Numerical study of the effect of the magnetic field on magnetoconvective flow of a Newtonian fluid confined between two vertically offset hemispheres.

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Abstract

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This work consists of numerically studying the effect of a uniform oblique magnetic field on the magnetoconvection of an electrically conductive Newtonian fluid. The system studied is a hemispherical cavity formed by two vertically eccentric hemispheres. The inner hemisphere is subjected to a constant density flow, while the outer hemisphere is maintained at a fixed temperature. The thermal and electrical boundary conditions of the system under study are combined to find the critical values of the parameters that indicate the onset of instability. The equations governing this fluid instability are first projected into a bispherical coordinate system and then discretized using the finite difference method. This made it possible to develop a computer code in FORTRAN. This code was used to determine the growth rates for different values of the Hartmann number. The eccentricity, Rayleigh number, radius ratio, and magnetic field inclination angle remain fixed in numerical simulations. The results show that the magnetic field has an effect on this transfer mode.

If the Hartmann number is low, the magnetic field is weak and the transfer is more convective, leading to an extended and chaotic convection structure, thus increasing the Nusselt number. A weak magnetic field has very little effect on magnetoconvective isotherms and isocurrents. On the other hand, when the Hartmann number is high, the magnetic field inhibits convection, thereby reducing the Nusselt number. Magnetoconvective isotherms and isocurrents are strongly affected, with a decrease in the size of the vortex and magnetoconvective motion. The results obtained at the end of the study are consistent and agree with those found in the literature.

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Introduction: -

The phenomenon of magnetoconvection, which describes the convective motion of an electrically conductive fluid subjected to both convective and magnetic forces, has been the subject of numerous studies in recent decades [1], [2]. These materials are of interest because of their implications in many natural and applied phenomena [3]. According to [4], magnetoconvection has various applications in fields such as geophysics, astrophysics, plasma physics, missile technology, medicine, and biology, among others. Consequently, numerous experimental and numerical studies have been carried out on the magnetoconvection of a fluid confined in enclosures of various configurations, in parallel with studies on pure natural convection [5]. These enclosures often have varied geometries, which can be parallelepiped [6], [7], cylindrical [8-9], or even spherical [10], [11], [12], [13].

Correlations giving the Nusselt and Rayleigh numbers are sometimes proposed. An increase in these numbers, reflecting an intensification of natural convection or the magnetic field, can influence the viscosity of the fluid and the stability of the flows depending on the geometry of the walls [14], [15]. In this context, modeling studies of magnetoconvection have shown that, in addition to stabilizing the main convection roll, a horizontal magnetic field leads to an increase in kinetic energy and heat transfer rate compared to a study without a magnetic field [7]. Furthermore, an analysis of the Hall effects of magnetoconvective instability and heat transfer conducted by [16] studies the parameters that can influence the flow field and temperature distribution. According to the results obtained, it can be seen that Hall currents significantly reduce the flow field. The studies conducted by references [17] and [18] sought to obtain a comprehensive and essential understanding of the characteristics of flows and heat transfer in an enclosure in the presence of a magnetic field, demonstrating that this magnetic field reduces the heat transfer rate. The impact of the magnetic field on mixed convection with an exponential temperature distribution, as well as on internal thermal and viscous dissipation, was examined by [19]. It was observed that increasing the Prandtl number decreases the skin friction coefficient, while increasing the magnetic field increases the local Nusselt number. A study conducted by [10] examined the transient regime of natural convection of a non-conductive Newtonian fluid between two vertical eccentric spheres, with the inner sphere exposed to a constant heat flux and the outer sphere maintained at a constant temperature. Their results show that increasing the modified Rayleigh number allows the steady state to be reached more quickly, and that eccentricity has a negligible influence on the establishment of equilibrium. Convective motion is amplified by positive eccentricities. Heat exchange, characterized by the Nusselt number, increases with the modified Rayleigh number. A study conducted by [20] focused on the case of a hemisphere, and the results indicate that the center of the vortex moves upward with greater eccentricities.The Nusselt number also increases with the modified Rayleigh number. When the latter increases, the temperature decreases for a given eccentricity. This extensive literature highlights the importance and scientific scope of thermal convection in an electrically conductive fluid subjected to a magnetic field [21]. It is precisely with this in mind that the present study was initiated. Our study focuses on natural convection between two eccentric hemispheres of a conductive fluid subjected to a magnetic field. In this study, we explore the dynamic interactions between magnetism and convection, highlighting the influences of the magnetic field on circulation and heat transfer patterns. By analyzing these phenomena, we seek to deepen our understanding of the underlying mechanisms and identify the practical implications of magnetoconvection in various contexts.

Materials and methods:

- Formulation of the problem

Figure 1 shows the displacement of an electrically conductive fluid with constant viscosity (ionized air) under the influence of an oblique magnetic field, which is located in an annular space bounded by two vertically eccentric hemispheres. The radii of the inner and outer hemispheres are denoted Ri and Re, respectively. The distance between the centers of the two hemispheres, called eccentricity e', is defined as the algebraic value of this distance. The temperature inside and on the walls of the enclosure is uniform at the start. A heat flux (q') of constant density is applied to the inner hemisphere, while the temperature of the outer hemisphere remains constant (T'). The walls separating the two hemispheres at angles $\theta=0$ and $\theta=\pi$ are adiabatic. The temperature difference between the two hemispheres will cause transient natural convection of the conductive fluid, which will develop inside the domain.

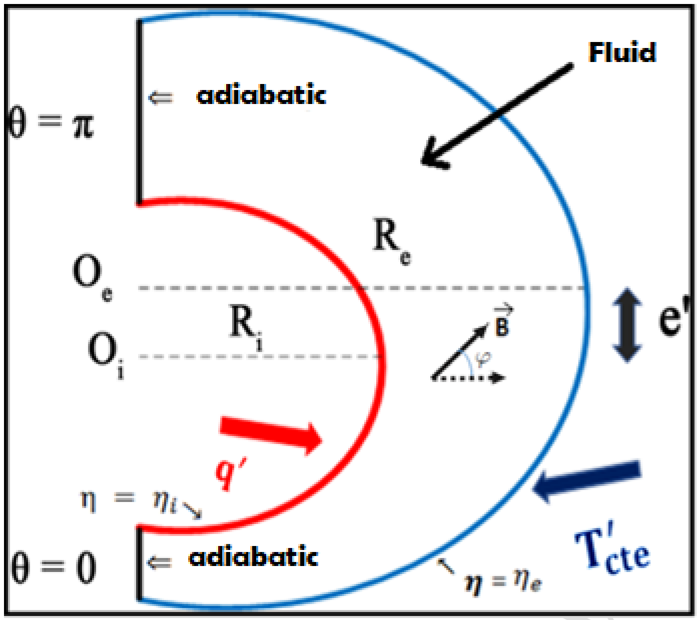

Figure 1 Geometry of the problem [2]

The physical properties of the fluid are constant, except for its density in the gravity term of the equation of motion, where it varies linearly with temperature according to Boussinesq's law. The fluid is Newtonian and the flow is laminar, incompressible, and two-dimensional. It is assumed that the magnetic field is constant and that the induced field is negligible. The viscous dissipation function, radiative effects, and pressure term are neglected. The boundaries of the system under study are considered to be electrically insulating. The walls of our enclosure are composed of two spherical parts and two others offset from the vertical. To simplify the boundary conditions, it is therefore necessary to find a curved coordinate system in which the boundaries of our domain are determined by lines of constant coordinates. Thus, given the geometry of the enclosure, the bispherical coordinate system is the most appropriate. For two-dimensional flow, the transition from Cartesian coordinates (x,y) to bispherical coordinates is given by relation (1):

$$x = \frac{a \sin\theta}{ch\eta - cos\theta} \quad ; \quad y = \frac{ash\eta}{ch\eta - cos\theta} \tag{1}$$

Along the vertical axis are two walls identified by θ = 0 and θ = π. The inner and outer hemispheres are represented by the coordinate lines h=hi and h=he, respectively. After introducing simplifying assumptions and the vorticity-current function formalism, we establish the various dimensionless equations needed to solve the problem considered in this study. The vortex-flux functions (vortex flow) are translated by the equations of momentum and heat translated by relation (2):

$$\partial_t F + A(U)\partial_\eta F + B(V)\partial_\theta F = P\left(\partial_\eta^2 F + \partial_\theta^2 F\right) + S\left(G_2\partial_\eta T - G_1\partial_\theta T\right) + R\left(\partial_\eta B_1 - \partial_\theta B_2\right) \tag{2}$$

The different values of variables F, A, B, P, S, and G are given in Table 1.

Table 1: Variables in the heat and vorticity equation

Equation	F	A(U)	B(V)	P	S	R
Heat	T	$\frac{1}{H}\left[U-\frac{G_2}{K}\right]$	$\frac{1}{H}\left[V+\frac{G_1}{K}\right]$	$\frac{1}{H^2}$	0	0
Movement	$\frac{\Omega}{K}$	$\frac{1}{H}\left[U-\frac{3PrG_2}{K}\right]$	$\frac{1}{H}\left[V+\frac{3PrG_1}{K}\right]$	$\frac{Pr}{H^2}$	$\frac{Ra.Pr}{KH}$	$\frac{Ha^2.Pr}{KH^2}$

With:

$$B_1 = H\left(UB_\eta B_\theta - VB_\eta^2\right) \quad ; \quad B_2 = H\left(VB_\eta B_\theta - UB_\theta^2\right) \tag{3}$$

$$B_\eta = \frac{B_\eta^*}{B_0} = G_2\, cos\varphi + G_1\, sin\varphi \quad and \quad B_\theta = \frac{B_\eta^*}{B_0} = G_2\, sin\varphi - G_1\, cos\varphi \tag{4}$$

Where the quantities U, V, G_1, G_2, K, H are defined by equations (5), (6) and (7)

$$U = \frac{1}{HK}\partial_\theta \Psi \quad ; \quad V = -\frac{1}{HK}\partial_\eta \Psi \tag{5}$$

$$G_1 = \frac{1-\cos\theta\, ch\,\eta}{ch\,\eta - \cos\theta} \quad ; \quad G_2 = -\frac{\sin\theta\, sh\,\eta}{ch\,\eta - \cos\theta} \tag{6}$$

$$K = \frac{asin\theta}{D(ch\eta - cos\theta)} \quad ; \quad H = \frac{a}{D(ch\eta - cos\theta)} \tag{7}$$

The incompressibility condition is verified by the equation of the current function given by relation (8):

$$\Omega = \frac{1}{K^2H}\left(G_2\partial_\eta\Psi - G_1\partial_\theta\Psi\right) - \frac{1}{KH^2}\left(\partial_\eta^2\Psi + \partial_\theta^2\Psi\right) \tag{8}$$

In addition to these different equations, there are boundary conditions and initial conditions.

At t = 0, the conditions are expressed by equation (9):

$$\Omega=\Psi=T=U=V=0 \tag{9}$$

At t > 0, the boundary conditions are expressed by equations (10), (11), and (12) depending on the location of the wall.

- On the inner spherical surface ($\eta = \eta i$)

$$\Psi = U = V = 0\,;\ \Omega = -\frac{1}{KH}\partial_\eta^2\Psi\,;\ \partial_\eta T = H_i = \frac{ch\eta_i}{sh^2\eta_i} \tag{10}$$

- On the outer spherical surface ($\eta=\eta e$)

$$\Psi = U = V = 0\,;\ \Omega = -\frac{1}{KH}\partial_\eta^2\Psi\,; T = 0 \tag{11}$$

- On the two vertical walls ($\theta = 0$, $\theta = \pi$)

$$\Psi = U = V = 0;\ \Omega = -\frac{1}{KH}\partial_\theta^2\Psi\ ;\ \partial_\eta T = 0 \quad (12)$$

The Nusselt number represents the thermal energy transmitted by a spherical wall. The local Nusselt number Nu and the average Nusselt number $\overline{Nu}$on spherical walls are defined by equations (13) and (14).

- For the inner spherical wall

$$Nu_i = \frac{1}{T_{i,m}} \quad ; \quad \overline{Nu_i} = \frac{1}{S_i}\int Nu_i dS_i \quad (13)$$

- For the outer spherical wall

$$Nu_e = \frac{1}{H_e T_{i,m}}\partial_\eta T \quad ; \quad \overline{Nu_e} = \frac{1}{S_e}\int Nu_e\, d\, S_e \quad (14)$$

- Numerical analysis

For the development of a numerical code simulating the magnetoconvection of a Newtonian fluid confined in an annular space, we used the following methods:

- The implicit alternating direction method (ADI) for the temporal resolution of the momentum and heat equations;

- The finite difference method for spatial integration.

The THOMAS algorithm will be used to solve the system of linear equations obtained by the ADI method. However, for the flow function equation, the solution will be obtained using the successive overrelaxation (SOR) method with an optimal relaxation parameter. In the iterative loop, the calculated result Znew for a quantity to be determined will be considered a convergent solution only if it satisfies the following relation (15) with the old value Zold.

$$\frac{|\boldsymbol{Z_{new}} - \boldsymbol{Z_{old}}|_{\boldsymbol{max}}}{|\boldsymbol{Z_{new}}|} \leq 10^{-5}$$

(15)Steady state is only achieved if this relative error between two consecutive time steps for all quantities obeys relation (16):

$$\frac{|Z^{n+1} - Z^n|_{max}}{|Z^{n+1}|_{max}} \leq 10^{-5} \quad (16)$$

Z^n represents Ω, Ψ, or T for the n^{th} time step.

Results and discussion

In this section, in order to better understand the effect of the magnetic field on magnetoconvective flow, we will discuss the results obtained by numerical simulations.

-Calculation conditions

The choice of a 51 x 51 grid and a 10^{-4}-time step was based on tests we conducted on the influence of these factors. The results of these simulation tests are presented in Tables 2 and 3 and prove that our choices are, among other things, a good compromise.

Table 2: Effects of time steps on the Nusselt number of the thermal wall for Ha=1, Ra=10^5, e=0, $\Delta t=10^{-4}$ and the grid system is 51x51

	Time steps		
	10^{-3}	**10^{-4}**	**10^{-5}**
Nu	4.7337	4.7298	4.7296
Difference (%)	0.087	0.004	0
Time computing (min)	5	124	802

Table 3: Effects of mesh refinement on the Nusselt number of the thermal wall for Ha=1, Ra=10^5, e=0, and $\Delta t=10^{-4}$

	Mesh grid							
	21*21	**21*41**	**41*41**	**41*51**	**41*81**	**51*51**	**51*81**	**81*81**
Nu	4.8750	4.8871	4.7515	4.7503	4.7502	4.7298	4.7297	4.7060
Difference (%)	3.59	3.85	0.97	0.94	0.94	0.51	0.50	0
Time computing (min)	9	97	225	261	362	348	447	604

-Validation

In the absence of a magnetic field, our problem is reduced to that of natural laminar convection. The data presented in Table 4 provide the values of the average Nusselt number calculated for different Rayleigh numbers. We compared our results with those from the study of transient laminar convection between two vertically offset hemispheres, as referenced in [20]. These comparisons indicate a relative difference of 2.72% for all situations examined, demonstrating excellent agreement between the results.

Table 4: Comparison of the average Nusselt number in the case of e = 0

	Ra				
	10^3	**10^4**	**10^5**	**10^6**	**10^7**
Nusselt number (our results)	2.0673	3.0379	4.8920	7.7680	11.708
Nusselt number (results of [20])	2.125	3.0651	4.982	7.6874	11.671
Difference (%)	2.72	0.89	1.81	1.05	0.32

-Influence of Hartmann's number

For this study, apart from the Hartmann number, which is directly proportional to the magnetic field, all other parameters are fixed. The eccentricity is set at 0.5, the magnetic field is inclined at an angle $\varphi = \pi/3$, and the Rayleigh number is set at $Ra=10^5$.

--Isotherms and current lines

Figures 2, 3, 4, and 5 show the temporal variations of isotherms and streamlines for different values of the Hartmann number (0.1, 1, 10, and 100). When the magnetic field is weak (Ha=0.1), the isotherms, as shown in Figure 2, are mainly distorted by fluid movements and temperature differences. Consequently, the magnetic effect has a relatively weak influence compared to other factors such as viscous flow and thermal convection, and the streamlines are only slightly affected by the magnetic field. The magnetic forces are therefore insufficiently powerful to cause significant disruption to the streamlines, although they do distort them slightly.

When the Hartmann number is equal to 1, Figures 3 show that the isotherms are influenced by both electromagnetic forces and convective forces in equal proportions. For example, in a free convection regime, where convective forces dominate, the isotherms may be more curved and distorted than those observed in a strictly electromagnetic regime. The Hartmann number also slows down convection and enhances magnetic diffusion. Current lines also undergo reorientation and deformation. The magnetic field suppresses vertical convection in the fluid, leading to more horizontal and less vertical current lines. This results in a more regular organization of the current lines. For a Hartmann number greater than 1 (Ha=10 or Ha=100), the isotherms in Figures 4 and 5 appear more uniform and less distorted. The magnetic field tends to suppress convective motions in directions perpendicular to its orientation. Thus, the isotherms align with the magnetic field, meaning that they are parallel to the magnetic lines of force. This results in a specific organization of isotherms, with elongated structures in the direction of the magnetic field. In addition, the current lines are also modified. The effects of the magnetic field lead to the suppression of turbulent convective motions and the formation of more organized convective cells, aligned along the magnetic field lines, which are more regular and less chaotic than in the case where the Hartmann number is less than 1.

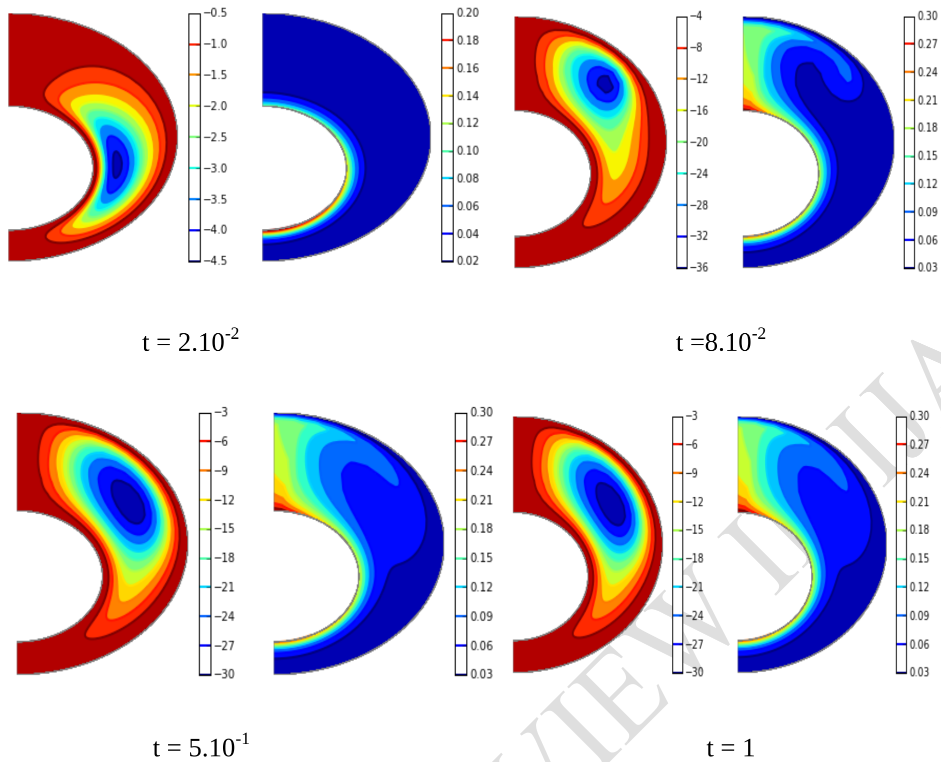

Figure 2: Streamlines and isotherms for Ra = 10^5; e = 0.5; Ha = 0.1; φ = π/3

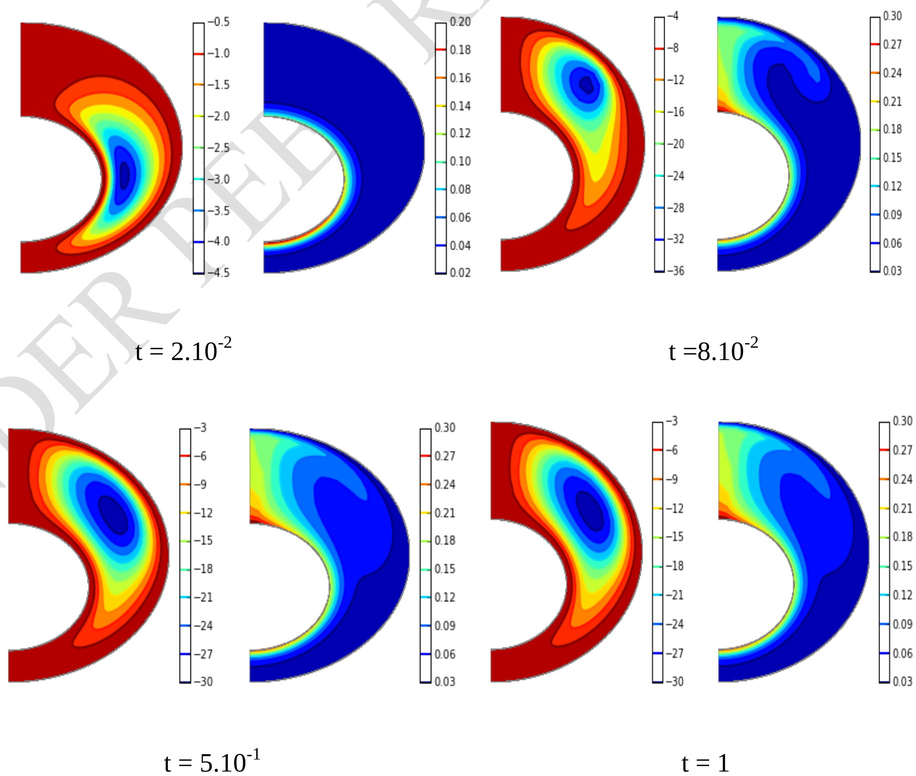

Figure 3: Streamlines and isotherms for Ra = 10^5; e = 0.5; Ha = 1; φ = π/3

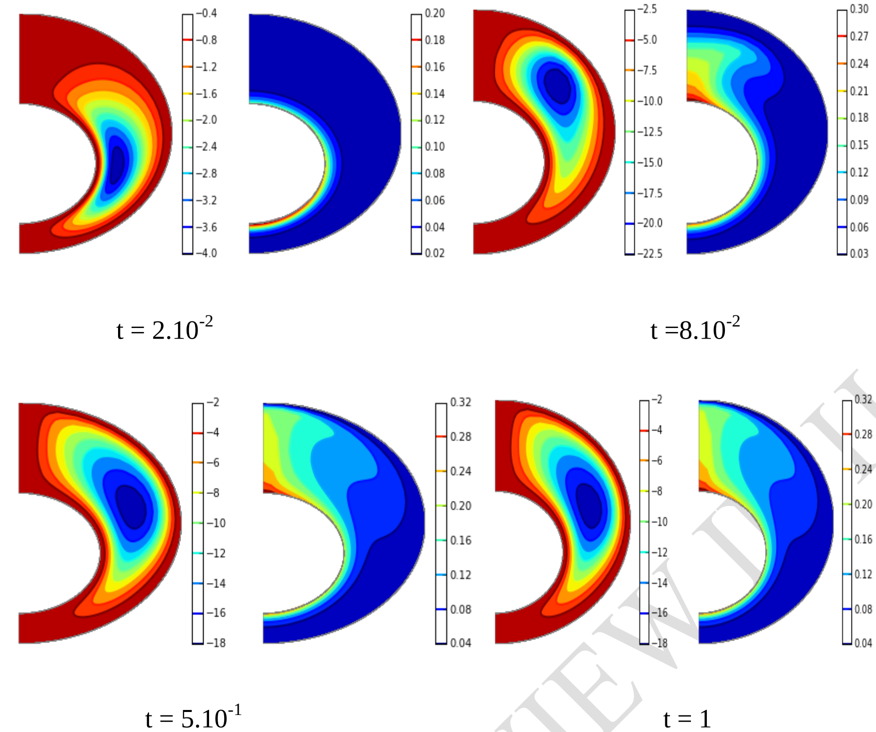

Figure 4: Streamlines and isotherms for Ra = $10^{5;}$ e = 0.5; Ha=10; φ = π/3

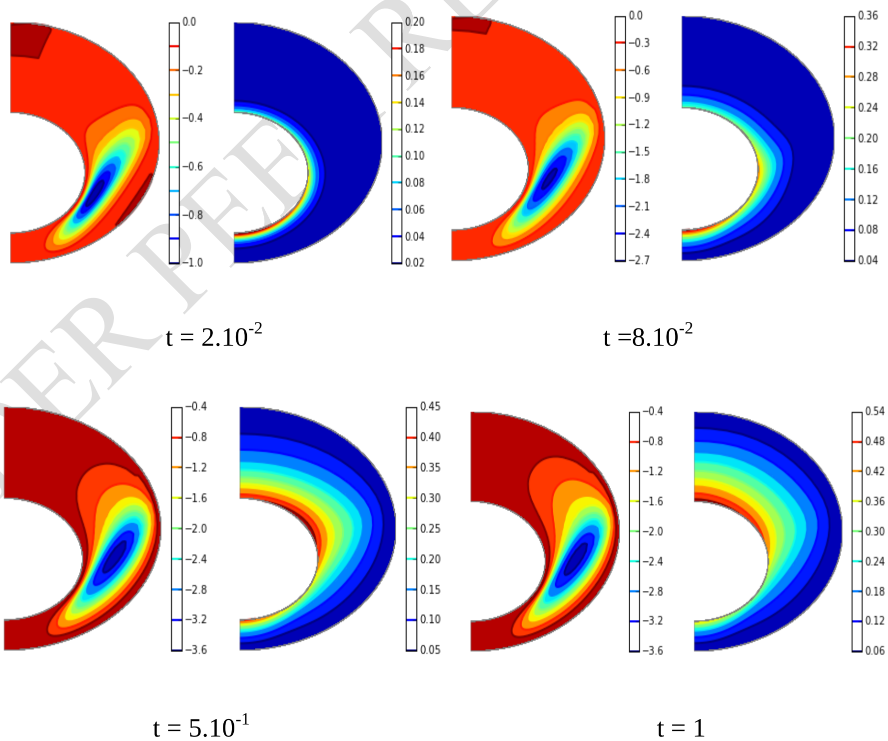

Figure 5: Streamlines and isotherms for Ra = $10^{5;}$ e = 0.5; Ha = 100; φ = π/3

--Minimum current function, Nusselt number, and internal wall temperature

Based on the observed evolution of the Nusselt number on the heated inner wall of the hemisphere in Figure 6, we can conclude that the Hartmann number has a significant influence on the Nusselt number in our magnetoconvection problem. Indeed, the Nusselt number decreases as the Hartmann number increases. This is because the magnetic field exerts a force on the charged particles in the fluid, which counteracts the natural convection motion. As a result, convection is inhibited and heat transfer becomes less efficient. When the Hartmann number is low, the magnetic field is also weak, thus promoting convection and leading to an extended and chaotic convection structure, which increases the Nusselt number. On the other hand, as the Hartmann number increases, the magnetic field inhibits convection, thus reducing the Nusselt number.

Furthermore, in the same figure, we can see that the Hartmann number influences the average temperature of the heated wall in a magnetoconvection problem. As the Hartmann number increases, the temperature of the heated wall also increases, probably due to a decrease in heat transfer efficiency caused by the magnetic field. Thus, the presence of the magnetic field has a more pronounced warming effect on the inner wall of the hemisphere.

The evolution of the minimum current function over dimensionless time also proves that this function is influenced by the Hartmann number. Initially, the minimum current function decreases before stabilizing for a long period, but it increases for high Hartmann number values. This sharp decrease observed for low Hartmann numbers is due to the fact that the initial fluid movements may be more turbulent, thus causing a loss of magnetic energy in the system. However, as the Hartmann number increases, the magnetic force becomes more predominant than convection. This has the effect of stabilizing the currents and organizing the convection structure. In other words, the magnetic field tends to suppress turbulent movements and promote more regular and orderly currents.

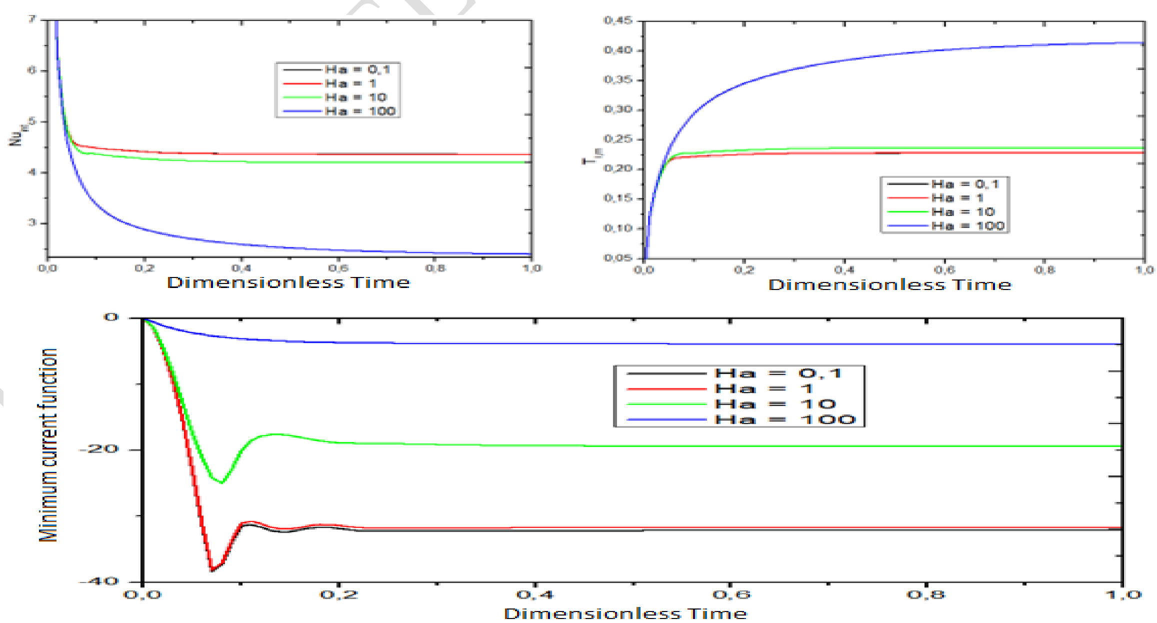

Figure 6: Influence of the Hartmann number on the various parameters

Conclusion

In this paper, we used numerical methods to study the magnetoconvection of a Newtonian fluid, in this case humid air, confined between two eccentric hemispheres. The hemispherical cavity studied is subject to thermal and electrical boundary conditions in order to determine the critical values of the parameters that mark the onset of instability. The objective is to highlight the impact of the Hartmann number on magnetoconvective transfer. To do this, a constant heat flux density is imposed on the inner hemispherical wall and a constant temperature on the outer hemisphere. The equations governing magnetoconvection are expressed in bispherical coordinates. Discretization using the finite difference method enabled the development of a computer code in Fortran. The equations are solved using the ADI and SOR methods. Assumptions are made about the vorticity and flux function variables. For very weak magnetic fields, magnetoconvection reduces to a natural convection problem. The results obtained at the end of the study are consistent and significant. The geometry of the system studied reveals that different values of the Hartmann number have effects on the magnetoconvection of a fluid subjected to a constant oblique magnetic field.

For a low Hartmann number (Ha=0.1), convection dominates and the magnetoconvective isotherms and isocurrents are similar to those obtained without a magnetic field.

For a large Hartmann number (Ha=10 or Ha=100), the magnetic field dominates and the isotherms and magnetoconvective isocurrents are strongly affected, with a decrease in the size of the vortex and magnetoconvective motion.

When the Hartmann number is low, the magnetic field is weak and the transfer is more convective, leading to an extended and chaotic convection structure, thus increasing the Nusselt number. On the other hand, when the Hartmann number is high, the magnetic field inhibits convection, thus reducing the Nusselt number.

The Hartmann number also affects the temperature of the heated wall in this magnetoconvection problem. As Ha increases, the temperature of the heated wall increases, due to a decrease in heat transfer efficiency caused by the magnetic field.

For a low Hartmann number, the minimum current function decreases rapidly over time, then increases before stabilizing when steady state is reached. This decrease becomes smaller as the Hartmann number increases, meaning that the minimum current function increases with the Hartmann number because the magnetic field reduces the convection current. Furthermore, our results are in agreement with solutions available in the literature, such as [10], [20], etc.

Nomenclature

Latin	Greek
a, parameter of the torus pole (m)	α, thermal diffusivity, ($m^2.s^{-1}$)
e, eccentricity	β, thermal expansion coefficient, (K^{-1})
g, gravitational intensity ($m.s^{-2}$)	σ, electrical conductivity, ($A.m.V^{-1}$)
Coefficients g_1 and g_2	Δt, time step, (s)
H and K: dimensionless metric coefficient	ΔT, temperature difference between the two hemispheres, (K)
B_0: magnetic field intensity ($N.A^{-1}.m^{-2}$)	η and θ, bispherical coordinates, (m)
Ha: Hartmann number	λ, thermal conductivity, ($W.K^{-1}.m^{-1}$)
Nu_e, Nusselt number for the outer hemisphere	ν, kinematic viscosity, ($m^3.s^{-1}$)
$Nu_{i:}$ Nusselt number for the inner hemisphere	Ψ, dimensionless flux function,
Oi and Oe: center of the inner and outer hemispheres, respectively	Ψ', dimensional flux function, ($m^3.s^{-1}$)
Pr: Prandtl number	Ω, dimensionless vorticity,
q: heat flux density ($W.m^{-2}$)	Ω', dimensional vorticity, ($m^3.s^{-2}$)
Ri and Re: radius of the inner and outer hemispheres, respectively	
Ra: Rayleigh number	
t: dimensionless time	
t': dimensional time (s)	
T: dimensionless temperature	
U and V: dimensionless components of velocity in the transformed planes	
x and y, Cartesian coordinates, (m)	

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