Journal Homepage: - www.journalijar.com
INTERNATIONAL JOURNAL OF ADVANCED RESEARCH (IJAR)
ISSN NO. 2320-5407
Article DOI: 10.21474/IJAR01/3780
DOI URL: http://dx.doi.org/10.21474/IJAR01/3780

RESEARCH ARTICLE

A CASCADE RELIABILITY MODEL FOR EXPONENTIAL AND LINDLEY DISTRIBUTIONS.

C. Doloi[1] and J. Gogoi[2]
1. Assistant Professor, Department of Statistics, Cotton College State University, Guwahati, Assam, India.
2. Assistant Professor, Department of Statistics, Assam University, Silchar, Assam, India.

Manuscript Info

Manuscript History

Received: 07 February 2017
Final Accepted: 01 March 2017
Published: April 2017

Key words:-
Cascade system, Stress-Strength, reliability, Exponential distribution, Lindley distribution.

Abstract

In the present paper we have considered two cases to obtain the system reliability of an n-cascade system. For the first case we consider one-parameter exponential strength and Lindley stress. Secondly, we consider Lindley strength and one-parameter exponential stress. Under this assumption the expression for reliability of an n-cascade system Rn is given. For both the cases all stress-strength are random variables with given density. Some numerical values of reliability have also been presented in tabular form for some selected values of the parameters.

Introduction:-

Cascade system were first developed and studied by Pandit and Sriwastav [6]. Many authors such as Kapur and Lamberson [5], Bhowal [1], Hanagal [3] to name a few have studied interference models in reliability without taking time into consideration. They have studied single impact systems. An n-Cascade system is a special type of n-standby system [6]. In a Cascade System the stresses on subsequent components are attenuated by a factor 'k', called attenuation factor. Attenuation factor is generally assumed to be a constant for all the components or to be a parameter having different fixed values for different components. But an attenuation factor may also be a random variable [2]. Most of the discussions of interference models assume that the parameters of stress and strength distributions are constants. But in many cases this assumption may not be true and the parameters may be assumed themselves (parameters) to be random variables. For example, solutions corrosive action may be highly influenced by variation in its temperature [4] and hence the distribution of stress (corrosive action) may have different parametric values which vary randomly with temperature or in other words, the stress parameter may be taken as a random variable.

Let $X_1, X_2, ..., X_n$ be the strengths of n-components in the order of activation and let $Y_1, Y_2, ..., Y_n$ are the stresses working on them. In Cascade system after every failure the stress is modified by a factor k (called attenuation factor) such that

$$Y_2 = kY_1, Y_3 = kY_2 = k^2 Y_1 ..., Y_i = k^{i-1} Y_1 \text{ etc.}$$

In stress strength model the reliability, R of a component (or system) is defined as he probability that its strength X, is not less than the stress Y working on it, where X and Y are random variables.

i.e. R = Pr(X ≥ Y)

Corresponding Author:- C. Doloi.
Address:- Assistant Professor, Department of Statistics, Cotton College State University, Guwahati, Assam, India .

In this paper we have considered an *n*-cascade system with this model. We have not come across any study where cascade model is considered for these two cases. The main aim of this paper is to obtain the system reliability R_n for this model where in the first case strength is considered as one-parameter exponential distribution and stress is Lindley distribution. Similarly in the second case strength is Lindley distribution and stress is one-parameter exponential distribution. The paper is organized as follows. In section 2 the general model is developed for an *n*-cascade system. In section 3 the reliability expressions of an *n*-cascade system is obtained when the stress-strength of the components follow particular distributions. In section 3.1 to 3.2 the expressions of R_n, is obtained when strength is considered as one-parameter exponential distribution and stress is Lindley distribution. Similarly strength is Lindley distribution and stress is one-parameter exponential distribution. Some numerical values of reliabilities $R(1)$, $R(2)$ and R_2 are tabulated for each cases in section 4. Results and Discussions are given at the end.

Mathematical Formulation:-

Let us consider an n-cascade system and suppose that n components are numbered from 1 to n in their order of activation. Let X_i be the strength of the ith component, in the order of activation, and when activated faces the stress Y_i, $i=1,2,\ldots,n$. For a cascade system with attenuation factor 'K' (constant).

$$Y_i = K^{i-1}Y_1, \quad i = 1,2,\ldots,n \tag{2.1}$$

The reliability of the system is given by

$$R_n = R(1) + R(2) + \ldots + R(n) \tag{2.2}$$

Now the marginal reliability $R(1)$, $R(2)$, $R(3)$,…, $R(n)$ may be obtained as

$$R(1) = \int_{-\infty}^{\infty} \overline{F}(y_1) g(y_1)\, dy_1 \tag{2.3}$$

$$R(2) = \int_{-\infty}^{\infty} F(y_1)\overline{F}(ky_1)\, g(y_1)\, dy_1 \tag{2.4}$$

$$R(3) = \int_{-\infty}^{\infty} F(y_1)\, F(ky_1)\overline{F}(k^2 y_1)\, g(y_1)\, dy_1 \tag{2.5}$$

Similarly,

$$R(n) = \int_{-\infty}^{\infty} F(y_1)F(ky_1)F(k^2 y_1)\ldots\overline{F}(k^{n-1}y_1)\; g(y_1)\, dy_1 \tag{2.6}$$

where r th component marginal reliability may be given as

$$R(r) = P[X_1 < Y_1, X_2 < kY_1, \ldots, X_{r-1} < k^{r-2}Y_1, X_r \geq k^{r-1}Y_1] \tag{2.7}$$

Stress-Strength follows Specific Distributions:-

When Stress-Strength follows particular distributions we can evaluate the expression (2.6) and thereby obtain the system reliability. In the following two sub-sections we assume different particular distributions of all the Stress-Strength involved and obtain expressions of system reliability.

Strength follows Exponential Distribution and Stress follows:-
Lindley Distribution:-

Let $f_i(x)$ be one-parameter exponential densities with parameters θ_i, and let $g_i(y_1)$ be Lindley densities with parameters γ_i respectively, $i=1,2,\ldots,n$ i.e.

$$f_i(x,\theta) = \theta_i\, e^{-\theta_i x_i}; \qquad x_i \geq 0, \theta_i \geq 0$$

$$g_i(y_1,\gamma) = \frac{\gamma^2}{(1+\gamma)}(1+y_1)e^{-\gamma y_1}; \qquad y_1 > 0, \gamma > 0$$

Then from (2.3) to (2.6) we have

$$R(1)=\frac{\gamma^2}{1+\gamma}\left[\frac{1}{\theta_1+\gamma}+\frac{1}{(\theta_1+\gamma)^2}\right]$$

$$R(2)=\frac{\gamma^2}{1+\gamma}\left[\left\{\frac{1}{\theta_2K+\gamma}+\frac{1}{(\theta_2K+\gamma)^2}\right\}-\left\{\frac{1}{\theta_1+\theta_2K+\gamma}+\frac{1}{(\theta_1+\theta_2K+\gamma)^2}\right\}\right]$$

$$R(3)=\frac{\gamma^2}{1+\gamma}\left[\begin{array}{l}\left\{\frac{1}{\theta_3K^2+\gamma}+\frac{1}{(\theta_3K^2+\gamma)^2}\right\}-\left\{\frac{1}{\theta_1+\theta_3K^2+\gamma}+\frac{1}{(\theta_1+\theta_3K^2+\gamma)^2}\right\}\\ -\left\{\frac{1}{\theta_2K+\theta_3K^2+\gamma}+\frac{1}{(\theta_2K+\theta_3K^2+\gamma)^2}\right\}+\\ \left\{\frac{1}{\theta_1+\theta_2K+\theta_3K^2+\gamma}+\frac{1}{(\theta_1+\theta_2K+\theta_3K^2+\gamma)^2}\right\}\end{array}\right]$$

Similarly,

$$R(n)=\frac{\gamma^2}{1+\gamma}\left[\begin{array}{l}\left\{\frac{1}{\theta_nk^{n-1}+\gamma}+\frac{1}{(\theta_nk^{n-1}+\gamma)^2}\right\}-\left\{\frac{1}{\theta_1+\theta_nk^{n-1}+\gamma}+\frac{1}{(\theta_1+\theta_nk^{n-1}+\gamma)^2}\right\}\\ -\ldots+(-1)^{n+1}\left\{\frac{1}{\theta_1+\theta_2k+\ldots+\theta_nk^{n-1}+\gamma}+\frac{1}{(\theta_1+\theta_2k+\ldots+\theta_nk^{n-1}+\gamma)^2}\right\}\end{array}\right]$$

Substituting the values of $R(1),R(2),R(3),\ldots,R(n)$ in (2.2) we can obtain R_n the reliability of the system.

Particular Case:-

Let the strengths of the n components be i.i.d. with p.d.f. $f(x)$ which follows one-parameter exponential with mean $1/\theta$ and the p.d.f. of Y_1 be Lindley density with parameter γ i.e

$$f(x,\theta)=\theta e^{-\theta x}; \qquad x\ge 0,\ \theta\ge 0$$

$$g(y_1,\gamma)=\frac{\gamma^2}{(1+\gamma)}(1+y_1)e^{-\gamma y_1}; \qquad y_1>0,\ \gamma>0$$

then from (2.3) to (2.6) we have

$$R(1)=\frac{\gamma^2}{1+\gamma}\left[\frac{1}{\theta+\gamma}+\frac{1}{(\theta+\gamma)^2}\right]$$

$$R(2)=\frac{\gamma^2}{1+\gamma}\left[\left\{\frac{1}{\theta K+\gamma}+\frac{1}{(\theta K+\gamma)^2}\right\}-\left\{\frac{1}{\theta+\theta K+\gamma}+\frac{1}{(\theta+\theta K+\gamma)^2}\right\}\right]$$

$$R(3)=\frac{\gamma^2}{1+\gamma}\left[\begin{array}{l}\left\{\frac{1}{\theta K^2+\gamma}+\frac{1}{(\theta K^2+\gamma)^2}\right\}-\left\{\frac{1}{\theta+\theta K^2+\gamma}+\frac{1}{(\theta+\theta K^2+\gamma)^2}\right\}\\ -\left\{\frac{1}{\theta K+\theta K^2+\gamma}+\frac{1}{(\theta K+\theta K^2+\gamma)^2}\right\}+\\ \left\{\frac{1}{\theta+\theta K+\theta K^2+\gamma}+\frac{1}{(\theta+\theta K+\theta K^2+\gamma)^2}\right\}\end{array}\right]$$

Similarly,

$$R(n)=\frac{\gamma^2}{1+\gamma}\left[\begin{array}{l}\left\{\frac{1}{\theta k^{n-1}+\gamma}+\frac{1}{(\theta k^{n-1}+\gamma)^2}\right\}-\left\{\frac{1}{\theta+\theta k^{n-1}+\gamma}+\frac{1}{(\theta+\theta k^{n-1}+\gamma)^2}\right\}\\ -...+(-1)^{n+1}\left\{\frac{1}{\theta+\theta k+...+\theta k^{n-1}+\gamma}+\frac{1}{(\theta+\theta k+...+\theta k^{n-1}+\gamma)^2}\right\}\end{array}\right]$$

A few numerical values of $R(1), R(2)\, and\, R_2$ are tabulated in Table 1 for different values of the parameters.

Strength follows Lindley Distributions and Stress follows:-
Exponential Distributions:-

Let $f_i(x)$ be Lindley densities with parameters θ_i and let $g_i(y_1)$ be one-parameter exponential densities with parameters λ_i, respectively, i =1,2,…, n i.e.

$$f_i(x,\theta)=\frac{\theta_i^{\,2}}{(1+\theta_i)}(1+y_1)e^{-\theta_i\, x_i}; \qquad x_i\ge 0,\ \theta_i\ge 0$$

$$g_i(y_1,\lambda)=\lambda e^{-\lambda\, y_i}; \qquad y_1>0,\ \lambda>0$$

then from (2.3) to (2.6) we have

$$R(1)=\frac{\lambda}{1+\theta_1}\left[\frac{1}{\theta_1+\lambda}+\frac{\theta_1}{\theta_1+\lambda}+\frac{\theta_1}{(\theta_1+\lambda)^2}\right]$$

$$R(2)=\frac{\lambda}{1+\theta_2}\left[\begin{array}{l}\frac{1}{\theta_2k+\lambda}+\frac{\theta_2}{\theta_2k+\lambda}+\frac{\theta_2k}{(\theta_2k+\lambda)^2}-\frac{1}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)^2}-\frac{\theta_1}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)}\\ -\frac{\theta_1}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)^2}-\frac{\theta_2}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)}-\frac{\theta_1\theta_2}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)}\\ -\frac{\theta_1\theta_2}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)^2}-\frac{\theta_2k}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)^2}-\frac{\theta_1\theta_2k}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)^2}\\ -\frac{2\theta_1\theta_2k}{(1+\theta_1)(\theta_2k+\theta_1+\lambda)^3}\end{array}\right]$$

$$R(3)=\frac{\lambda}{1+\theta_3}\left[\begin{array}{l}\frac{1}{\theta_3k^2+\lambda}+\frac{\theta_3}{\theta_3k^2+\lambda}+\frac{\theta_3k^2}{(\theta_3k^2+\lambda)^2}-\frac{1}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}-\\ \frac{\theta_1}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}-\frac{\theta_1}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}-\frac{\theta_3}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}\\ -\frac{\theta_1\theta_3}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}-\frac{\theta_1\theta_3}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}-\frac{\theta_3k^2}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}\\ -\frac{\theta_1\theta_3k^2}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}-\frac{2\theta_1\theta_3k^2}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^3}-\frac{1}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}\\ -\frac{\theta_2}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}-\frac{\theta_2k}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^2}-\frac{\theta_3}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}\\ -\frac{\theta_2\theta_3}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}-\frac{\theta_2\theta_3k}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^2}-\frac{\theta_3k^2}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^2}\\ -\frac{\theta_2\theta_3k^2}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}-\frac{2\theta_2\theta_3k^3}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^3}+\\ \frac{1}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\frac{\theta_1}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\\ \frac{\theta_1}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\frac{\theta_2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\\ \frac{\theta_1\theta_2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\frac{\theta_1\theta_2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\\ \frac{\theta_2k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\frac{\theta_1\theta_2k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}\end{array}\right]$$

$$R(3)=\frac{\lambda}{1+\theta_3}\left[\begin{array}{l}
\frac{1}{\theta_3k^2+\lambda}+\frac{\theta_3}{\theta_3k^2+\lambda}+\frac{\theta_3k^2}{(\theta_3k^2+\lambda)^2}-\frac{1}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}-\\
\frac{\theta_1}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}-\frac{\theta_1}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}-\frac{\theta_3}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}\\
-\frac{\theta_1\theta_3}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)}-\frac{\theta_1\theta_3}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}-\frac{\theta_3k^2}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}\\
-\frac{\theta_1\theta_3k^2}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^2}-\frac{2\theta_1\theta_3k^2}{(1+\theta_1)(\theta_3k^2+\theta_1+\lambda)^3}-\frac{1}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}\\
-\frac{\theta_2}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}-\frac{\theta_2k}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^2}-\frac{\theta_3}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}\\
-\frac{\theta_2\theta_3}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}-\frac{\theta_2\theta_3k}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^2}-\frac{\theta_3k^2}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^2}\\
-\frac{\theta_2\theta_3k^2}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)}-\frac{2\theta_2\theta_3k^3}{(1+\theta_2)(\theta_3k^2+\theta_2k+\lambda)^3}+\\
\frac{1}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\frac{\theta_1}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\\
\frac{\theta_1}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\frac{\theta_2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\\
\frac{\theta_1\theta_2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\frac{\theta_1\theta_2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\\
\frac{\theta_2k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\frac{\theta_1\theta_2k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}
\end{array}\right]$$

$$+\frac{\lambda}{1+\theta_3}\left[\begin{array}{l}\frac{2\theta_1\theta_2 k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^3}+\frac{\theta_3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\\ \frac{\theta_1\theta_3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\frac{\theta_1\theta_3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\\ \frac{\theta_2\theta_3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\frac{\theta_1\theta_2\theta_3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\\ \backslash\frac{\theta_1\theta_2\theta_3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\frac{\theta_2\theta_3 k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\\ \frac{\theta_1\theta_2\theta_3 k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\frac{2\theta_1\theta_2\theta_3 k}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^3}+\\ \frac{\theta_3 k^2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)}+\frac{\theta_1\theta_3 k^2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\\ \frac{2\theta_1\theta_3 k^2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^3}+\frac{\theta_2\theta_3 k^2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\\ \frac{\theta_1\theta_2\theta_3 k^2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^2}+\frac{2\theta_1\theta_2\theta_3 k^2}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^3}+\\ \frac{2\theta_1\theta_3 k^3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^3}+\frac{2\theta_1\theta_2\theta_3 k^3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^3}+\\ \frac{2\theta_1\theta_2\theta_3 k^3}{(1+\theta_1)(1+\theta_2)(\theta_3k^2+\theta_2k+\theta_1+\lambda)^3}\end{array}\right]$$

Similarly,

$$R(n)=\frac{\lambda}{1+\theta_n}\left[\begin{array}{l}\frac{1}{\theta_nk^{n-1}+\lambda}+\frac{\theta_n}{\theta_nk^{n-1}+\lambda}+\frac{\theta_nk^{n-1}}{(\theta_nk^{n-1}+\lambda)^2}-\frac{1}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)}-\\ \frac{\theta_1}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)}-\frac{\theta_1}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)^2}-\frac{\theta_n}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)}\\ -\frac{\theta_1\theta_n}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)}-\frac{\theta_1\theta_n}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)^2}\\ -\frac{\theta_nk^{n-1}}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)^2}-\frac{\theta_1\theta_nk^{n-1}}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)^2}\\ -\frac{2\theta_1\theta_nk^{n-1}}{(1+\theta_1)(\theta_nk^{n-1}+\theta_1+\lambda)^3}-\ldots\\ (-1)^{n-1}\frac{(n-1)\theta_1\theta_2\ldots\theta_{n-1}k^n}{(1+\theta_1)(1+\theta_2)\ldots(1+\theta_{n-1})(\theta_nk^{n-1}+\ldots\theta_2k+\theta_1+\lambda)}\end{array}\right]$$

Substituting the values of $R(1), R(2), R(3), ..., R(n)$ in (2.2) we can obtain R_n, the reliability of the system.

Particular Case:-

Let the strengths of the n components be i.i.d. with p.d.f. $f(x)$ which follows Lindley density with parameter θ and the p.d.f. of Y_1 be one-parameter exponential with mean $1/\lambda$ i.e.

$$f(x,\theta) = \frac{\theta^2}{(1+\theta)}(1+x)e^{-\theta x}; \qquad x \ge 0,\ \theta \ge 0$$

$$g(y_1,\lambda) = \lambda e^{-\lambda y_i}; \qquad y_1 > 0,\ \lambda > 0$$

then from (2.3) to (2.6) we have

$$R(1) = \frac{\lambda}{1+\theta}\left[\frac{1}{\theta+\lambda} + \frac{\theta}{\theta+\lambda} + \frac{\theta}{(\theta+\lambda)^2}\right]$$

$$R(2) = \frac{\lambda}{1+\theta}\left[\begin{array}{l} \frac{1}{\theta k+\lambda} + \frac{\theta}{\theta k+\lambda} + \frac{\theta k}{(\theta k+\lambda)^2} - \frac{1}{(1+\theta)(\theta k+\theta+\lambda)^2} - \frac{\theta}{(1+\theta)(\theta k+\theta+\lambda)} \\ - \frac{\theta}{(1+\theta)(\theta k+\theta+\lambda)^2} - \frac{\theta}{(1+\theta)(\theta k+\theta+\lambda)} - \frac{\theta^2}{(1+\theta)(\theta k+\theta+\lambda)} \\ - \frac{\theta^2}{(1+\theta)(\theta k+\theta+\lambda)^2} - \frac{\theta k}{(1+\theta)(\theta k+\theta+\lambda)^2} - \frac{\theta^2 k}{(1+\theta)(\theta k+\theta+\lambda)^2} \\ - \frac{2\theta^2 k}{(1+\theta)(\theta k+\theta+\lambda)^3} \end{array}\right]$$

$$R(3)=\frac{\lambda}{1+\theta}\left[\begin{aligned}
&\frac{1}{\theta k^2+\lambda}+\frac{\theta}{\theta k^2+\lambda}+\frac{\theta k^2}{(\theta k^2+\lambda)^2}-\frac{1}{(1+\theta)(\theta k^2+\theta+\lambda)}-\\
&\frac{\theta}{(1+\theta)(\theta k^2+\theta+\lambda)}-\frac{\theta}{(1+\theta)(\theta k^2+\theta+\lambda)^2}-\frac{\theta}{(1+\theta)(\theta k^2+\theta+\lambda)}\\
&-\frac{\theta^2}{(1+\theta)(\theta k^2+\theta+\lambda)}-\frac{\theta^2}{(1+\theta)(\theta k^2+\theta+\lambda)^2}-\frac{\theta k^2}{(1+\theta)(\theta k^2+\theta+\lambda)^2}\\
&-\frac{\theta^2 k^2}{(1+\theta)(\theta k^2+\theta+\lambda)^2}-\frac{2\theta^2 k^2}{(1+\theta)(\theta k^2+\theta+\lambda)^3}-\frac{1}{(1+\theta)(\theta k^2+\theta k+\lambda)}\\
&-\frac{\theta}{(1+\theta)(\theta k^2+\theta k+\lambda)}-\frac{\theta k}{(1+\theta)(\theta k^2+\theta k+\lambda)^2}-\frac{\theta}{(1+\theta)(\theta k^2+\theta k+\lambda)}\\
&-\frac{\theta^2}{(1+\theta)(\theta k^2+\theta k+\lambda)}-\frac{\theta^2 k}{(1+\theta)(\theta k^2+\theta k+\lambda)^2}-\frac{\theta k^2}{(1+\theta)(\theta k^2+\theta k+\lambda)^2}\\
&-\frac{\theta^2 k^2}{(1+\theta)(\theta k^2+\theta k+\lambda)}-\frac{2\theta^2 k^3}{(1+\theta)(\theta k^2+\theta k+\lambda)^3}+\\
&\frac{1}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+\frac{\theta}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+\\
&\frac{\theta}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+\frac{\theta}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+\\
&\frac{\theta^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+\frac{\theta^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+\\
&\frac{\theta k}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+\frac{\theta^2 k}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}
\end{aligned}\right]$$

$$+\frac{\lambda}{1+\theta}\left[\begin{array}{l}\frac{2\theta^2 k}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^3}+\frac{\theta}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+ \\ \frac{\theta^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+\frac{\theta^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+ \\ \frac{\theta^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+\frac{\theta^3}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+ \\ \backslash\frac{\theta^3}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+\frac{\theta^2 k}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+ \\ \frac{\theta^3 k}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+\frac{2\theta^3 k}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^3}+ \\ \frac{\theta k^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)}+\frac{\theta^2 k^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+ \\ \frac{2\theta^2 k^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^3}+\frac{\theta^2 k^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+ \\ \frac{\theta^3 k^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^2}+\frac{2\theta^3 k^2}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^3}+ \\ \frac{2\theta^2 k^3}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^3}+\frac{2\theta^3 k^3}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^3}+ \\ \frac{2\theta^3 k^3}{(1+\theta)^2(\theta k^2+\theta k+\theta+\lambda)^3}\end{array}\right]$$

Similarly,

$$R(n)=\frac{\lambda}{1+\theta}\left[\begin{array}{l}\frac{1}{\theta k^{n-1}+\lambda}+\frac{\theta}{\theta k^{n-1}+\lambda}+\frac{\theta k^{n-1}}{(\theta k^{n-1}+\lambda)^2}-\frac{1}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)}- \\ \frac{\theta}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)}-\frac{\theta}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)^2}-\frac{\theta}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)} \\ -\frac{\theta^2}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)}-\frac{\theta^2}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)^2} \\ -\frac{\theta k^{n-1}}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)^2}-\frac{\theta^2 k^{n-1}}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)^2} \\ -\frac{2\theta^2 k^{n-1}}{(1+\theta)(\theta k^{n-1}+\theta+\lambda)^3}-\ldots \\ (-1)^{n-1}\frac{(n-1)!\,\theta^n k^n}{(1+\theta)^{n-1}(\theta k^{n-1}+\ldots\theta k+\theta+\lambda)^n}\end{array}\right]$$

A few numerical values of $R(1)$, $R(2)$ *and* R_2 are tabulated in Table 2 for different values of the parameters.

Numerical Evaluation:-

For some specific values of the parameters we evaluate the marginal reliabilities $R(1)$, $R(2)$ and system reliability R_2 for the above two particular cases from their expressions obtained in the last section.

Table 1:- Values of $R(1)$, $R(2)$ and R_2 when strength and stress are Exponential and Lindley variates.

θ	γ	*K*	*R(1)*	*R(2)*	R_2
1	1	1	.3750	.1528	.5278
1	2	2	.5926	.0967	.6893
1	3	3	.7031	.0702	.7733
2	*1*	*1*	*.2222*	*.1022*	*.3244*
2	*2*	*2*	*.4167*	*.0718*	*.4884*
2	*3*	*3*	*.5400*	*.0546*	*.5946*
3	*1*	*1*	*.1563*	*.0746*	*.2309*
3	*2*	*2*	*.3200*	*.0553*	*.3753*
3	*3*	*3*	*.4375*	*.0431*	*.4806*

Table 2:- Values of $R(1)$, $R(2)$ and R_2 when strength and stress are Lindley and Exponential variates

θ	λ	K	R(1)	R(2)	R_2
1	1	1	.6250	.2176	.8426
1	2	2	.7778	.1690	.9468
1	3	3	.8438	.1527	.9964
2	1	1	.4074	.1647	.5721
2	2	2	.5833	.1123	.6956
2	3	3	.6800	.0901	.7701
3	1	1	.2969	.1278	.4247
3	2	2	.4600	.0848	.5448
3	3	3	.5625	.0655	.6280

Results and Discussions:-

For some specific values of the parameters we evaluate the marginal reliabilities $R(1)$, $R(2)$ and system reliability R_2 for the above two particular cases from their expressions obtained in Sub-Section 3.1 and 3.2.

From the **Table 1**, we notice that if the stress parameter γ increases then the system reliability R_2 increase. When the strength parameter θ remain constant then $R(1)$ increases but $R(2)$ decreases. For instance, if γ =1, $R(1) = 0.3750$ and if $\gamma = 2$, $R(1) = 0.5926$. In general we see that when γ, k increases and for fix value θ then $R(1)$ and R_2 will also increases i.e. when the attenuation factor k increases then the marginal reliability $R(1)$ and system reliability R_2 increases but $R(2)$ decrease.

Table 2 shows that with some set of values of the parameters, if the stress parameter λ increases then the system reliability R_2 increase. When the strength parameter θ remain constant then $R(1)$ increases but $R(2)$ decreases. For instance, if λ =2, $R(1) = 0.7778$ and if $\lambda = 3$, $R(1) = 0.8438$. In general we see that when λ, k increases and for fix value θ then $R(1)$ and R_2 will also increases i.e. when the attenuation factor k increases then the marginal reliability $R(1)$ and system reliability R_2 increases but $R(2)$ decrease.

References:-

1. Bhowal, M. K.(1999): *A Study of Cascade Reliability and Other Interference Models*, Ph.DThesis, Dibrugarh University.
2. Doloi. C, Borah. M, and Sriwastav G. L. (2010): Cascade System with Random Attenuation Factor, *IAPQR Transactions*, **35(2)**, 81-90.
3. Hanagal, D. D.(2003): Estimation of System Reliability in Multicomponent Series Stress-Strength Models, *Journal of the Indian Statistical Association*, **41**, 1-7.
4. Kakati, M. C. (1983): *Interference Theory and Reliability*, Ph.D Thesis (Unpublished), Dibrugarh University, India.
5. Kapur, K. C. and Lamberson, L. R. (1977): *Reliability in Engineering Design*, John Wiley and Sons, New York.
6. Pandit, S. N. N. and Sriwastav G. L.(1975): Studies in Cascade Reliability I, *IEEE Trans. on Reliability*, **24(1)**, 53-56.