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REVIEWER'S REPORT

Manuscript No.: **IJAR-52495** Date: 27-06-2025

Title: Cyclotomic Cosets in The Ring R_(2p^n q^m)=GF(l)[x]/(x^(2p^n q^m)-1)

Recommendation:
Accept as it is .....
Accept after minor revision.....
Accept after major revision .....
Do not accept (*Reasons below*) .....

Rating	Excel.	Good	Fair	Poor
Originality		✓		
Techn. Quality			✓	
Clarity			✓	
Significance		✓		

Reviewer Name: Mir Tanveer

Reviewer's Comment for Publication.

General Assessment:
This manuscript presents a rigorous and mathematically elegant treatment of cyclotomic cosets in a specialized ring structure over a finite field. The focus on R2pnqm=GF(l)[x]/(x2pnqm−1)R_{2p^n q^m} = GF(l)[x]/(x^{2p^n q^m} - 1), where p,q,lp, q, l are distinct odd primes and ll is a primitive root modulo pnp^n and qmq^m, situates the work within the intersection of algebraic number theory and coding theory. The work builds meaningfully upon prior contributions by Bakshi, Raka, Sahni, and Sehgal, and extends the known structure of primitive idempotents to more complex ring configurations with greater generality.

Abstract Evaluation:
The abstract concisely conveys the scope and result of the study, specifically the enumeration and explicit construction of all 2(m·n·d+m+n+1)2(m \cdot n \cdot d + m + n + 1) cyclotomic cosets. The assumptions are clearly stated, including the primitive root condition and the greatest common divisor constraint on Euler's totient functions. The relevance to cyclic codes and generating polynomials is briefly noted, supporting the mathematical and applied importance of the work.

Introduction Evaluation:
The introduction effectively sets the context by discussing the construction and importance of the ring RzR_z and its connection to minimal cyclic codes. References to foundational works ([4] and [5]) are well-integrated, and the logical progression from earlier cases z=pnqz = p^n q to the more general case z=2pnqmz = 2p^n q^m is well-motivated. The clarification that previous results become special cases of this work strengthens the scholarly continuity.

Mathematical Depth and Contribution:
The manuscript demonstrates deep engagement with finite field theory, ring theory, and the theory of cyclotomic cosets. The generalization from prior two-prime cases to three-factor ring orders of the form

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2pnqm2p^n q^m with detailed enumeration of cosets contributes significantly to the structure theory of cyclic codes. The result concerning 2(m·n·d+m+n+1)2(m \cdot n \cdot d + m + n + 1) cosets provides both explicit construction and theoretical completeness, which is of practical relevance in coding theory applications.

Originality and Theoretical Significance:
The originality lies in the systematic and general construction of cyclotomic cosets under specific modular and algebraic conditions. By involving the factor 2 and extending the powers of both primes pp and qq, this work significantly broadens the landscape for applications in cyclic code generation and algebraic coding theory.

Mathematical Notation and Style:
The mathematical notation is clear, concise, and consistent with standard conventions in algebra and coding theory. The use of symbols like ϕ\phi (Euler's function), GF(l)GF(l), and exponents are properly formatted. The structure of the paper is logically sequenced, and the assumptions are explicitly stated where necessary.

Relevance to Mathematical Subject Classification (MSC):
The MSC codes are appropriately assigned:

- 11T30 (Cyclotomy, primitive roots, residue systems)
- 94B15 (Linear codes, general)
- 11T71 (Algebraic coding theory; cryptography)

Overall Verdict:
This paper presents a solid and important contribution to the study of cyclotomic cosets and their applications in cyclic code construction over finite fields. It is grounded in strong theoretical foundations, extends prior literature, and offers precise and well-documented results that will be of interest to mathematicians and coding theorists alike.