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RESEARCH ARTICLE

Stress Distribution between Gauss Quadrature Method and Alternate Method for Functionally Graded Material.

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Abstract

The present study aims to compare the nodal stresses for Gauss Quadrature method and alternate method. The alternate method represents the corner point and midpoint of a quadrilateral as a sampling points of the element. It is extrapolated using Richardson extrapolation in the finite element analysis. To investigate the influence of material property variation like Young's modulus, numerical example for the family of quadrilateral element (Q4 and Q8) are solved and compare the results of functionally graded elements with the homogeneous elements. The result of nodal stresses of the alternate method are compared to the conventional Gauss Quadrature method.

Introduction: -

Functionally Graded Material (FGM) is one of the most widely used materials because the material constituents are changedfor different situation. It is a composite in which the material property varying as a functional form. The overall properties of FGM are unique and different from other. It eliminates the sharp interfaces existing in graded material where failure occurs. It replaces this sharp interface with a gradient interface to transmit from one material to the other [1]. Most structural components used in the field of engineering.

FGM are divided into two groups namely thin and bulk. Thin FGM are thin sections or thin surface coating.Tokita et al [2] discussed about bulk FGM are volume of materials but more labour intensive processes are required. The various process of technique for thin FGM is produced by physical vapour deposition (PVD), chemical vapour deposition (CVD), and plasma spraying. Bulk FGM is produced by powder technology, centrifugal casting, solid freeform technology and spark plasma sintering (SPS) technique may be modeled as an isotropic material [2]. It has been used extensively in gas turbine engines and rocket nozzles. The graded elements are obtained with the isoparametric formulation are compared with conventional homogeneous elements [4].

The alternate method is introduced in the finite element analysis. It represents the corner point and midpoint are taken as a sampling point and then extrapolate using Richardson extrapolation. Richardson extrapolation combined with Runga-kutta method for convergence of numerical problems [6]. In this present work focus is on family of Quadrilateral element to get the result using Richardson Extrapolation. Shyy and Garbey [7] discussed about the least square extrapolation method for improving solution accuracy of PDE. The required number of sampling points depend upon the order of approximation function results are sampled. This paper explained about the family of quadrilateral element using Gauss method and alternate method in Functionally Graded Material is validated by solving a numerical problem.

Formulation: -

The general stiffness matrix for the family of Quadrilateral element

$$[K] = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]|J|\, t d\xi d\eta \qquad (1)$$

where $[B]$ is the strain-displacement shape function derivatives, $[D]$ is the stress-strain or material matrix, $|J|$ is the determinant of Jacobian matrix,T is the transpose,ξ and η are the natural coordinates.The nodal displacement can be calculated by

$$[K]\{u\} = \{F\} \qquad (2)$$

where {u} is the displacement vector, {F} is the applied force.
The element stress can be calculated by

$$\{\sigma\} = [D][B]\{u\} \qquad (3)$$

The material property variation like Young's modulus and Poisson's ratio in FGM and HGM are

$$E = \sum_{i=1}^{n} N_i E_i \ , \ \ \nu = \sum_{i=1}^{n} N_i \nu_i \qquad (4)$$

$$E = \sum_{i=1}^{n} E_i \ , \ \ \nu = \sum_{i=1}^{n} \nu_i \qquad (5)$$

where N_i is the shape function of an element corresponding to the node i, and n is the number of nodal points of an element.

A. Gauss Quadrature method

The stiffness matrix can be obtained by using Gauss Quadrature rule,

$$[\mathrm{K}] = \sum_{\mathrm{k}=1}^{\mathrm{m}}\sum_{\mathrm{l}=1}^{\mathrm{m}}([\mathrm{B}]^{\mathrm{T}}[\mathrm{D}][\mathrm{B}]|\mathrm{J}|\ \mathrm{t})\ \mathrm{w_k w_l} \qquad (6)$$

where w_k and w_l are the weights of the Gauss points, m is the number of gauss points for each element [5].The element stress can be calculated for each Gauss nodes of an element and then extrapolate into the corner nodes of an element.

B. *Alternate method*

The corner point and midpoint of an element are taken as a sampling point and the weights can calculate by the sampling points. The complete cubic polynomial function to get the weights of the respective sampling points.

$$f(\xi\ ,\eta) = a_1 + a_2\xi + a_3\eta + a_4\xi^2 + a_5\xi\eta + a_6\eta^2 + a_7\xi^3 + a_8\xi^2\eta + a_9\xi\eta^2 + a_{10}\eta^3 \qquad (7)$$

The weights can be calculating by the function,

$$\int_{-1}^{1}\int_{-1}^{1} f(\xi,\eta)\, d\xi d\eta = w_1 f(\xi_1,\eta_1) + w_2 f(\xi_2,\eta_2) + w_3 f(\xi_3,\eta_3) + w_4 f(\xi_4,\eta_4) + w_5 f(\xi_5,\eta_5) \qquad (8)$$

By integrating the cubic function,

$$\int_{-1}^{1}\int_{-1}^{1} f(\xi,\eta)\, d\xi d\eta = 4a_1 + \left(\frac{4}{3}\right)a_4 + \left(\frac{4}{3}\right)a_6 \qquad (9)$$

The alternate sampling point are (-1, -1), (-1, 1), (1, 1), (1, -1), (0, 0) and to attain the weights w_1, w_2, w_3, w_4, w_5 respectively.

$$w_1 f(-1,-1) = w_1(-a_1 + a_2 + a_3 - a_4 - a_5 - a_6 + a_7 + a_8 + a_9 + a_{10}) (10)$$

$$w_2 f(1,-1) = w_2(a_1 + a_2 - a_3 + a_4 - a_5 + a_6 + a_7 - a_8 + a_9 - a_{10}) \qquad (11)$$

$$w_3 f(1,1) = w_3(a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10}) \qquad (12)$$

$$w_4 f(-1,1) = w_4(a_1 - a_2 + a_3 + a_4 + a_5 + a_6 - a_7 + a_8 - a_9 + a_{10}) \quad (13)$$

$$w_5 f(0,0) = w_5 a_1 \quad (14)$$

By substituting (10) to (14) in (8) and solving the equation. The weights are$w_1 = w_2 = w_3 = w_4 = \frac{1}{3}$ and $w_5 = \frac{8}{3}$ .Here the nodal stresses can be calculated using corner points itself. For Q8 and Q9 elements, the sampling points are taken from the midpoint of the corner node.

I. PROBLEM

Kim and Paulino et al [4] shows figure 1 as an isotropic FGM plate with theYoung's modulusvary along the x direction for both exponential and linear graded material. The Poisson's ratio is constant $\gamma = 0.3$. Assume $E_1 = 1$, $E_2 = 8$.

$$E_1 = E_0 = E(0) to E_2 = E(W) \quad (15)$$

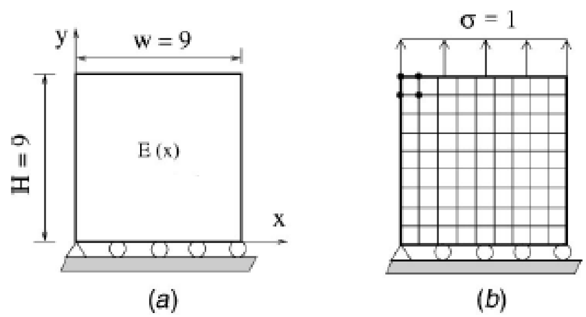

Figure 1: (a) Geometry and boundary condition; (b) Geometry meshed with 9X9 element and tension load applied perpendicular to material gradation

1. Exponential function

$$E(x) = E_0 e^{\beta x} \quad (16)$$

$$\beta = \frac{1}{W} log\left[\frac{E(W)}{E(0)}\right] \quad (17)$$

2. Linear function

$$E(x) = E_0 + \gamma x \quad (18)$$

$$\gamma = \frac{E(W) - E(0)}{W} \quad (19)$$

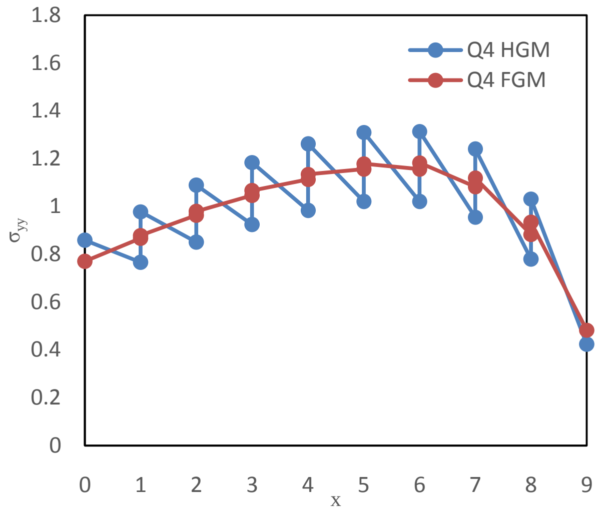

Figure 2: Stress distribution (σ_{yy}) of Q4 element for exponential material gradation using Gauss method

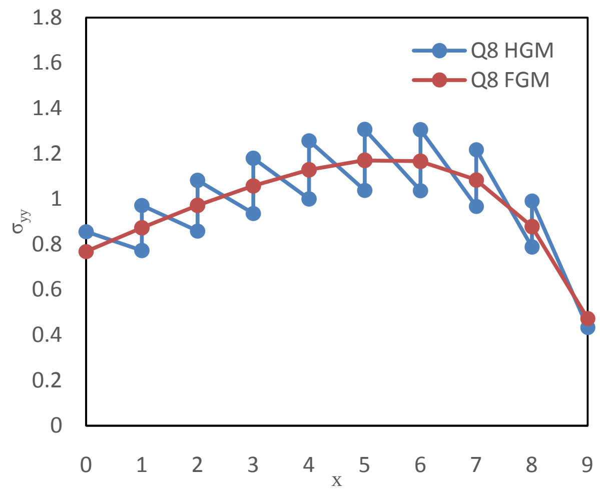

Figure 3: Stress distribution (σ_{yy}) of Q8 element for exponential material gradation using Gauss method

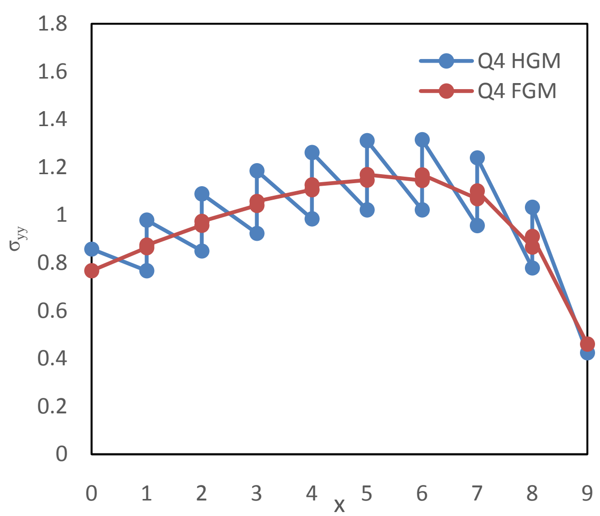

Figure 4: Stress distribution (σ_{yy}) of Q4 element for linear material gradation using Gauss method

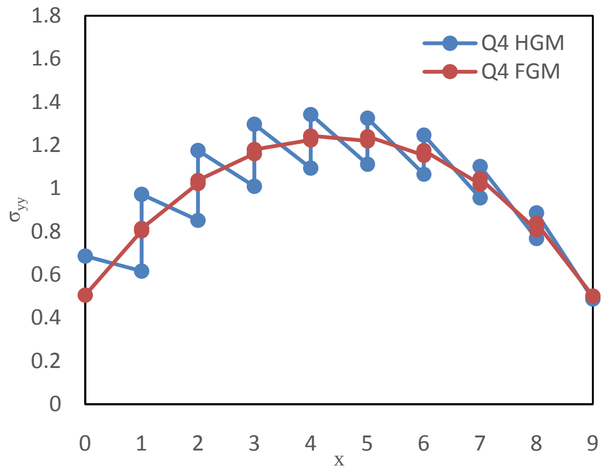

Figure 5:Stress distribution (σ_{yy}) of Q4 element for exponential material gradation using alternate method

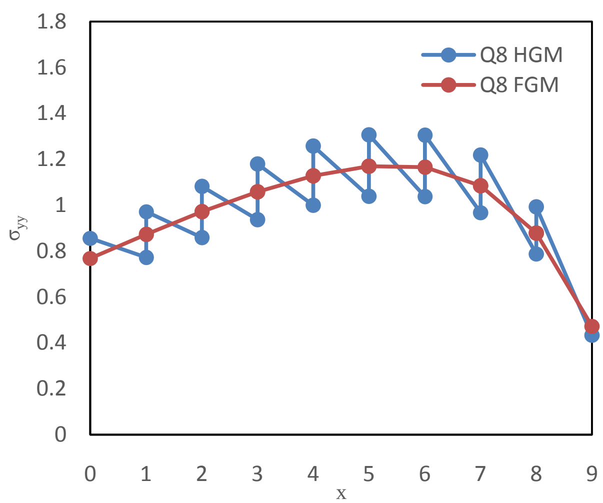

Figure 6: Stress distribution (σ_{yy}) of Q8 element for exponential material gradation using alternate method

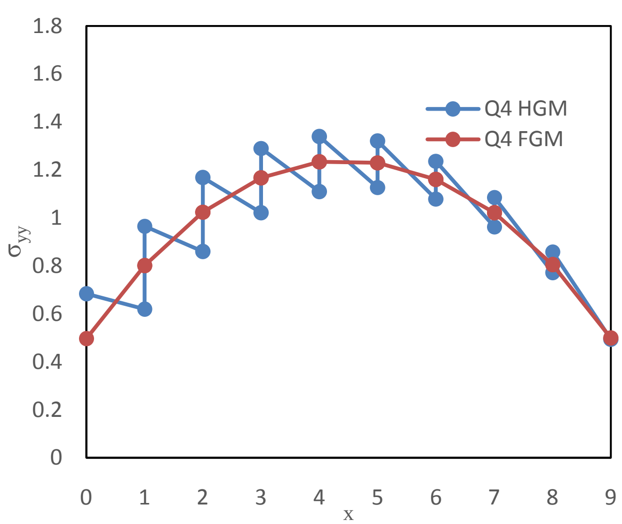

Figure 7: Stress distribution (σ_{yy}) of Q4 element for linear material gradation using alternate method

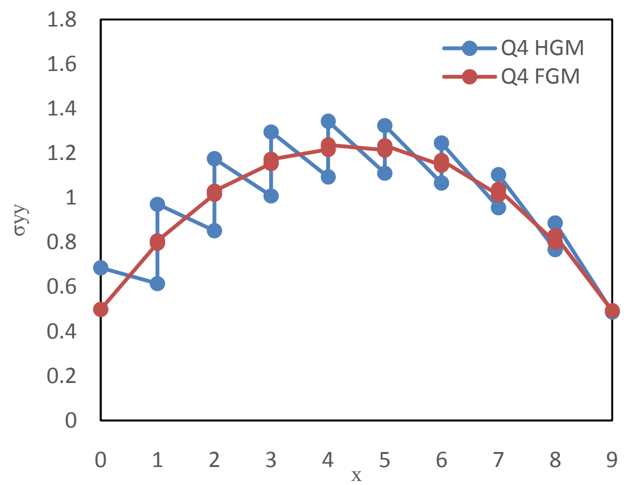

Figure 8: Stress distribution (σ_{yy}) of Q8 element for linear material gradation using Gauss method

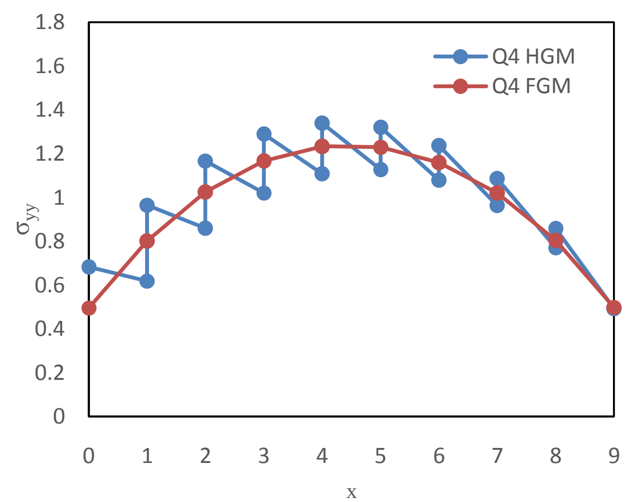

Figure 9: Stress distribution (σ_{yy}) of Q8 element for linear material gradation using alternate method

Results and discussion: -

The results can be obtained by using MATLAB. Thegraphs are plotted (x vs σ_{yy}) above as shown in figure 2 to 9. Figure 2 and 3 shows the nodal stresses (σ_{yy}) ofQ4 and Q8 foreach node in HGM as a decreasing function of x for each element while the FGM is very close for each node because it varies as a function. Figure 4 and 8 shows the nodal stress for Q4 and Q8 elements of linear material variation. The graded elements are superior than the homogeneous elements. The nodal stresses of the alternate method are compared with the Gauss method. Figure 5, 6, 7 and 9 shows the nodal stress results of Q4 and Q8 elements for exponential and linear material variationis equal to the Gauss Quadrature method.

Conclusion: -

The graded elements for the family of quadrilateral element have been investigated. The material property variation like Young's modulus, both exponential and linear function are used in the graded elements have been considered and compared. Averaging the nodal stress values of the functionally graded element which would convert into a regular homogeneous element. The same process can be done for the alternate method i.e. corner points and midpoint of the element as a sampling points have been calculated and compared with the conventional Gauss method. The results of nodal stresses of the alternate method is equal to the conventional Gauss Quadrature method.

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