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RESEARCH ARTICLE

DRAG ON AN MICROPOLAR FLOW PAST A SPHERE SPECIFYING UNIFORM VELOCITY AWAY FROM THE BOUNDARIES.

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Abstract

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A study of the effect of drag force on anmicropolar flow past a sphere specifying uniform velocity away from the boundaries. We find a similarity solution, assuming the fluid outside the sphere and satisfies the Eringen's micro polar equations and applying no slip condition at the sphere of the surface. An appearance for drag force is obtained. It is found that the increase in the coupling parameter with fixed coupling stress parameter is to decreases drag. Further a reversed behavior is noticed that the drag is increases andthe same is represented graphically.

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Introduction:-

The earliest formulation of a general theory of microcontinua is accredited to Eringen [1] has considered as fluids with deformable microelements. Eringen's [2] 'micropolar fluid theory' is based on the assumptions that the deformation of the fluid microelements is very small. This theory is still capable of taking into account the effect of microrotational surface and body couples. The evaluation of uniform flow past a spherical shell in Newtonian stokes flow has been extensively investigated in the literature, because of its application in lubrication theory, transpiration cooling and other important applications. The stokes uniform flow past a porous sphere has also been investigated by several authors with the assumsion of axisymmtric flow (Padmavati et al [3], Berman [4], Rudraiah et al [5]).
However in environmental pollution problems, particularly in water pollution problem, it is central to consider the effects of suspended particles on the flow past a sphere. The effect of these suspended particles may be taken into account either using Eringen'smicropolar fluid model or using Saffman dusty fluid model. The Saffman dusty fluid model does not much importance of the effect of micro rotation of balanced particles unless we consider principal of angular momentum in addition to linear momentum. The micropolar fluid model has built in mechanism of taking care of micro rotation.

The recently K. Ramalashmi and PankajShukla[6] investigated Drag on a porous sphere embedded in micropolar fluid.Jize Sui, et al [7] is investigation for the shear flow and heat transfer of a micropolar fluid by means of novel constitutive models is valuable .Jian-Jun Shu and JennShiun Lee [8] they obtain fundamental Stokes and Oseen solutions for micropolar flow in three dimensions, so that the point force and point couple can be prescribed in any direction. Karl-Heinz Hoffmann et al [9] they studied the resistant force exerted on a sphere moving with a constant velocity in a micro-polar fluid.

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Presently the analytical study of micropolar fluid flow past an impermeable sphere specifying identical Velocity far From the Boundaries. The expression for drag force is determined. The evaluation of drag coefficient on non - dimensional coupling parameter N_1 and coupling stress parameter N_3 is discussed and presented graphically.

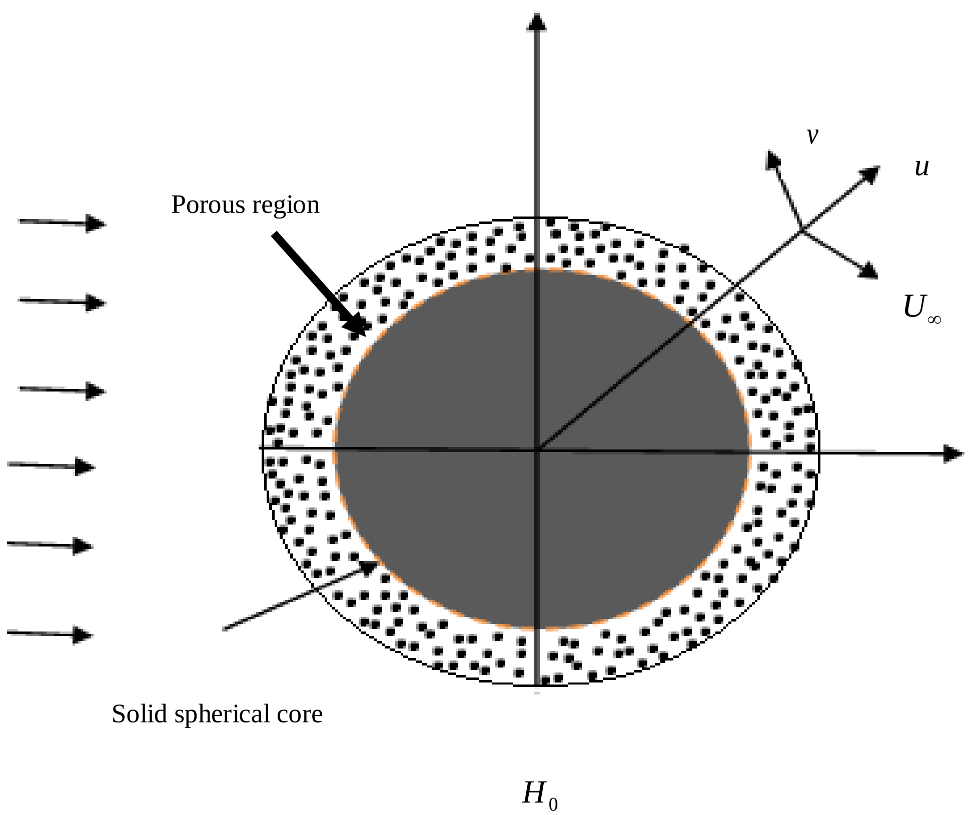
2. Mathematical Formulation

Consider a steady incompressible micropolar fluid flow past an impervious sphere of radius 'a' embedded in a sparsely packed porous medium. The schematic representation is show in diagram under assumptions and approximations made together with governing by the equations of continuity, conservation of momentum and Conservation of angular momentum

$$\nabla \cdot \vec{q} = 0. \tag{1}$$

$$-\nabla p + \varsigma\left(\nabla \times \vec{\omega}\right) + (\varsigma + \eta)\nabla^2 \vec{q} = 0. \tag{2}$$

$$\varsigma\left(\nabla \times \vec{q}\right) - 2\varsigma\vec{\omega} + (\lambda' + \eta')\nabla(\nabla \cdot \vec{\omega}) + \eta'\nabla^2 \vec{\omega} = 0. \tag{3}$$



Physical configuration

where $\vec{q}$ is the velocity, ς is the coupling viscosity , $\vec{\omega}$ is the angular velocity, p is the pressure, ς is the coupling viscosity, η is the bulk viscosity co-efficients and λ is the shear viscosity co-efficients λ' is the bulk spin viscosity co-efficients and η' is the shear spin viscosity co-efficients.

Stream function $\psi(r,\theta)$ is introduced, such that the equation of continuity is satisfied in spherical polar co-ordinate system for porous regions and defined as:

$$u = \frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}; \quad v = \frac{-1}{r\sin\theta}\frac{\partial \psi}{\partial r}. \tag{4}$$

By removing the pressure term from equations (2) and (3) will obtained as:

$$\left(E^4 - N^2 E^2\right)E^2\psi = 0. \tag{5}$$

Where $E^2 = \frac{\partial^2}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right)$ is the Laplacian operator in spherical co-ordinate system.

$N^2 = \frac{(2-N_1)}{N_3}N_1 \quad N_1 = \frac{\varsigma}{\varsigma+\eta}$ Coupling parameter $0 \le N_1 \le 1$

$N_3 = \frac{\eta'}{(\varsigma+\eta)a^2}$ Coupling stress parameter $0 \le N_3 \le m$ (m: positive real).

We now assume $\psi = \psi_0 + \psi_1$, equation (5) using this resolution we get

$$E^2\psi_0 = 0 \tag{6}$$

$$E^4\psi_1 - N^2E^2\psi_1 = 0 \tag{7}$$

3. Boundary conditions

To solve the above governing equation we considered the boundary conditions are no-slip condition given by

$$\frac{\partial}{\partial\theta}\psi(r,\theta) = \frac{\partial}{\partial r}\psi(r,\theta) = 0 \qquad at \quad r = 1. \tag{8}$$

The flow has uniform velocity when Far away from the sphere given by

$$\psi(r,\theta) \sim \frac{r^2}{2}\sin^2\theta, \text{ as } \quad r \to \infty. \tag{9}$$

4. Method of solution

The boundary condition from equation (9) suggests the following similarity solution

$$\psi(r,\theta) = f(r)\sin^2\theta. \tag{10}$$

Substituting equation (10) in (7), then functions $\psi(r,\theta)$ reduces to fourth order ordinary differential equation in $f(r)$ as follows:

$$f^{iv}(r) - \frac{4}{r^2}f''(r) + \frac{8}{r^3}f'(r) - \frac{8}{r^4}f(r) - N^2\left(f''(r) - \frac{2}{r^2}f(r)\right) = 0. \tag{11}$$

The corresponding $f(r)$, from equation (8) and (9) reduces to: No-slip condition of solid sphere at the surface is given by

$$f(1) = 0, \qquad f'(1) = 0. \text{ at } r=1 \tag{12}$$

Further, the uniform velocity extreme away from the boundary, from equation (10) reduces to:

$$f(r) \sim \frac{r^2}{2} \text{ as } r \to \infty. \tag{13}$$

The solution for the equations (11) is obtained analytically by using thetransformation

$$g(r) = f''(r) - \frac{2}{r^2} f(r). \tag{14}$$

Substituting equation (14) in equation (11), it reduces to second order ordinary differential equation in $g(r)$ as,

$$g''(r) - \left(N^2 + \frac{2}{r^2}\right) g(r) = 0. \tag{15}$$

Further, consider the transformation function $g(r)$ as

$$g(r) = \sqrt{r}\, w(r). \tag{16}$$

Where $w(r)$ the arbitrary function. Thereby, equation (15) reduces to:

$$r^2 w''(r) + r\, w'(r) - \left[\left(\frac{3}{2}\right)^2 + (rN)^2\right] w(r) = 0. \tag{17}$$

Which is the modified Bessel's differential equation, and its solution in terms of modified Bessel's function is given as:

$$w(r) = C_1 I_{3/2}(rN) + D_1 K_{3/2}(rN). \tag{18}$$

Where, C_1 and D_1 are arbitrary constants. Thus, from equation (18) we have:

$$g(r) = C_1 \sqrt{r}\, I_{3/2}(rN) + D_1 \sqrt{r}\, K_{3/2}(rN). \tag{19}$$

Further, equation (14) reduces to a second order ordinary differential equation with variable co-efficient as:

$$f''(r) - \frac{2}{r^2} f(r) = C_1 \sqrt{r}\, I_{3/2}(rN) + D_1 \sqrt{r}\, K_{3/2}(rN). \tag{20}$$

Equation (20) is an ordinary differential equation of order two with variable co-efficient; its general solution can be obtained by the method of variation of parameters and is given by:

$$f(r) = \frac{A_1}{r} + B_1 r^2 + C_1 \sqrt{rN}\, I_{3/2}(rN) + D_1 \sqrt{rN}\, K_{3/2}(rN) \tag{21}$$

Where A_1 and B_1 are arbitrary constants.

as $r \to \infty$ then $I_{3/2}(rN) \to \infty$. Therefore, the abovesolution is valid if and only if $C_1 = 0$. Hence equation (21) reduces to:

$$f(r) = \frac{A_1}{r} + B_1 r^2 + D_1 \sqrt{rN}\, K_{3/2}(rN). \tag{22}$$

Where A_1, B_1, D_1 are constants to be evaluate using the boundary conditions (8) and (9). Determining these constants and on substitution, equation (22) reduces to:

$$f(r) = \frac{r^2}{2} - \frac{N^2 + 3N + 3}{N^2} \frac{1}{2r} + \frac{3}{2N}\left(1 + \frac{1}{rN}\right) e^{-(r-1)N} \tag{23}$$

In terms of stream function, from the equation (10) we get:

$$\psi(r,\theta)=\left(\frac{r^2}{2}-\frac{N^2+3N+3}{N^2}\frac{1}{2r}+\frac{3}{2N}\left(1+\frac{1}{rN}\right)e^{-(r-1)N}\right)\sin^2\theta. \quad (24)$$

This equations shows the function of coupling parameter and coupling stress parameter
The shearing stress is given by

$$\tau_{r\theta}=\bar{\mu}\left(\frac{1}{r}\frac{\partial u}{\partial\theta}+r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right) \quad (25)$$

On non – dimensionlisingequation (25) reduces to

$$\frac{\tau_{r\theta}}{\frac{\bar{\mu}U_\infty}{a}}=-\left(\frac{1}{r}\frac{\partial u}{\partial\theta}+r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right) \quad (26)$$

When the sphere on the surface (i.e r =1), the shearing stress becomes

$$\frac{\tau_{r\theta}}{\frac{\bar{\mu}U_\infty}{a}}=-r\frac{\partial}{\partial r}\left(\frac{v}{r}\right) \quad (27)$$

$$\tau_{r\theta}=\frac{\bar{\mu}U_\infty}{a}\frac{3}{2}(1+N)\sin\theta. \quad (28)$$

5. Determination of the Drag force

The drag force F experienced by an impermeable sphere of radius ‘a’ is defined by

$$F_t=\int_0^{2\pi}\int_0^{\pi}\left\{(\tau_{r\theta})_{r=a}\ \sin\theta\right\}R^2\sin\theta\, d\theta\, d\phi. \quad (29)$$

On evaluation, equation (29) reduces to as:

$$F_t=4\pi(\bar{\mu}U_\infty a)(1+N). \quad (30)$$

Further, the drag coefficient can be written as

$$C_D=\frac{-F_t}{\frac{1}{2}\rho U_\infty^2 a^2\pi} \quad (31)$$

Substitution of equation (30) in equation (31), itreduces to:

$$C_D=\frac{-16}{R_e}(1+N)\ and\ \ R_e=\frac{2\rho U_\infty a}{\bar{\mu}} \quad (32)$$

Where R_e is the Reynolds number and if S=0, then equation (30) reduces to:

$$F_t=4\pi\bar{\mu}U_\infty a. \quad (33)$$

One of the result of Happel and Brenner [10]

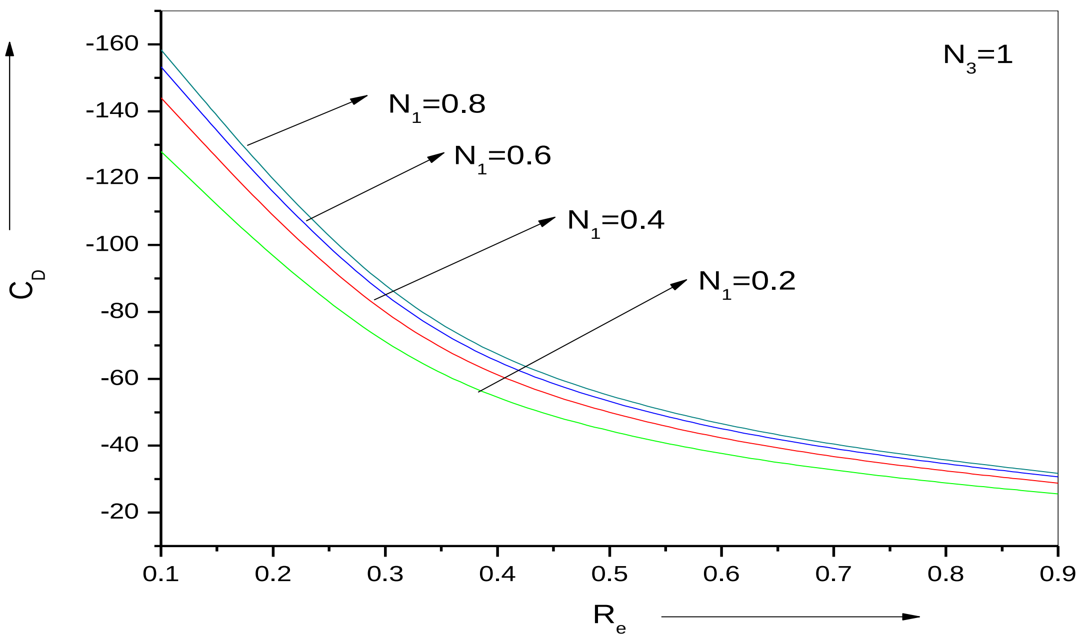


Fig 1:-Dependence of the Drag coefficient on coupling stress parameter N_3=1 for various values of coupling parameter N_1

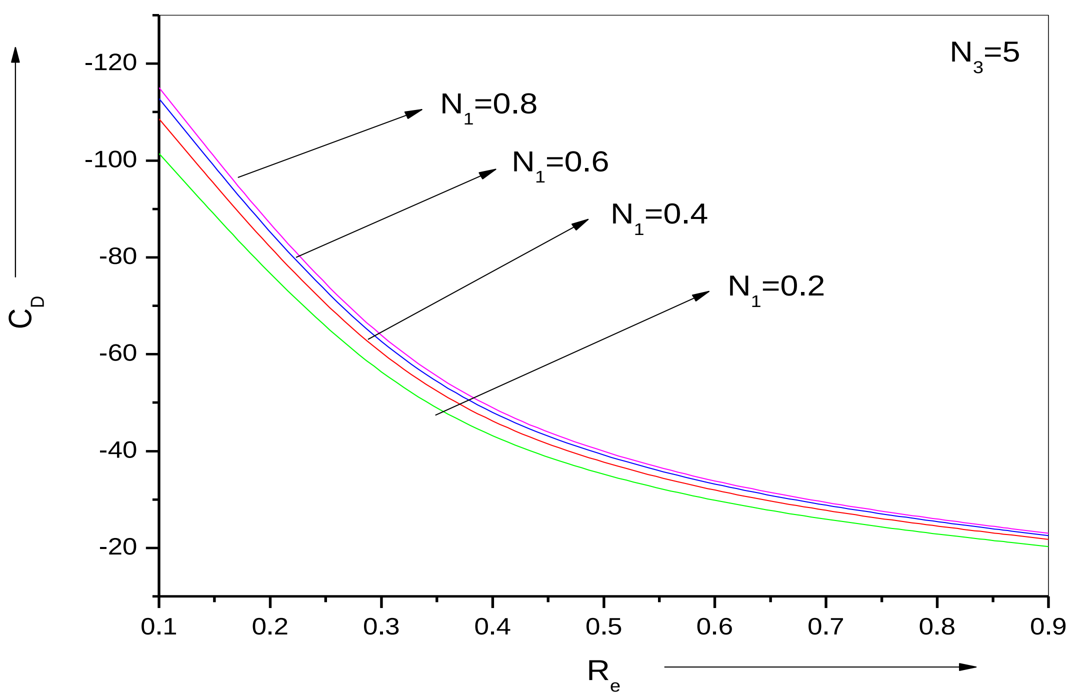


Fig 2:-Dependence of the Drag coefficient on coupling stress parameter N_3=5 for various values of coupling parameter N_1

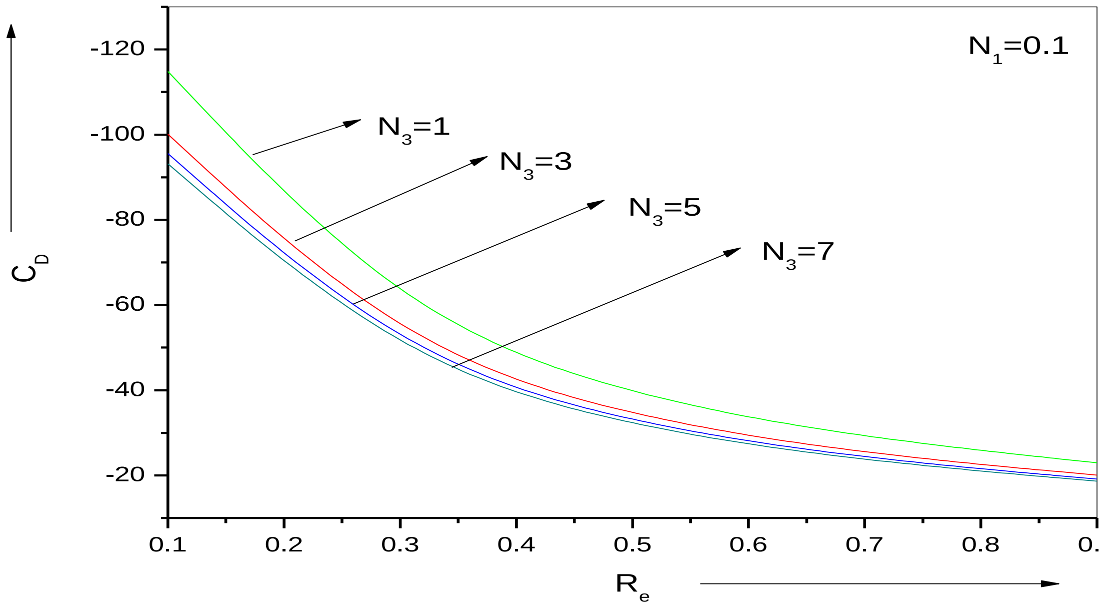


Fig 3:-Dependence of the Drag coefficient on coupling parameter N_1 =0.1 for various values of coupling stress parameter N_3

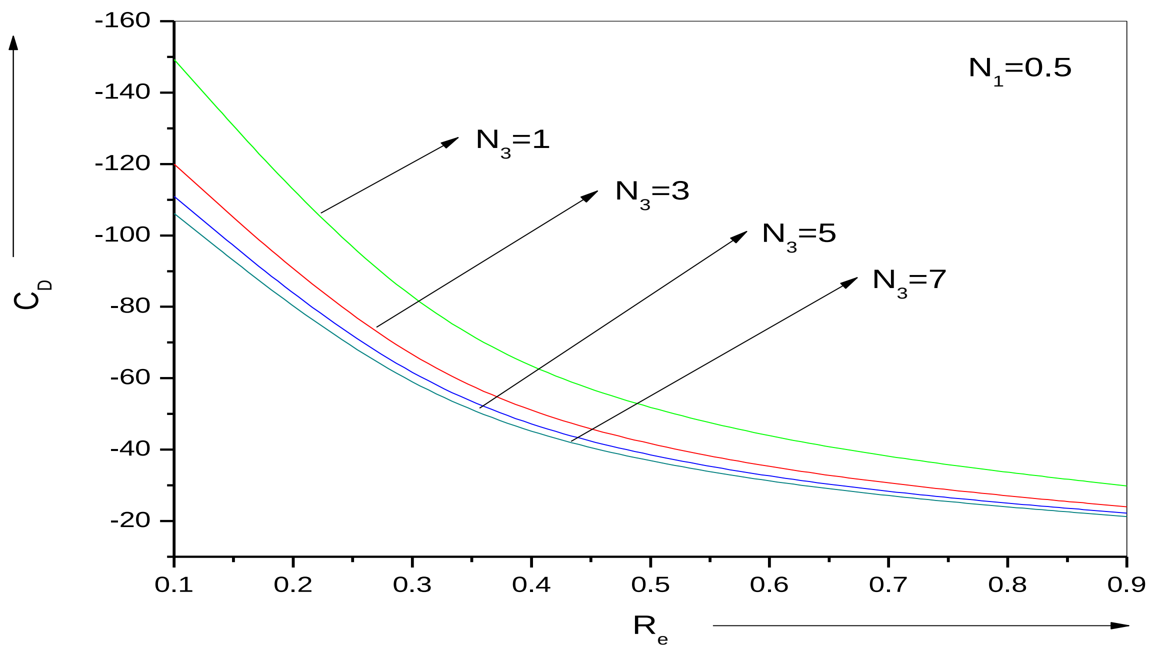


Fig 4:-Dependence of the Drag coefficient on coupling parameter N_1 =0.5 for various values of coupling stress parameter N_3

Results:-

In this paper we study dependence of drag coefficient with variation of coupling parameter and coupling stress parameter for the steady flow of viscous. The drag experienced by a sphere embedded in porous medium, using the no-slip condition at the solid surface and uniform shear flow far away from the region as the boundary conditions. Also, the expression for the drag co-efficient C_D is obtained.

From figures 1 and 2, we noticed that the drag co-efficient C_D decreases with increase coupling parameter N_1 for fixed coupling stress parameter N_3 =1 and N_3 =5 near the solid surface and maintains asymptotic behavior away from the surface. Further, figures 3 and 4 shows that the increase coupling stress parameter N_3, drag co-efficient C_D is increases for fixed coupling parameter N_1 =0.1 and N_1= 0.5.

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