}^n \mu^2(u_i, v_j)}{2}} + \sqrt{(\sum_{i=1}^n d[\sigma(u_i)] - 1) (\sum_{i=1}^n d[\sigma(u_i)] - \frac{\sum_{i,j=1}^n \mu^2(u_i, v_j)}{2})}$$

**Proof**

$$\lambda_1 \geq \sqrt{\frac{\sum_{i,j=1}^n \mu^2(u_i, v_j)}{2}}$$

By Cauchy-Schwartz inequality,

$$\begin{aligned}
\sum_{i=1}^n |\lambda_i| &\leq \sqrt{(\sum_{i=1}^n d[\sigma(u_i)] - 1) \sum_{i=1}^n \lambda_i^2} \\
&= \sqrt{(\sum_{i=1}^n d[\sigma(u_i)] - 1) (2 \sum_{i,j=1}^n \mu(u_i, u_j) - \lambda_1^2)} \\
&\leq \lambda_1 + \sqrt{(\sum_{i=1}^n d[\sigma(u_i)] - 1) (2 \sum_{i,j=1}^n \mu(u_i, u_j) - \lambda_1^2)} \\
E_{FG} &\leq \sqrt{\frac{\sum_{i,j=1}^n \mu^2(u_i, v_j)}{2}} + \sqrt{(\sum_{i=1}^n d[\sigma(u_i)] - 1) (\sum_{i,j=1}^n d[\sigma(u_i)] - \frac{\sum_{i,j=1}^n \mu^2(u_i, v_j)}{2})}
\end{aligned}$$

**Theorem:-**

Let  $G: (V, \sigma, \mu)$  be a fuzzy graph with  $|V| = n$  and  $\mu = (e_1, e_2, e_3, \dots, e_m)$ . Then

- (i)  $(\sum_{i=1}^n d[\sigma(u_i)]) \bullet (\sum_{i=1}^n d[\sigma(u_i)] - 1) = (\sum_{i=1}^n d[\sigma(u_i)] - 1)$  if  $(\sum_{i=1}^n d[\sigma(u_i)] < 1)$   
(ii)  $(\sum_{i=1}^n d[\sigma(u_i)]) \bullet (\sum_{i=1}^n d[\sigma(u_i)] - 1) = 0$  if  $(\sum_{i=1}^n d[\sigma(u_i)] > 1)$

where  $\bullet$  is the max- min composition.

**Proof:-**

$$\begin{aligned}
&(\sum_{i=1}^n d[\sigma(u_i)]) \bullet (\sum_{i=1}^n d[\sigma(u_i)] - 1) = \\
&\sum_{i=1}^n d[\sigma(u_i)] \bullet \sum_{i=1}^n d[\sigma(u_i)] - \sum_{i=1}^n d[\sigma(u_i)] \bullet 1 \\
&= \max[\min(\sum_{i=1}^n d[\sigma(u_i)], \sum_{i=1}^n d[\sigma(u_i)])] - \max[\min(\sum_{i=1}^n d[\sigma(u_i)], 1)] \\
&= (\sum_{i=1}^n d[\sigma(u_i)] - 1) \text{ if } (\sum_{i=1}^n d[\sigma(u_i)] < 1) \\
&= 0 \text{ if } (\sum_{i=1}^n d[\sigma(u_i)] > 1)
\end{aligned}$$

**Spectra of fuzzy graph:-**

**Theorem:-**

Let  $FG_1$  and  $FG_2$  be any two fuzzy graphs with adjacency matrices  $A_{FG_1}$  and  $A_{FG_2}$  respectively then

- (i)  $E(FG_1 + FG_2) = E(FG_1) + E(FG_2)$   
(ii)  $E(FG_1 - FG_2) = E(FG_1) - E(FG_2)$

**Proof:-**

Let the spectra of  $FG_1$  and  $FG_2$  be  $\{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n\}$  and  $\{\eta_1, \eta_2, \eta_3, \dots, \eta_n\}$  respectively.

$$\begin{aligned}
E(FG_1 + FG_2) &= \sum_{j,k} |\lambda_j + \eta_k| \\
&= \sum_j |\lambda_j| + \sum_k |\eta_k| \\
&= E(FG_1) + E(FG_2)
\end{aligned}$$

Similarly one can prove that  $E(FG_1 - FG_2) = E(FG_1) - E(FG_2)$ .

### Corollary:-

Let  $FG_1$  and  $FG_2$  be any two fuzzy graphs with adjacency matrices  $A_{FG1}$  and  $A_{FG2}$  respectively

then (i)  $A(FG_1 + FG_2) = A(FG_1) + A(FG_2)$

(ii)  $A(FG_1 - FG_2) = A(FG_1) - A(FG_2)$

where  $A(FG_1 + FG_2) = \max\{A(FG_1), A(FG_2)\}$  and  $A(FG_1 - FG_2) = \min\{A(FG_1), A(FG_2)\}$ .

### Theorem:-

If  $A_{FG1}$  and  $A_{FG2}$  fuzzy adjacent matrices of the fuzzy graphs  $FG_1$  and  $FG_2$

with spectrum  $\{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_r\}$  and  $\{\eta_1, \eta_2, \eta_3, \dots, \eta_s\}$  respectively then the spectrum of  $A(FG_1 + FG_2)$

and  $A(FG_1 - FG_2)$  is  $\{\lambda_j \eta_k : 1 \leq j \leq r ; 1 \leq k \leq s\}$ .

### Proof:-

Let  $X$  and  $Y$  be eigen vectors corresponding to the eigen values  $\lambda$  and  $\eta$  of the fuzzy graphs  $FG_1$  and  $FG_2$  respectively. Then

$$\begin{aligned}
A_{FG1}X &= \lambda X \text{ and } A_{FG2}Y = \eta Y \\
(A_{FG1} + A_{FG2})(X + Y) &= A_{FG1}X + A_{FG2}Y \\
&= \lambda X + \eta Y \\
&= \lambda \eta (X + Y)
\end{aligned}$$

Since  $(X+Y)$  is non zero,  $(A_{FG1} + A_{FG2}) = \lambda \eta$

Similarly one can prove that  $(A_{FG1} - A_{FG2}) = \lambda \eta$

### Theorem:-

Let  $G: (V, \sigma, \mu)$  be a fuzzy graph with  $|V| = n$  and  $\mu = (e_1, e_2, e_3, \dots, e_m)$  then  $E_{FG} \leq M_k(FG)$ ,  
 $k = 2, 3, \dots, n$ .

### Proof:-

Let  $E_{FG} = \sum |\lambda_i|$  and  $M_k(FG) = \sum_{i=1}^n (\lambda_i)^k$

But  $M_k(FG) = 0$  for  $k = 1$ .

Hence  $M_k(FG) = \sum_{i=1}^n (\lambda_i)^k$  for  $k = 2, 3, \dots, n$

$$\geq \sum |\lambda_i|$$

$$= E_{FG}$$

Therefore  $E_{FG} \leq M_k(FG)$

### Conclusion:-

In this paper fuzzy graph is represented by fuzzy adjacent matrix. Basic characteristics of the matrix are highlighted including the adjacent eigen values. The fuzzy adjacent matrix provides the sound interpretation on the bounds. The main focus is made on the energy of the fuzzy graph thereby enabling to calculate the bounds and spectra of the fuzzy adjacent matrix. The specialty of this paper is the computational procedure is very simple.

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