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RESEARCH ARTICLE

ON rgwαlc-SEPARATION AXIOMS IN TOPOLOGICAL SPACES.

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Abstract

The aim of this paper is to introduce and study two new classes of spaces, namely rgwαlc-τ_0, rgwαlc-τ_1, rgwαlc-τ_2,rgwαlc-regular and rgwαlc-normal spaces and obtained their properties by utilizing rgwαlc-closed sets. Also we will present some characterizations of these spaces.

Introduction:-

Munshi [1], introduced g–regular and g– normal spaces using g–closed sets of Levine [4]. Maheshwari and Prasad [7] introduced the new class of spaces called s–normal spaces using semi–open sets. It was further studied by Noiri and Popa[10],Dorsett[2] and Arya[8]. Later, Benchalli et al [9] and Shik John[3] studied the concept of g* – pre regular, g* – pre normal and w–normal, w–regular spaces in topological spaces. Recently, Wali et al [5,6] introduced and studied the properties of rgwαlc-closed sets and RGWαLC–continuous andRGWαLC-irresolute maps.

Preliminaries:-

Throughout this paper (X, τ), (Y, τ) (or simply X, Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X the closure, interior and rα–closure of A with respect to τ are denoted by cl(A), int(A) and rαcl(A) respectively

Definition 2.1: A subset A of a topological space X is called a
(1)semi–open set [9] if A ⊆cl(int(A)).
(2)w–closed set [3] if cl(A)⊆U whenever A ⊆ U and U is semi–open in X.
(3)g–closed set [4] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition 2.2: A topological space X is called
i) a α– τ_0 [11] if for each pair of distinct points x, y of X, there exists a α–open sets G in X containing one of them and not the other.
ii) a α– τ_1 [11] if for each pair of distinct points x, y of X , there exists two α–open sets G1, G2 in X such that x∈G1 , y∉ G1, and y∈ G2 , x∉ G2.

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iii) a α– τ_2 [11] (α– Hausdorff) if for each pair of distinct points x, y of X there exists distinct α–open sets H1 and H2 such that H1 containing x but not y and H2 containing y but not x.

Definition 2.3: A topological space X is said to be a
(1)g–regular[10],if for each g–closed set F of X and each point x∉F, there exists disjoint open sets U and V such that F ⊆ U and x∈V .
(2)α–regular [4], if for each closed set F of X and each point x∉F, there exists disjoint α– open sets U and V such that F ⊆ V and x∈ U.
(3)w–regular[3], if for each w–closed set F of X and each point x∉F, there exists disjoint open sets U and V such that F ⊆ U and x∈V .

Definition 2.4: A topological space X is said to be a
(1) g–normal [10], if for any pair of disjoint g–closed sets A and B, there exists disjoint open sets U and V such that A ⊆ U and B ⊆ V .
(2) α–normal [4], if for any pair of disjoint closed sets A and B, there exists disjoint α–open sets U and V such that A ⊆ U and B ⊆ V .
(3) w–normal [3], if for any pair of disjoint w–closed sets A and B, there exists disjoint open sets U and V such that A ⊆ U and B ⊆ V .

Definition 2.5: A subset A of a topological space X is called a
(i)regular generalized weakly α-locally closed [5] (briefly rgwαlc- closed) if A=U∩F where U is rgwα-open in (X, τ) and F is rgwα-closed in (X, τ).
(ii)regular generalized weakly α-locally open [5] (briefly rgwαlc- open) if A^c is rgwα-locally closed.

Definition 2.6: [5] A topological space X is called τrgwαlc–space if every rgwαlc-closed set in it is closed set.

Definition 2.7: A function f: X→Y is called
(1) RGWαLC-continuous [6] (resp. w–continuous [12]) if f^{-1}(F) is RGWαLC-closed (resp. w–closed) set in X for every closed set F of Y.
(2) RGWαLC -irresolute [6] (resp. w–irresolute [12]) if f^{-1} (F) is RGWαLC-closed (resp. w–closed set in X for every RGWαLC-closed (resp. w– closed) set F of Y.

rgwαlc–τ_k Space (k=0, 1, 2).

Definition 3.1: A topological space X is called
i) a **rgwαlc –τ_0** if for each pair of distinct points x, y of X, there exists a rgwαlc–open set G in X containing one of them and not the other.
ii) a **rgwαlc–τ_1** if for each pair of distinct points x, y of X , there exists two rgwαlc– open sets G1, G2 in X such that x∈G1 , y∉G1, and y∈G2 , x∉G2.
iii) a **rgwαlc–τ_2**(rgwαlc– Hausdorff) if for each pair of distinct points x, y of X there exists distinct rgwαlc–open sets H_1 and H_2 such that H_1 containing x but not y and H2 containing y but not x.

Theorem 3.2:
(i) Every τ_0 space is rgwαlc – τ_0 space.
(ii) Every τ_1 space is rgwαlc – τ_0 space.
(iii) Every τ_1 space is rgwαlc –τ_1 space.
(iv) Every τ_2 space is rgwαlc – τ_2 space.
(v) Every rgwαlc – τ_1 space is rgwαlc – τ_0 space.
(vi) Every rgwαlc –τ_2 space is rgwαlc – τ_1 space.
Proof: Straight forward.
The converse of the theorem need not be true as in the examples.

Example 3.3: Let X= {a, b, c, d} and τ ={X, Φ, {a}, {a, b}}. Then rgwαlcC(X) = rgwαlcO(X)= P(X). Here (X, τ) is rgwαlc– τ_0 and rgwαlc– τ_1 space but not τ_0 space and not τ_1 space.

Example 3.4: Let X= {a, b, c} and τ ={X, Φ, {a},{b},{a, b}}. Then rgwαlcC(X) = rgwαlcO(X)= P(X). Here (X, τ) is rgwαlc– τ_2 space but not τ_2 space.

Theorem 3.5:
(i) Every α– τ_0 space is rgwαlc– τ_0 space.
(ii) Every α– τ_1 space is rgwαlc– τ_0 pace.
(iii) Every α– τ_1 space is rgwαlc– τ_1 space.
(iv) Every α– τ_2 space is rgwαlc– τ_2 space.
Proof: i) For each pair of distinct points x, y of X. Since α– τ_0 space, there exists a α– open sets G in X containing one of them and not the other. But every α–open is rgwαlc– open then there exists a α–open sets G in X containing one of them and not the other. Therefore rgwαlc– τ_0 space.
ii) Since α– τ_1 space, but every α– τ_1 space is α– τ_0 space and also from Theorem 5.5(i). Therefore rgwαlc– τ_0 space.
iii) and (iv) similarly we can prove.

Theorem 3.6: Let X be a topological space and Y is an rgwαlc– τ_0 space. If f: X → Y is injective and rgwαlc–irresolute then X is rgwαlc– τ_0 space.
Proof: Suppose x, y∈X such that x≠y. Since f is injective then f(x)≠f(y). Since Y is rgwαlc– τ_0 space then there exists a rgwαlc–open sets U in Y such that f(x)∈U , f(y) U or there exists a rgwαlc–open sets V in Y such that f(y)∈V , f(x) V with f(x)≠f(y). Since f is rgwαlc– irresolute then f^{-1}(U) is a rgwαlc–open sets in X such that x∈f^{-1}(U), y f^{-1}(U) or f^{-1}(V) is a rgwαlc–open sets in X such that y∈f^{-1}(V), x f^{-1}(V). Hence X is rgwαlc– τ_0 space.

Theorem 3.7: Let X be a topological space and Y is an rgwαlc– τ_2 space. If f: X → Y is injective and rgwαlc–irresolute then X is rgwαlc– τ_2 space.
Proof: Suppose x, y∈X such that x≠y. Since f is injective then f(x)≠f(y). Since Y is rgwαlc– τ_2 space then there are two rgwαlc–open sets U and V in Y such that f(x)∈U , f(y)∈V and U∩V=Φ. Since f is rgwαlc– irresolute then f^{-1}(U), f^{-1}(V) are two rgwαlc– open sets in X, x∈f^{-1}(U), y∈f^{-1}(V), f^{-1}(U)∩f^{-1}(V) =Φ. Hence X is rgwαlc– τ_2 space.

Theorem 3.8: Let X be a topological space and Y is an rgwαlc– τ_1 space. If f: X → Y is injective and rgwαlc–irresolute then X is rgwαlc– τ_1 space.
Proof: Similarly to Theorem 3.7.

Theorem 3.9: Let X be a topological space and Y is an τ_2 space. If f: X → Y is injective and rgwαlc– continuous then X is rgwαlc– τ_2 space.
Proof: Suppose x, y∈X such that x≠y. Since f is injective, then f(x)≠f(y). Since Y is an τ_2 space, then there are two open sets U and V in Y such that f(x)∈U, f(y)∈V and U∩V =Φ. Since f is rgwαlc– continuous then f^{-1}(U), f^{-1}(V) are two rgwαlc– open sets in X. Then x∈f^{-1}(U), y∈f^{-1}(V), f^{-1}(U)∩f^{-1}(V) =Φ. Hence X is rgwαlc– τ_2 space.

Theorem 3.10: (X, τ) is rgwαlc– τ_0 space if and only if for each pair of distinct x, y of X, rgwαlc–cl({x}) ≠ rgwαlc-cl({y}) .
Proof: Let (X,τ) be a rgwαlc– τ_0 space. Let x, y ∈ X such that x≠y, then there exists a rgwαlc– open set V containing one of the points but not the other, say x∈V and y∉ V. Then V^c is a rgwαlc–closed containing y but not x. But rgwαlc–cl({y}) is the smallest rgwαlc–closed set containing y. Therefore rgwαlc–cl({y})⊂V^c and hence x∉rgwαlc– cl({y}). Thus rgwαlc–cl({x}) ≠ rgwαlc–cl({y}).
Conversely, suppose x, y ∈ X, x≠ y and rgwαlc–cl({x}) ≠ rgwαlc–cl({y}). Let z∈ X such that z ∈rgwαlc–cl({x}) but z ∉rgwαlc–cl({y}). If x ∈rgwαlc–cl({y}) then rgwαlc–cl({x}) ⊂rgwαlc–cl({y}) and hence z∈rgwαlc–cl({y}). This is a contradiction. Therefore x∉rgwαlc–cl({y}). That is x ∈ $(\text{rgwαlc–cl(y)})^c$. Therefore $(\text{rgwαlc–cl(\{y\})})^c$ is a rgwαlc– open set containing x but not y. Hence (X,τ) is rgwαlc– τ_0 space.

Theorem 3.11: A topological space X is rgwαlc– τ_1 space if and only if for every x∈ X singleton {x} is rgwαlc–closed set in X.
Proof: Let X be rgwαlc– τ_1 space and let x∈X, to prove that {x} is rgwαlc–closed set. We will prove X– {x} is rgwαlc- open set in X. Let y ∈ X–{x}, implies x≠y ∈ and since X is rgwαlc– τ_1 space then their exit two rgwαlc–open sets G1, G2 such that x∉G1, y∈ G2 ⊆ X–{x}. Since y∈ G2 ⊆ X–{x} then X–{x} is rgwαlc– open set. Hence {x} is rgwαlc–closed set.

Conversely, Let x≠y ∈X then {x}, {y} are rgwαlc– closed sets. That is X–{x} is rgwαlc–open set. Clearly, x∉ X–{x} and y∈ X–{x}.Similarly X–{y} is rgwαlc– open set, y∉X–{y} and x∈X–{y}. Hence X is rgwαlc– τ_1 space.

Theorem 3.12: For a topological space (X, τ), the following are equivalent
(i) (X, τ) is rgwαlc– τ_2 space.
(ii) If x∈X , then for each y≠ x , there is a rgwαlc–open set U containing x such that y∉rgwαlc–cl(U)
Proof: (i)⟹(ii) Let x∈X. If y∈X is such that y≠x there exists disjoint rgwαlc–open sets U and V such that x∈U and y∈V. Then x∈U ⊂X–V which implies X–V is rgwαlc– open and y∉X–V. Therefore y∉rgwαlc–cl(U).
(ii) ⟹(i) Let x, y∈ X and x ≠y. By (ii), there exists a rgwαlc– open U containing x such that y∉rgwαlc–cl(U). Therefore y∈ X–(rgwαlc–cl(U)). X–(rgwαlc–cl(U)) is rgwαlc–open and x∉X– (rgwαlc–cl(U)). Also U∩X–(rgwαlc– cl(U))= Φ. Hence (X,τ) is rgwαlc– τ_2 space.

rgwαlc- Regular Space:-
In this section, we introduce a new class of spaces called rgwαlc-regular spaces using rgwαlc-closed sets and obtain some of their characterizations.

Definition 4.1: A topological space X is said to be **rgwαlc-regular** if for each rgwαlc-closed set F and a point x∉F, there exist disjoint open sets G and H such that F ⊆ G and x ∈H.
We have the following interrelationship between rgwαlc –regularity and regularity.

Theorem 4.2: Every rgwαlc-regular space is regular.
Proof: Let X be a rgwαlc-regular space. Let F be any closed set in X and a point x∈X such that x∉F. By [2], F is rgwαlc-closed and x∉F. Since X is a rgwαlc-regular space, there exists a pair of disjoint open sets G and H such that F ⊆ G and x∈H. Hence X is a regular space.

Theorem 4.3: If X is a regular space and τrgwα– space, then X is rgwαlc- regular.
Proof: Let X be a regular space and τrgwα– space. Let F be any rgwαlc –closed set in X and a point x∈X such that x∉F. Since X is τrgwα– space, F is closed and x∉F. Since X is a regular space, there exists a pair of disjoint open sets G and H such that F ⊆ G and x∈H. Hence X is a rgwαlc–regular space

Theorem 4.4: Every rgwαlc–regular space is α–regular.
Proof: Let X be a rgwαlc –regular space. Let F be any α–closed set in X and a point x∈X such that x∉F. By [2], F is rgwαlc–closed and x∉F. Since X is a rgwαlc–regular space, there exists a pair of disjoint open sets G and H such that F ⊆ G and x∈H. Hence X is a α- regular space.
We have the following characterization.

Theorem 4.5: The following statements are equivalent for a topological space X
i) X is a rgwαlc–regular space.
ii) For each x∈X and each rgwαlc open neighbourhood U of x there exists an open neighbourhood N of x such that cl(N) ⊆ U.
Proof: (i) => (ii): Suppose X is a rgwαlc –regular space. Let U be any rgwαlc – neighbourhood of x. Then there exists rgwαlc -open set G such that x∈G ⊆ U. Now X – G is rgwαlc –closed set and x∉ X – G. Since X is rgwαlc –regular, there exist open sets M and N such that X – G ⊆ M , x∈N and M ∩N = ϕ and so N ⊆ X –M. Now cl(N) ⊆ cl (X – M) = X –M and X – G⊆ M. This implies X – M ⊆ G ⊆ U. Therefore cl(N) ⊆ U.
(ii) => (i): Let F be any rgwαlc– closed set in X and x∉ F or x∈X –F and X –F is a rgwαlc–open and so X –F is a rgwαlc – neighbourhood of x. By hypothesis, there exists an open neighbourhood N of x such that x∈ N and cl(N) ⊆ X –F. This implies F ⊆ X – cl(N) is an open set containing F and N ∩ {(X –cl(N)} = ϕ. Hence X is rgwαlc – regular space.
We have another characterization of rgwαlc – regularity in the following.

Theorem 4.6: A topological space X is rgwαlc –regular if and only if for each rgwαlc –closed set F of X and each x∈ X –F there exist open sets G and H of X such that x∈ G, F ⊆ H and cl(G) ∩ cl(H) = ϕ.
Proof: Suppose X is rgwαlc – regular space. Let F be a rgwαlc –closed set in X with x∉F. Then there exists open sets M and H of X such that x∈ M, F ⊆ H and M ∩H = ϕ. This implies M ∩ cl(H) = ϕ. As X is rgwαlc –regular,

there exist open sets U and V such that x∈U, cl(H) ⊆ V and U ∩ V = Φ, so cl(U) ∩ V = Φ. Let G = M ∩ U, then G and H are open sets of X such that x ∈G, F ⊆ H and cl(H) ∩ cl(H) = ϕ.
Conversely, if for each rgwαlc –closed set F of X and each x ∈ X– F there exists open sets
G and H such that x ∈ G, F ⊆ H and cl(H) ∩ cl(H) = Φ. This implies x ∈ G, F ⊆ H and G ∩ H = ϕ. Hence X is rgwαlc – regular.
Now we prove that rgwαlc –regularity is a hereditary property.

Theorem 4.7: Every subspace of a rgwαlc- regular space is rgwαlc –regular.
Proof: Let X be a rgwαlc –regular space. Let Y be a subspace of X. Let x∈Y and F be a rgwαlc– closed set in Y such that x∉F. Then there is a closed set and so rgwαlc –closed set A of X with F = Y ∩ A and x∉A. Therefore we have x∈X, A is rgwαlc–closed in X such that x∉A. Since X is rgwαlc–regular, there exist open sets G and H such that x∈G, A ⊆ H and G ∩ H = Φ. Note that Y ∩ G and Y ∩ H are open sets in Y. Also x∈G and x∈Y, which implies x∈Y ∩ G and A ⊆ H implies Y ∩ A ⊆ Y ∩ H, F ⊆ Y ∩ H. Also (Y ∩ G) ∩ (Y ∩ H) = Φ. Hence Y is rgwαlc–regular space.
We have yet another characterization of rgwαlc–regularity in the following.

Theorem 4.8: The following statements about a topological space X are equivalent:
(i) X is rgwαlc–regular
(ii) For each x∈X and each rgwαlc–open set U in X such that x∈U there exists an open set V in X such that x∈V ⊆ cl(V) ⊆ U
(iii) For each point x∈X and for each rgwαlc–closed set A with x∉A, there exists an open set V containing x such that cl(V) ∩ A = Φ.
Proof: (i)=> (ii): Follows from Theorem 3.5.
(ii) => (iii): Suppose (ii) holds. Let x∈X and A be an rgwαlc– closed set of X such that x∉A. Then X –A is a rgwαlc–open set with x∈X –A. By hypothesis, there exists an open set V such that x∈V ⊆ cl(V) ⊆ X –A. That is x∈V,V⊆ cl(A) and cl(A) ⊆ X –A. So x∈V and cl(V)∩ A= Φ.
(iii) => (ii): Let x∈X and U be an rgwαlc–open set in X such that x∈U. Then X –U is an rgwαlc –closed set and x∈X –U. Then by hypothesis, there exists an open set V containing x such that cl(V) ∩ (X –U) = Φ. Therefore x∈V, cl(V) ⊆ U so x∈V ⊆ cl(V) ⊆ U.
The invariance of rgwαlc– regularity is given in the following.

Theorem 4.9: Let f: X → Y be a bijective, rgwαlc –irresolute and open map from a rgwαlc– regular space X into a topological space Y, then Y is rgwαlc–regular.
Proof: Let y∈Y and F be a rgwαlc–closed set in Y with y∉F. Since f is rgwαlc–irresolute, $f^{-1}(F)$ is rgwαlc–closed set in X. Let f(x) = y so that x = $f^{-1}(y)$ and x∉$f^{-1}(F)$. Again X is rgwαlc–regular space, there exist open sets U and V such that x∈U and f^{-1} (F) ⊆ G, U ∩ V = Φ. Since f is open and bijective, we have y∈f(U) , F ⊆ f(V) and f(U) ∩ f(V) = f(U ∩ V) = f(Φ) = Φ. Hence Y is rgwαlc–regular space.

Theorem 4.10: Let f: X → Y be a bijective, rgwαlc –closed map from a topological space X into a rgwαlc–regular space Y. If X is τrgwα–space, then X is rgwαlc–regular.
Proof: Let x ϵ X and F be an rgwαlc–closed set in X with x∉F. Since X is τrgwα–space, F is closed in X. Then f(F) is rgwαlc–closed set with f(x)∉f(F) in Y , since f is rgwαlc–closed. As Y is rgwαlc–regular, there exist disjoint open sets U and V such that f(x)∈ U and f(F) ⊆ V . Therefore x∈f^{-1}(U) and F ⊆ f^{-1} (V). Hence X is rgwαlc–regular space.

Theorem 4.11: Let X be a topological space. If X is a rgwαlc–regular and a τ_1 space then X is an rgwαlc– τ_2 space.
Proof: Suppose x, y X such that x≠y. Since X is τ_1 – space then there is an open set U such that x∈ U, y∉U. Since X is rgwαlc–regular space and U is an open set which contains x, then there is rgwαlc–open set V such that x∈V⊂rgwαlc–cl(V)⊆U. Since y U, hence y rgwαlc–cl(V). Therefore y∈X–(rgwαlc–cl(V)). Hence there are rgwαlc–open sets V and X–(rgwαlc–cl(V)) such that (X– (rgwαlc–cl(V)))∩V = Φ. Hence X is rgwαlc– τ_2 space.

rgwαlc–Normal Spaces;-
In this section, we introduce the concept of rgwαlc– normal spaces and study some of their characterizations.
Definition 5.1: A topological space X is said to be **rgwαlc–normal** if for each pair of disjoint rgwαlc– closed sets A and B in X, there exists a pair of disjoint open sets U and V in X such that A ⊆ U and B ⊆ V .
We have the following interrelationship.

Theorem 5.2: Every rgwαlc–normal space is normal.
Proof: Let X be a rgwαlc–normal space. Let A and B be a pair of disjoint closed sets in X. From [2], A and B are rgwαlc–closed sets in X. Since X is rgwαlc–normal, there exists a pair of disjoint open sets G and H in X such that A ⊆ G and B ⊆ H. Hence X is normal.

Remark 5.3: The converse need not be true in general as seen from the following example.

Example 5.4: Let Let X={a,b,c}, τ= {X, ϕ , {a},{b},{a,b}}. Then the space X is normal but not rgwαlc –normal, since the pair of disjoint rgwαlc–closed sets namely, A = {b} and B = {c} for which there do not exists disjoint open sets G and H such that A ⊆ G and B ⊆ H.

Theorem 5.5: If X is normal and τrgwα–space, then X is rgwαlc–normal.
Proof: Let X be a normal space. Let A and B be a pair of disjoint rgwαlc–closed sets in X. since τrgwα–space, A and B are closed sets in X. Since X is normal, there exists a pair of disjoint open sets G and H in X such that A ⊆ G and B ⊆ H. Hence X is rgwαlc– normal.

Theorem 5.6: Every rgwαlc–normal space is w–normal.
Proof: Let X be a rgwαlc–normal space. Let A and B be a pair of disjoint w–closed sets in X. From [2], A and B are rgwαlc-closed sets in X. Since X is rgwαlc–normal, there exists a pair of disjoint open sets G and H in X such that A ⊆ G and B ⊆ H. Hence X is w– normal.
Hereditary property of rgwαlc–normality is given in the following.

Theorem 5.7: A rgwαlc–closed subspace of a rgwαlc–normal space is rgwαlc–normal.
Proof: Let X a be rgwαlc–normal space. Let Y be a rgwαlc-closed subspace of X. Let A and B be pair of disjoint rgwαlc–closed sets in Y. Then A and B be pair of disjoint rgwαlc –closed sets in X. Since X is rgwαlc–normal, there exist disjoint open sets G and H in X such that A ⊆ G and B ⊆ H. Since G and H are open in X, Y ∩ G and Y ∩ H are open in Y. Also we have A ⊆ G and B ⊆ H implies Y ∩ A ⊆ Y ∩ G, Y ∩ B ⊆ Y ∩ H . So A ⊆ Y ∩ G and B ⊆ Y ∩ H and (Y ∩ G) ∩ (Y ∩ H) = Y ∩ (G∩ H) = ϕ. Hence Y is rgwαlc -normal.
We have the following characterization.

Theorem 5.8: The following statements for a topological space X are equivalent:
i) X is rgwαlc–normal.
ii) For each rgwαlc–closed set A and each rgwαlc–open set U such that A ⊆ U, there exists an open set V such that A ⊆ V ⊆ cl(V) ⊆ U
iii) For any disjoint rgwαlc–closed sets A, B, there exists an open set V such that A ⊆ V and cl(V) ∩ B = Φ
iv) For each pair A, B of disjoint rgwαlc–closed sets there exist open sets U and V such that A ⊆ U, B ⊆ V and cl(U) ∩ cl(V) = Φ.
Proof: (i) => (ii): Let A be a rgwαlc–closed set and U be a rgwαlc–open set such that A ⊆ U. Then A and X –U are disjoint rgwαlc–closed sets in X. Since X is rgwαlc–normal , there exists a pair of disjoint open sets V and W in X such that A ⊆ V and X –U ⊆ W. Now X –W ⊆ X – (X –U), so X –W ⊆ U also V ∩W = Φ implies V ⊆ X –W, so cl (V) ⊆ cl(X –W) which implies cl(V) ⊆ X – W. Therefore cl(V) ⊆ X –W ⊆ U. So cl (V) ⊆ U. Hence A ⊆ V ⊆ cl(V) ⊆ U.
(ii)=>(iii): Let A and B be a pair of disjoint rgwαlc– closed sets in X. Now A∩B = Φ, so A ⊆ X –B, where A is rgwαlc–closed and X – B is rgwαlc–open . Then by (ii) there exists an open set V such that A ⊆ V ⊆ cl(V)⊆ X –B. Now cl(V) ⊆ X –B implies cl(V) ∩ B = Φ. Thus A ⊆ V and cl(V) ∩ B = Φ
(iii) =>(iv): Let A and B be a pair of disjoint rgwαlc-closed sets in X. Then from (iii) there exists an open set U such that A ⊆ U and cl(U)∩B = Φ. Since cl(V) is closed, so rgwαlc–closed set. Therefore cl(V) and B are disjoint rgwαlc–closed sets in X. By hypothesis, there exists an open set V, such that B ⊆ V and cl(U) ∩ cl(V) = Φ.
(iv) => (i): Let A and B be a pair of disjoint rgwαlc–closed sets in X. Then from (iv) there exist an open sets U and V in X such that A ⊆ U, B ⊆ V and cl(U)∩cl(V) = Φ. So A ⊆ U, B ⊆ V and U ∩ V = Φ. Hence X rgwαlc– normal.

Theorem 5.9: Let X be a topological space. Then X is rgwαlc–normal if and only if for any pair A, B of disjoint rgwαlc–closed sets there exist open sets U and V of X such that A ⊆U, B ⊆V and cl(U)∩cl(V)= Φ.
Proof: Follows from Theorem 5.8.

Theorem 5.10: Let X be a topological space. Then the following are equivalent:
(i) X is normal
(ii) For any disjoint closed sets A and B, there exist disjoint rgwαlc–open sets U and V such that A ⊆ U, B ⊆ V.
(iii) For any closed set A and any open set V such that A ⊆ V , there exists an rgwαlc–open set U of X such that A ⊆ U ⊆ rαcl(U) ⊆ V.
Proof: (i) =>(ii): Suppose X is normal. Since every open set is rgwαlc–open [2], (ii) follows.
(ii)=>(iii): Suppose (ii) holds. Let A be a closed set and V be an open set containing A. Then A and X –V are disjoint closed sets. By (ii), there exist disjoint rgwαlc–open sets U and W such that A ⊆ U and X–V ⊆ W, since X –V is closed, so rgwαlc–closed. From Theorem 2.3.14 [2], we have X –V ⊆ rαint(W) and U ∩ rαint(W) = Φ and so we have cl(U) ∩ rαint(W) = Φ. Hence A ⊆ U ⊆ rαcl(U) ⊆ X –rαint(W) ⊆ V . Thus A ⊆ U ⊆ rαcl(U) ⊆V.
(iii) =>(i): Let A and B be a pair of disjoint closed sets of X. Then A ⊆ X–B and X–B is open. There exists a rgwαlc-open set G of X such that A ⊆ G ⊆ rαcl(G) ⊆ X–B. Since A is closed, it is rgwαlc–closed, we have A ⊆ int(G). Take U = int(cl(int(rαint(G)))) and V = int(cl(int(X rαcl(G)))). Then U and V are disjoint open sets of X such that A ⊆ U and B ⊆ V. Hence X is normal.

Theorem 5.11: If f: X → Y is bijective, open, rgwαlc–irresolute from a rgwαlc–normal space X onto Y then is rgwαlc–normal.
Proof: Let A and B be disjoint rgwαlc–closed sets in Y. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint rgwαlc –closed sets in X as f is rgwαlc–irresolute. Since X is rgwαlc–normal, there exist disjoint open sets G and H in X such that $f^{-1}(A)$ ⊆ G and $f^{-1}(B)$ ⊆ H. As f is bijective and open, f(G) and f(H) are disjoint open sets in Y such that A ⊆ f(G) and B ⊆ f(H). Hence Y is rgwαlc–normal.

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