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## RESEARCH ARTICLE

### WEIGHTED LEAST SQUARE SAR AUTOFOCUSING TECHNIQUE (WLS).

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#### *Abstract*

The Weighted Least Squares (WLS) autofocus algorithm has been widely used in spotlight Synthetic Aperture Radar (SAR) for phase error estimation to compensate motion-induced blurs in the images. The WLS autofocus algorithm has proven to be a superior method for higher order autofocus because it can be used to estimate all kinds of phase errors, no matter whether they are of low order, high order or random. Compared with other methods, the WLS estimation is optimal in the scene that it has the minimum variance of the estimated error. In this paper, a proposed PC-based software model of the WLS autofocus algorithm is introduced. Simulation results between blurred and focused images, or between the input phase error function and the estimated one carried out by calculating the root mean square (rms) value of the residual phase error, show significant removal of high-order phase errors and better removal of the low-order phase errors. Finally, a suggested Graphical User Interface (GUI) of the proposed algorithm for easier handling on PC-based equipment is introduced.

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#### **Introduction:-**

The Synthetic aperture radar (SAR) has the capability of producing high-resolution images in all weather conditions. It is rapidly becoming a key technology in modern remote-sensing applications. SAR is a coherent imaging system, which means that it is necessary to maintain the correct and precise phase relationship between return signals in order to perform coherent summation [1]. This requires precise motion compensation (MOCO) because uncompensated motion between the SAR antenna phase center and the scene being imaged causes phase errors that may blur the image severely. Even in a SAR equipped with modern electronic navigation systems, determining the platform position to the required tolerance over the entire synthetic aperture can prove to be a difficult task. This is especially true for the high-resolution imaging system since a long synthetic aperture is required. So it is necessary to adopt a data-driven autofocus technique to eliminate these phase errors. One technique that has been used to focus spotlight images is the WLS autofocus algorithm [7].

#### **WLS Fundamentals:-**

An autofocus algorithm tries to estimate the unknown phase error function (PEF) by using the radar data themselves. The WLS algorithm estimates the phase error for each azimuth position along the synthetic aperture [3]. This technique is flexible in the sense that there is no limit to the order of the phase correction function. In principle, an arbitrary phase error curve can be estimated. In the literature it is claimed that this technique is quite robust and should give good results even with modest image contrast. Its basic premise is that all image points share the same underlying phase error, and that averaging the (residual) phase histories of many selected points will reveal the common error. WLS is an iterative technique that estimates the phase error from the data, applies this error to the data (corrects the phase) and again calculates the remaining phase error [4]. When the subsequent errors become smaller than a certain limit, the loop is ended and the PEF selected.

### WLS estimation of phase errors:-

WLS estimation algorithm begins in range-compressed phase history domain [5]. The received signal at the  $n^{\text{th}}$  range bin can be written as

$$F_n(m) = |F_n(m)| e^{j(\phi_n(m) + \phi_e(m))} \quad (1)$$

The subscript  $n$  refers to the range bin index, where  $m$  refers to azimuth line.  $|F_n(m)|$  and  $\phi_n(m)$  are the magnitude and phase of the range-compressed data for the  $n^{\text{th}}$  range bin respectively. The uncompensated phase error  $\phi_e(m)$  along the synthetic aperture is the same for all range bins of interest and independent of  $n$  [6].

Suppose that the strongest scatterer in the  $n^{\text{th}}$  range bin has a Doppler frequency  $f_n$  and an initial phase  $\psi_{0,n}$ . Other smaller scatterers are considered as clutters. The received signal then can be written as [2], where

$$F_n(m) = |F_n(m)| \exp\{j[2\pi f_n m + \psi_{0,n} + \alpha_n(m) + \phi_e(m)]\} \quad (2)$$

$2\pi f_n m$  Linear phase term caused by the Doppler frequency of the strongest scatterer.

$\psi_{0,n}$  Initial phase term (constant along the azimuth direction but has different values in each range bin).

$\alpha_n(m)$  Random phase caused by clutter in range bin  $n$  (it is a random disturbance for estimating the phase error).

$\phi_e(m)$  Phase error to be estimated.

$n, m$  Subscript  $n$  refers to the range bin index,  $m$  is the azimuth line index,  $n = 1, 2, 3, \dots, N$ ,  $m = 1, 2, 3, \dots, M$

The Fourier transform is used to transform the range-compressed phase history data into the image domain. The strongest scatterer in each range bin will be moved to the center of the image to remove the Doppler frequency offset of the scatterer. The initial phase  $\psi_{0,n}$  is a constant along the azimuth direction but has different values in each range bin. The  $\psi_{0,n}$  adds a random constant in each phase signal  $\phi_n(m)$  and makes them randomly distributed within  $[-\pi, \pi]$ .

To restore the original phase signal  $\phi_n(m)$ , phase unwrapping process as well as circular shift are applied for each range bin to remove the initial phase term  $\psi_{0,n}$  and Doppler frequency offset of the strongest scatterer respectively [2].

The phase of the received signal becomes,  $\Phi_n(m) = \alpha_n(m) + \phi_e(m)$  (3)

It is common to assume that the clutters in each range bin are independent, so the phase fluctuation  $\alpha_n(m)$  caused by the clutter is also independent from bin to bin. Also, as shown before, the phase error to be estimated  $\phi_e(m)$  is the same for all range bins, and it varies only with azimuth lines.

Since the phase fluctuation caused by the clutter  $\alpha_n(m)$  will cause errors in the estimation process, and the objective of this algorithm is to minimize the variance of the estimation process. The WLS estimation algorithm will be used to estimate the phase error  $\phi_e(m)$  from the received signal, rewriting (3) using vector notations,

$$\Phi(m) = \alpha(m) + H \phi_e(m)$$

Where

$$\Phi(m) = [\Phi_1(m), \Phi_2(m), \dots, \Phi_N(m)]^T, \alpha(m) = [\alpha_1(m), \alpha_2(m), \dots, \alpha_N(m)]^T, H = [1, 1, \dots, 1]^T \quad (4)$$

Denote the variance of the phase term  $\alpha_N(m)$  in the  $n$ -th range bin as  $\sigma_n^2$ . The covariance matrix of  $\alpha(m)$  can be expressed as,

$$\Sigma = \text{diag}[\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2]. \quad (5)$$

the WLS estimate of  $\phi_e(m)$  is, [2]

$$\hat{\phi}_{wls}(m) = (H^T \sum^{-1} H)^{-1} H^T \sum^{-1} \Phi(m) \quad (6)$$

substituting (4) & (5) into (6) then

$$\hat{\phi}_{wls}(m) = \sum_{n=1}^N w_n \cdot \Phi_n(m) \quad (7)$$

Equation (7) shows that the WLS estimate of  $\phi_e(m)$  is the weighted average of the phase signals  $\Phi_n(m)$ ,  $n = 1, 2, \dots, N$ .

The weights and the variance of the estimation error using WLS are calculated respectively as,

$$w_n = (1/\sigma_n^2) / \sum_{l=1}^N \frac{1}{\sigma_l^2} \quad \sigma_e^2 = (H^T \sum^{-1} H)^{-1} = \frac{1}{\sum_{n=1}^N \frac{1}{\sigma_n^2}}$$

So,  $\sigma_e^2$  is smaller than the phase variance  $\sigma_n^2$  of any range bin. The more range bins used, the smaller the value of  $\sigma_e^2$ . The WLS is an optimal estimation in the sense that it minimizes  $\sigma_e^2$  [2].

In the above analysis, there is no any assumption on the clutter model, the WLS estimation does not require the clutter to be of a certain model, so this estimation is very robust and can be applied to most kinds of scene contents.

### Calculation of weights:-

Equation 4-16 shows that the WLS estimate of  $\phi_e(m)$  is the weighted average of the phase signal  $\phi_n(m)$  of all range bins, that weight is inversely proportional to the variance of the phase term  $\alpha_n(m)$  caused by clutter in that bin  $\sigma_n^2$ .

Let the signal of the strongest scatterer in range bin  $n$  is simplified to  $f_0(t) = a_0$ , and the signal of clutter in this range bin is  $f_1(t) = \sum_i a_i e^{j\phi_i(t)}$  where  $(a_i \ll a_0)$ .

So the overall phase error-free signal is,  $f_n(t) = a_0 + \sum_i a_i e^{j\phi_i(t)} = A_n e^{j\phi_n(m)}$

The  $\sigma_n^2$  will be calculated using the following steps [2], denote  $\mu_c$  and  $\mu_d$  as the mean and the mean square value of the amplitude of the echo signal respectively, then

$$\mu_c = E[A_n(m)] = \frac{1}{M} \sum_{m=1}^M A_n(m) \text{ and } \mu_d = E[A_n^2(m)] = \frac{1}{M} \sum_{m=1}^M A_n^2(m) \\ R \approx \frac{1}{\mu_d} \left[ 4(2\mu_c^2 - \mu_d) - 4\mu_c \sqrt{4\mu_c^2 - 3\mu_d} \right] \quad (8)$$

where the variable  $R$  is the reciprocal of signal to clutter ratio (SCR) =  $\frac{1}{R} = \frac{a_0^2}{\sum_i a_i^2}$

The signal variance for  $n^{\text{th}}$  range bin is denoted as [2]  $\sigma_n^2 \approx \frac{1}{2} R + \frac{5}{24} R^2$  (9)

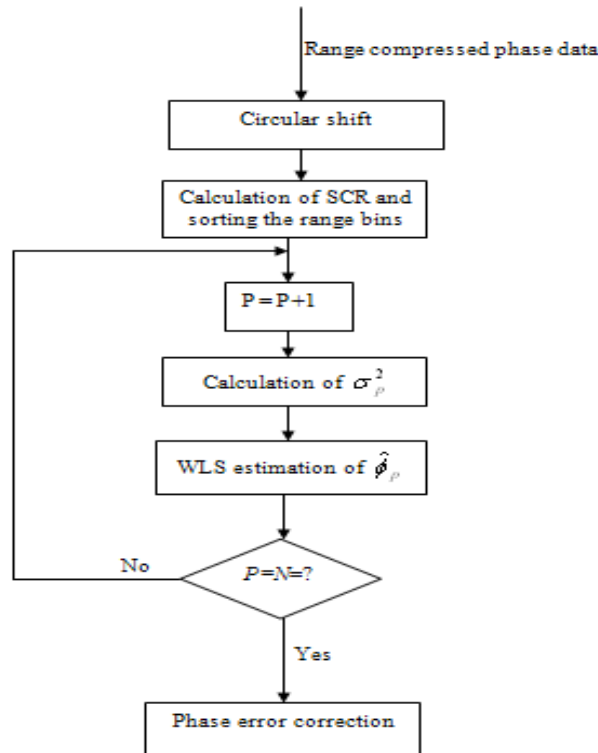
Many approximations of Taylor series expansions have been used in  $\sigma_n^2$  calculation. These approximations have small errors when the SCR is large. In order to testify the accuracy of this approach to estimate the  $\sigma_n^2$ , a Mont Carlo simulation is carried out for various SCR. The results show that for large and medium SCR (larger than 1 dB) the estimation error is small. But for very small SCR, the results will have a large estimation error and this approach tends to be inaccurate to estimate  $\sigma_n^2$ , when SCR is small (SCR smaller than 1 dB) the signal variance will be calculated by the following equation [2].

$$\sigma_n^2 = \frac{1}{M} \sum_{m=1}^M [\Phi_n(m) - \hat{\phi}_{e_{n-1}}(m)]^2 \quad (10)$$

The block diagram in Fig.1 outlines the procedure of the WLS autofocus [8]. The range bins after performing the circular shift, and phase unwrapped are sorted according to their SCR in descending order and they are included in the computations in that order one by one. The SCR for each range bin is tested, if it larger than 1 dB the phase variance  $\sigma_n^2$  will be calculated by (9) otherwise (10) will be used to calculate phase variance. Then the phase error

estimate of the  $P^{\text{th}}$  range bin and  $m^{\text{th}}$  azimuth line is calculated as,  $\hat{\phi}_p(m) = \sum_{n=1}^P [1/\sigma_n^2 / \sum_{l=1}^P \frac{1}{\sigma_l^2}] \Phi_n(m)$

This process is repeated until all range bins are used ( $p=N$ ).



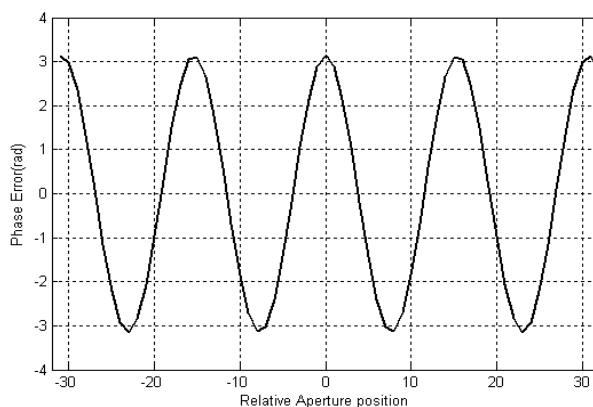
**Fig. 1:-** Block diagram of the WLS estimation Autofocus Algorithm

### Simulation Results:-

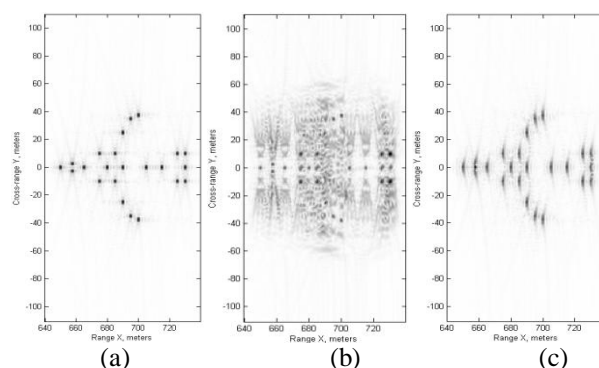
The performance of WLS autofocus algorithm is evaluated using a simulated SAR image shown in Fig. 3(a), the WLS algorithm was applied over the SAR signal that corrupted by the phase error function, shown in Fig.2.

The simulated SAR image are generated by using a MATLAB programs for pulsed spotlight SAR simulation and reconstruction, this simulated SAR image of area size (100m x 200m). Besides the visual comparison between the blurred and focused image, or between the input phase error function and the estimated one, the performances of autofocus algorithms are evaluated also by calculating the rms value of the residual phase error. Simulated SAR image are generated, it represents 23-point scatterers distributed in arbitrary manner as shown in Fig. 3(a). This image is corrupted by a multi-cycles sinusoidal phase error with amplitude  $1.5\pi$  rad peak-to-peak, (3.3322 rad rms) as shown in Fig.2.

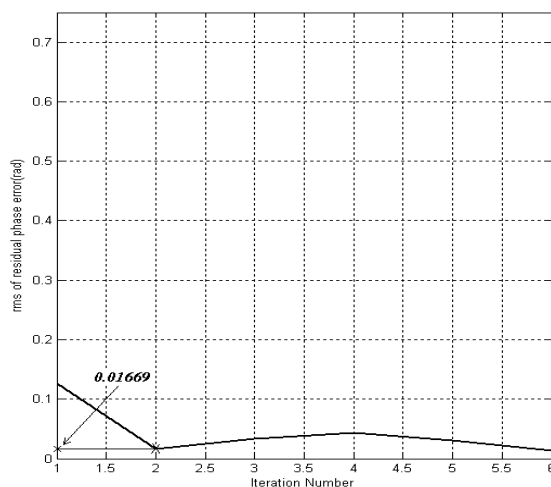
The blurred image is shown in Fig.3(b). WLS algorithm is used to focus this blurred image; Fig.4 shows the rms of the residual phase error versus the number of WLS iterations. It is quite clear that the WLS reaches a minimum rms after two iterations (0.01669 rad), and its performance will not improve after more iteration. Finally, Fig.3(c) shows the focused image obtained after two iterations of the WLS algorithm. This experiment shows that, WLS algorithm requires two iterations to focus the image.



**Fig. 2:-** Sinusoidal phase error function.

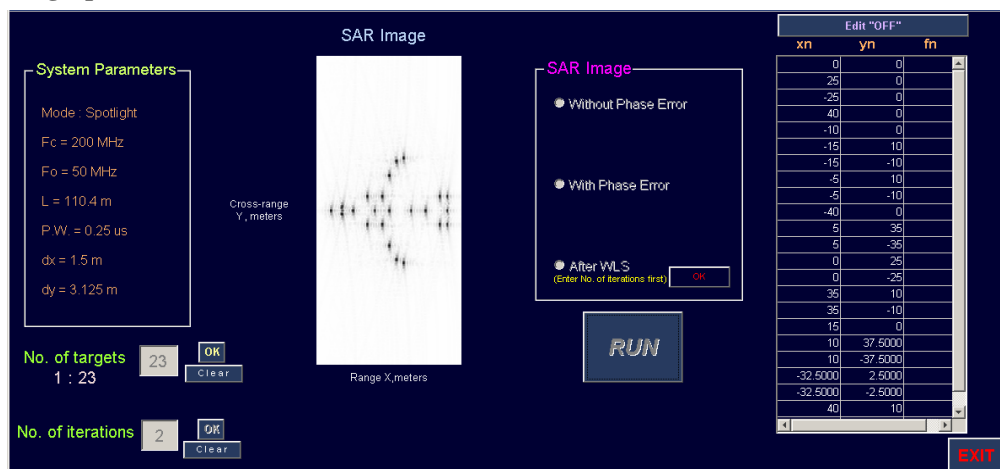


**Fig. 3:-** (a) Simulated images for WLS performance test; (b) Image blurred with sinusoidal phase error; (c) WLS autofocus results after two iterations.



**Fig. 4:-** The rms of residual phase error versus the iteration number for the simulated SAR image.

### A suggested graphical user interface (GUI):-



**Fig. 5:-** Graphical User Interface (GUI)

From the GUI, the user can enter, edit, and modify SAR and scene parameters, number of targets, number of iterations, and target position as shown in Fig. 5. Desired SAR image can be viewed with/without phase error. The corrected output image is obtained by selecting "After WLS" button, which will appear at the middle of the GUI window.

### Conclusions:-

The WLS is a robust technique for SAR autofocus for different reasons, its nonparametric method (different forms of phase errors), has a good performance over a wide range of applications (independent on SAR scene), does not need for a strong isolated scatterer in image, works well on both high and low signal-to-clutter ratio images, and no human intervention.

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