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REVIEWER'S REPORT

Manuscript No.: **IJAR-52557** Date: 02-07-2025

Title: Cyclotomic Cosets in The Ring R_(4p^n q^m)=GF(l)[x]/(x^(4p^n q^m)-1)

Recommendation:
Accept as it is .....YES.....
Accept after minor revision.....
Accept after major revision .....
Do not accept (*Reasons below*) .....

Rating	Excel.	Good	Fair	Poor
Originality			✓	
Techn. Quality			✓	
Clarity		✓		
Significance		✓		

Reviewer Name: Mir Tanveer

Reviewer's Comment for Publication.

Overall Evaluation:
This paper addresses an important and mathematically rich topic within algebraic coding theory and finite field structures by examining cyclotomic cosets in a generalized ring setting. It successfully extends prior foundational work and contributes significantly to the theoretical framework of minimal cyclic codes and idempotent generators over generalized composite modulus.

Strengths:

1. **Mathematical Depth and Rigor**:
   The study demonstrates a high level of theoretical rigor, particularly in its use of group theory, number theory, and finite field algebra. The derivation of the number and explicit structure of cyclotomic cosets in R4pnqmR_{4p^n q^m} is conducted with mathematical precision and attention to detail.
2. **Generalization of Prior Results**:
   The paper clearly builds upon and extends the work of Bakshi, Raka, Sahni, and Sehgal by generalizing their results to the setting where z=4pnqmz = 4p^n q^m. It appropriately situates this generalization within the broader literature, establishing continuity with earlier findings and highlighting the significance of the current advancement.
3. **Clarity in Definitions and Lemmas**:
   The formal presentation of definitions (e.g., cyclotomic cosets), notations, and Lemma 2.1 is clear and mathematically consistent. Lemma 2.1 is central to the paper's analytical core and is stated and proved with completeness.
4. **Effective Use of Euler's Totient Function**:
   The role of Euler's totient function and its properties in defining the order of elements in modular arithmetic is well employed throughout the analysis. The distinction between modular orders over 2pn−j2p^{n-j}, 2qm−k2q^{m-k}, and 4pn−jqm−k4p^{n-j} q^{m-k} is properly handled with justified algebraic reasoning.

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5. **Well-Chosen Conditions**:
   The assumptions and conditions—such as ll being a primitive root modulo pnp^n and qmq^m, and gcd(φ(pn),φ(qm))=d\gcd(\varphi(p^n), \varphi(q^m)) = d, with p∤(q−1)p \nmid (q-1)—are logically consistent and ensure the applicability of results. These conditions add depth to the analysis while keeping the scope well-defined.
6. **Quantification of Cyclotomic Cosets**:
   The expression for the total number of cyclotomic cosets, namely 4(m×n×d+m+n+1)4(m \times n \times d + m + n + 1), is explicitly presented and forms a significant contribution. It indicates both theoretical completeness and computational relevance for further applications in coding theory.
7. **Relevance to Algebraic Coding Theory**:
   The study provides insights directly applicable to the construction of minimal cyclic codes over composite lengths, enriching the algebraic toolkit available to researchers working in code theory and related algebraic structures.

Conclusion:
This paper constitutes a substantial theoretical contribution to the study of cyclotomic cosets over composite ring structures and finite fields. It methodically extends previous results to a more generalized setting and maintains mathematical rigor throughout. The content is highly relevant to both pure mathematical interests in ring theory and number theory, as well as to practical domains in algebraic coding theory.