CONTRIBUTION TO MODELING AND IMPROVING QUALITY CONTROL OF FINISHED PRODUCTS IN PRODUCTION SYSTEMS BY USING BAYESIAN NETWORKS AND LEAN SIX SIGMA.

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Abstract

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Industrial production systems in the Sahelian region of sub-Saharan Africa and the Central African sub-region face numerous challenges, including the lack of control over customer satisfaction levels and the instability and variability of operational quality control processes. This often leads to consumer dissatisfaction and an insufficient product conformity rate. To address this issue, we propose a methodology aimed at reducing variability and improving the operational quality control process of industrial production systems. This methodology combines the use of Lean Six Sigma (LSS) tools, Bayesian Networks (BNs), and multilinear regression analysis. Our combined approach consists of six stages. To implement this combined approach, we selected a tissue production system from the SITRACEL industrial company based in Cameroon. This implementation revealed an insufficient conformity rate of 3.727σ, customer dissatisfaction of 16.25% compared to benchmarks, dominant quality defect causes directly related to the machine, and modeled quality control indicators to track variability in scrap and waste rates.

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Introduction:-

Industrial manufacturing companies in the Sahelian region of sub-Saharan Africa and the Central African sub-region face numerous daily challenges in producing finished products that meet the quality and needs of consumers. The main challenges include electrical energy instability [1], insufficient quality controls, lack of standardization, reactive rather than preventive maintenance, and lack of monitoring systems. In the face of these numerous challenges, we focused on those related to product variability and operational quality control processes. These issues lead to financial losses and decreased customer satisfaction [2].To address these challenges, Lean Six Sigma (LSS) tools and Bayesian Networks (BNs) are often used.

While Lean Six Sigma is gaining popularity for process optimization [3], [4], [5], [6], [7], the integration of Bayesian networks for modeling and diagnosing quality defect causes remains marginal. The scientific literature [8], [9], [10], [11], [12], [13]highlights the use of Bayesian networks, often combined with other methods such as FMEA, logical diagnostic approach, HAZOP, and fault tree analysis, for diagnosis and prediction in industrial maintenance, but rarely in conjunction with Lean Six Sigma. S. WAHABI et al [14],discuss LSS in Africa, but without detailing its coupling with Bayesian networks. This observation has allowed us to highlight a knowledge and practice gap regarding the combined approach of Bayesian networks and Lean Six Sigma in the sub-region. To help manufacturing industries in the sub-region face these challenges, we propose a methodology that we call a combined approach, which is both operational and analytical. This combined approach results from the combination of LSS tools, Bayesian networks, and multilinear regression analysis.

These studies aim to propose a promising solution to industries for modeling interactions between different production factors, diagnosing the root causes of defects, and predicting the impact of improvements, thereby addressing the issue of non-quality and product variability. In this methodology, Lean Six Sigma will be used to reduce variability and eliminate waste in the operational processes of industrial production systems, while Bayesian Networks will be used for graphical and analytical modeling of industrial systems. We will implement this methodology on a concrete case study from an industry in the sub-region. This work is organized as follows: on one hand, we have the materials and methods, and on the other hand, the results and discussions.

1. MATERIALS ET METHODS

1.1 Lean Six-Sigma

Lean Six Sigma (LSS) is a process improvement method that combines the principles of Lean Manufacturing and Six Sigma. It is the alliance of two concepts that link the notions of productivity (Lean) and quality (Six Sigma) [15]. The implementation steps of Lean Six Sigma through the DMAIC process (Define, Measure, Analyze, Improve, and Control) are presented in Table 1. The common tools necessary for implementing each step are also presented in Table 1.

Tableau 1 : Implementation Steps of Lean Six Sigma (LSS)

Steps	**Overview**	**Implementation Tools**
Define	This step aims to precisely identify the problem to be solved. We clearly define the objectives, scope, deliverables. The goal is to have a shared and precise understanding of the problem and how success will be measured.	- CTQ Diagram - Process Black Box - SIPOC Diagram - 5 Whys Diagram - Project Charter
Measurer	This step focuses on collecting data to quantify the extent of the problem and establish a baseline for future improvements. The goal is to obtain a clear and precise picture of the current situation.	- CTQ measurement - Ishikawa giagram - Conformity rate measurement - Sigma performance levelMeasurement
Analyze	This step aims to identify the root causes of the problem. We use analysis tools to examine the collected data and determine the factors that contribute most to the problem. The goal is to understand « why » the problemoccurs.	- Pareto analysis - Causes / Effets table
Improve	The improvement step involves developing and implementing solutions to eliminate the root causes identified during the analysis step. The goal is to find effective and sustainable solutions.	(No specifictoolsmentioned)
Control	This final step aims to ensure that the improvements achieved are maintained over time. The goal is to prevent the problem from recurring and anchor performance gains.	(No specifictoolsmentioned)

1.2 Bayesian Networks

Bayesian networks are a graphical representation through a directed acyclic graph of the relationships between variables. They are also a graphical probabilistic model that represents the links or conditions between variables, allowing for better visualization of root variables and dependent variables. Bayesian networks enable the representation of knowledge and uncertainties, modeling probabilistic reasoning through inferences. Their application allows for diagnostics, prediction, and causality studies to provide better guidance in certain decision-making frameworks. Mathematically, Bayesian networks are defined as follows [16]:

- A directed acyclic graph, G = (V, E), where V is the set of nodes of G, and E the set of arcs of G;
- A finite probability space (Ω, Z, P);
- A set of random variables associated with the nodes of the graph and defined on (Ω, Z, P), such that:

$$P(V_1, V_2, \dots, V_n) = \prod_{i=1}^{n} P(V_i | C(V_i)) \qquad \textbf{(1)}$$

Where $C(V_i)$ represents the set of causes (parents) of V_i in the graph G.

1.3 Multiple Regression Analysis

Multiple regression analysis is the study of a relationship between a dependent variable and two or more independent variables. For the case of samples, the estimation model [17] is defined by equation (2). Regression analysis allows predicting the behavior of the dependent variable Y as a function of the variables $X_1, X_2, \ldots, X_k$ on which it depends.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k \qquad (2)$$

Where:

- Y is the dependent variable.
- $X_1, X_2, \ldots, X_k$ are the independent variables.
- β_0 is the intercept (the value of Y when all independent variables are zero).
- $\beta_1, \beta_2, \ldots, \beta_k$ are the regression coefficients, which represent the effect of each independent variable on the dependent variable, all else being equal.

The estimation model (Equation (2)) thus obtained must undergo two tests to be validated: the T-test and the F-test.

1.4 Combined Approach Methodology

The combined approach we propose in this research work is the combination of LSS and RNs tools, to which a prediction tool is associated, namely multilinear regression analysis, because these tools have complementarities for improving quality in an industrial setting. This combined approach is implemented through six major steps, each with a specific objective, as presented in Figure 1.

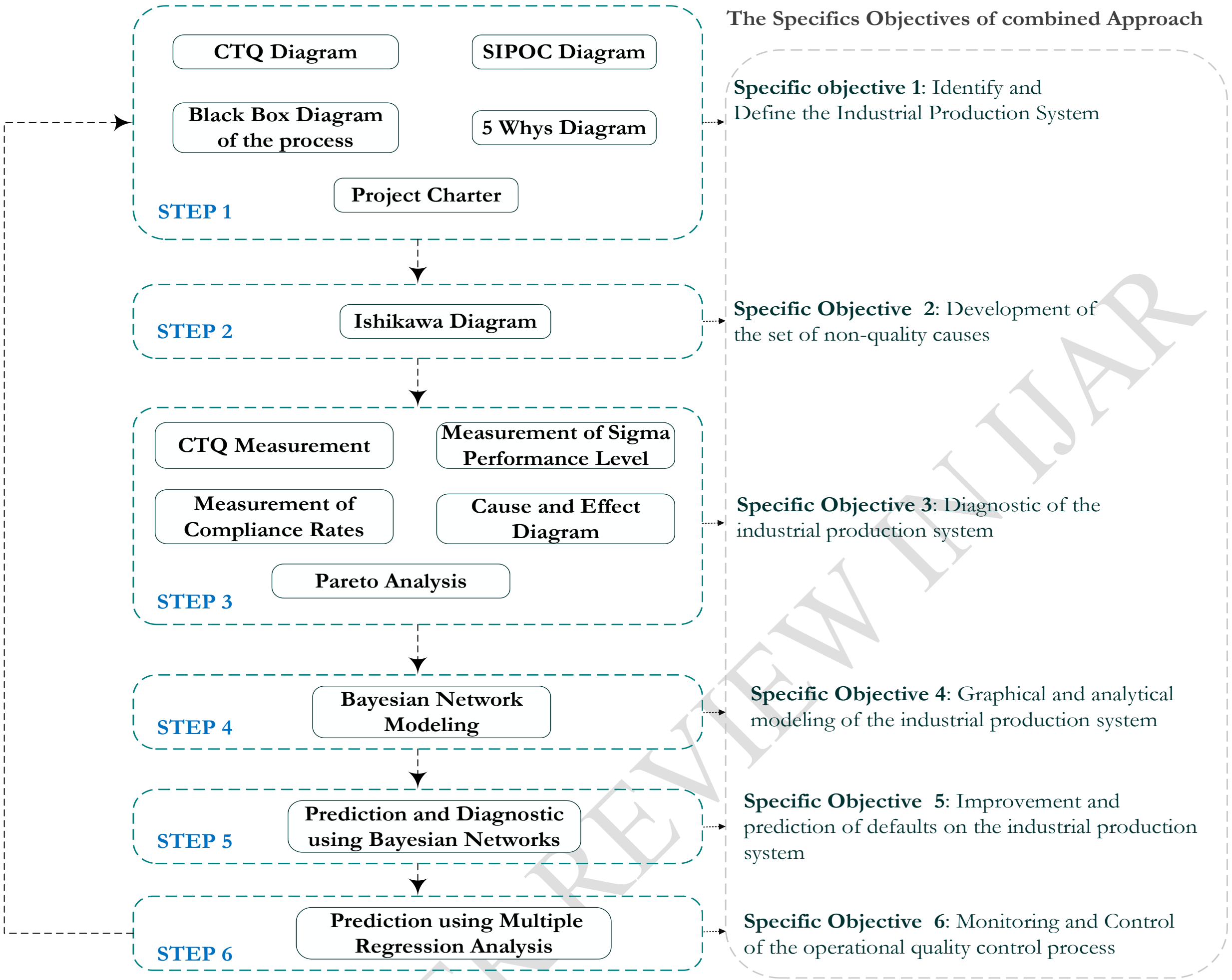


Figure 1 : Structuring and Specific Objectives of the Combined Approach

1.4.1 Step 1: Identify and define the industrial production system

The main objective of this step is to lay the foundations for the quality improvement project. It involves understanding in depth the production system, customer needs, and the quality problem to be solved. This allows framing the project and ensuring that improvement efforts are focused on the right objectives. This step consists of five main tools, each playing a specific role. **These include the CTQ diagram, SIPOC diagram, black box diagram, 5 whys diagram, and project charter.** The combination of these tools provides a holistic view of the production system.

CTQ Diagram (Critical to Quality): This tool translates customer needs into measurable and specific requirements. It breaks down customer expectations into critical characteristics for quality. It ensures that process improvements meet customer needs and focus on aspects of the process that have the most impact on customer satisfaction.

SIPOC Diagram (Suppliers, Inputs, Process, Outputs, Customers): This tool provides a high-level visual representation of the production process, identifying suppliers, inputs, the process itself, outputs, and customers. It provides an overview of the process and its environment and helps understand interactions between different parts of the process.

Black Box Diagram: This tool provides a simplified representation of the production process, considering it as a « black box » with inputs and outputs, without detailing internal operations. It clarifies the boundaries of the production process and its interactions with its environment and facilitates understanding of the process as a whole.

5 Whys Diagram: This tool analyzes root causes by repeatedly asking « why? » to identify the fundamental cause of a problem. It digs beyond symptoms to identify deep causes of quality problems.

Project Charter: This is a formal document that defines objectives, scope, deliverables, resources, and project schedule. It frames the project and ensures that all stakeholders are aligned on the same objectives.

1.4.2 Step 2: Develop the set of causes of non-quality

In this second step, we need to identify and list all potential causes that contribute to the non-quality problem observed in the production system. The main recommended tool is the Ishikawa Diagram.

Ishikawa Diagram: it is specifically designed to facilitate the identification and organization of potential causes of a problem. Its visual structure allows grouping causes by categories, which facilitates analysis and understanding of cause-and-effect relationships. This diagram also provides an overview of potential causes, promotes collaboration, and helps identify areas where further investigation is necessary.

1.4.3 Step 3 : Diagnose the industrial production system

The objective of this third step is to measure and analyze the production system to confirm which causes have a significant impact on the quality problem. It involves obtaining concrete data to support the analysis and prioritize improvement actions. The tools used in this step are: **CTQ measurement, Sigma performance level measurement, conformity rate measurement, Pareto analysis, and cause-and-effect diagram.** These tools provide a complementary approach to diagnose the production system.

Sigma Performance Level Measurement: it quantifies the process performance in terms of defects per million opportunities (DPMO). A higher Sigma level indicates better performance. It also allows evaluating the current quality level of the process and setting improvement objectives.

Conformity Rate Measurement: this tool measures the proportion of products that meet defined specifications, evaluates the quality of process outputs, and identifies areas of non-conformity. It provides a direct measure of quality and allows tracking performance evolution over time.

Pareto Analysis (Pareto Diagram): it represents the frequency of different types of defects or causes of non-quality, classified in descending order of importance. It allows focusing improvement efforts on the most impactful causes for maximum return on investment.

Cause-and-Effect Diagram (Ishikawa): Ishikawa is used here in a more analytical way, taking into account data obtained at the end of step 2. In this step, it allows validating hypotheses formulated in step 2 and better understanding the mechanisms that lead to non-quality.

1.4.4 Step 4: Graphical and analytical modeling of the industrial production system

The objective of this fourth step is to build a mathematical model that represents the cause-and-effect relationships between the different variables of the process. This modeling will enable simulating the behavior of the industrial production system, predicting the impact of changes, and optimizing improvement actions on it. The modeling should be done using Bayesian Networks.

Bayesian Network modeling: Bayesian Networks allow graphically representing relationships and quantifying the influence of each variable on the final outcome, managing uncertainty, and predicting the impact of changes in the production system.

1.4.5 Step 5: Improvement and prediction of defects on the industrial production system

The objective of this fifth step is to use the model developed in step 4 to simulate the impact of different corrective actions and predict their effectiveness in reducing defects. It involves optimizing interventions to achieve the best possible outcome in terms of quality. The tool we highlight here is again Bayesian networks.

Prediction and diagnosis using Bayesian networks: The Bayesian network modeling we built in the previous step allows us to simulate different intervention scenarios without physically implementing them. This enables us to virtually test multiple solutions and choose those that maximize quality improvement, evaluate their potential impact, and select the most promising ones before deploying them in a real-world situation on the production system.

1.4.6 Step 6: Monitoring and control of the operational quality control process

Through this sixth step, we aim to maintain the gains achieved and ensure that the process remains under control. Indeed, it involves setting up monitoring mechanisms to quickly detect any drift and adjust the process if necessary. To this end, we use multilinear regression analysis.

Prediction using multilinear regression analysis: This tool is used to model the influence of multiple variables on quality. Recall that previous steps have identified key factors influencing quality. Multiple regression, in turn, allows formalizing these relationships and building a predictive model. We can then use this model to continuously monitor the process and quickly detect any deviation from target performance. Note also that while Bayesian networks are excellent for modeling and prediction in complex and uncertain contexts, multiple regression provides a robust statistical framework for continuous monitoring and long-term control.

2 RESULTS AND DISCUSSION: CASE STUDY OF AN INDUSTRIAL POCKET TISSUES MANUFACTURING SYSTEM

2.1 Presentation of the Industrial Production System

The RN 04 machine we will be using is an automatic production line for manufacturing pocket tissues (PT) by transforming cellulose wadding rolls. It has a capacity of 70 pieces per minute (Pcs/min). Its finished products are PT SITA and PT JOHN F. The RN 04 consists of four units (see Figure 2): a PT production unit, a PT packaging unit in packets of 10 PT, a packet labeling unit for packets of 10 PT, and a unit for packaging 10 packets of 10 PT (10x10 PT). Each production unit consists of one or more stations, as shown in Table 2. The structural and functional diagram of the RN04 production line is provided in Appendix 1 [18].

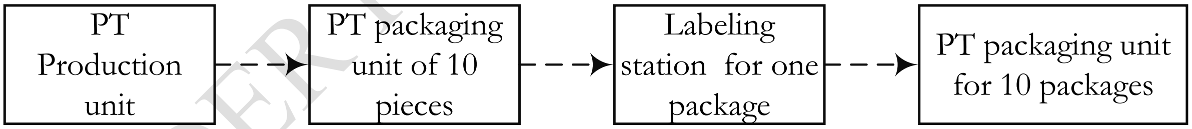


Figure 2 : Process for Pooling Production Units of RN 04

Tableau 2 : Functional Structuring of RN04 Machine

Machine	Block Machine	Reference	STATION	N°
RN-04 (M)	Production Unit of PT (B1)	DMO	Motorized Cotton Unwinder	A1
		SDG	Embossing Station	A2
		SPC	Folding and Cutting Station for Cotton	A3
	Packaging Unit for 10 PT (B2)	SDT	Unloading and Transport Station for 10 PT	A4
		SDP	Polyethylene Unwinding and Perforation Station	A5
		SEP	Packaging, Folding, and Longitudinal/Lateral Sealing Station for PT	A6
	Labeling Unit for Packs of 10 PT (B3)	SET	Labeling Station for PT Packs	A7

	Packaging Unit for 10x10 PT (B4)	STE	Transport Station for Packs of 10 PT to Packaging Machine	A8
		SPP	Polypropylene Unwinding Station	A9
		SPS	Folding and Longitudinal/Lateral Sealing Station for 10 x 10 PT	A10

2.2 Collection and Processing of Data

In implementing the combined approach, we collected both qualitative and quantitative data. The qualitative data focused on customer satisfaction, while the quantitative data concerned daily production activity indicators at the factory. Data collection and recording were carried out over a four-month period, from February to June 2024. Qualitative data were collected through a customer satisfaction survey, and quantitative data were extracted from the SURAP production management tool, which allows for permanent recording of production activity data within the company. A summary of the collected qualitative data is represented in Table 8. For the implementation of Bayesian networks, the quantitative data used for experience feedback are represented in Table 3, and regarding the prediction of the quantity of defective Pocket Tissues produced, a summary of recorded observations is presented in Table 4.

Tableau 3 : Frequency of Failures and Cumulative Downtime per Station

Station	N°	Failure Frequency	Downtime (min)	Availability
DMO	A1	3	13	99,77%
SDG	A2	8	16	99,72%
SPC	A3	49	285	95,05%
SDP	A4	397	268	95,35%
SDP	A5	9	93	98,39%
SEP	A6	461	1034	82,05%
SET	A7	8	31	99,46%
STE	A8	3	6	99,90%
SPP	A9	35	194	96,63%
SPS	A10	43	139	97,59%
Total		1016	2079	

Tableau 4 : Summary of Some Production Indicators

Production Factors	Rebus (Packets)	Waste(Kg)	Quality Defects Frequency	Production Stops Frequency	Downtime (min)	Availabilit y	Production
Energy (M1)	11	0,24	1	3	22	99,62%	132 330
Machine (M2)	7 129	185	1 012	1 016	2 079	63,91%	
Raw Material (M3)	559	14,55	8	76	339	94,11%	
Operator (M4)	931	22,18	4	86	639	88,91%	
Environement (M5)	0	0	0	0	0	100,00%	
Total	**8630**	**221,69**	**1025**	**1181**	**3079**		**132 0**

2.3 Implementation of the Combined Approach

2.3.1 Step 1: Identification and Definition of the Industrial Production System

a) CTQ (Critical to Quality) Diagram

Using the CTQ diagram, we broke down the customer's needs (what drives them to use the products) into requirements (quality, cost, delivery time) that were matched with characteristics that we evaluated through measurements. For each of these characteristics, we determined limit specifications defined as the company's standards. These specifications are presented in Table 5.

Tableau 5 : CTQ Diagram of Finished Products from the Machine

Customer	Needs	Requirements	Characteristics	Specification

Service Commercial	Customer or Consumer Satisfaction	Quality Products	Conformity Rate	$\geq 4,5\sigma$ (99,87%)
		ProductsDelivered on Time	On-time Delivery Rate	$\geq$ 95%
		Products Delivered in Requested Quantity	Service Level	$\geq$ 95%

b) Process Mapping: Black Box and SIPOC Diagram

We carried out process mapping of the production process on the industrial production system, which is represented by the Black Box and SIPOC diagrams shown in Figures 3 and 4.

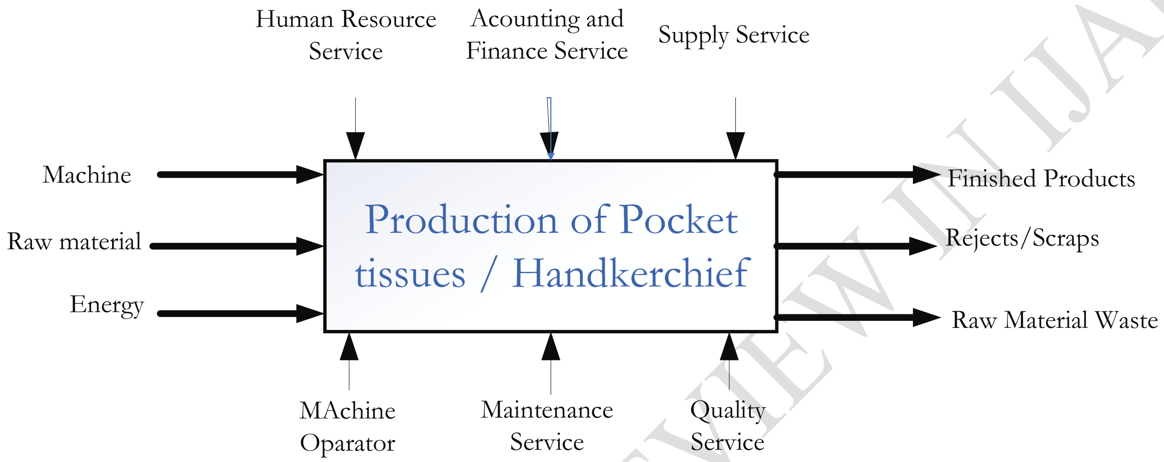


Figure 3 : Black Box of the RN-04 Production Process

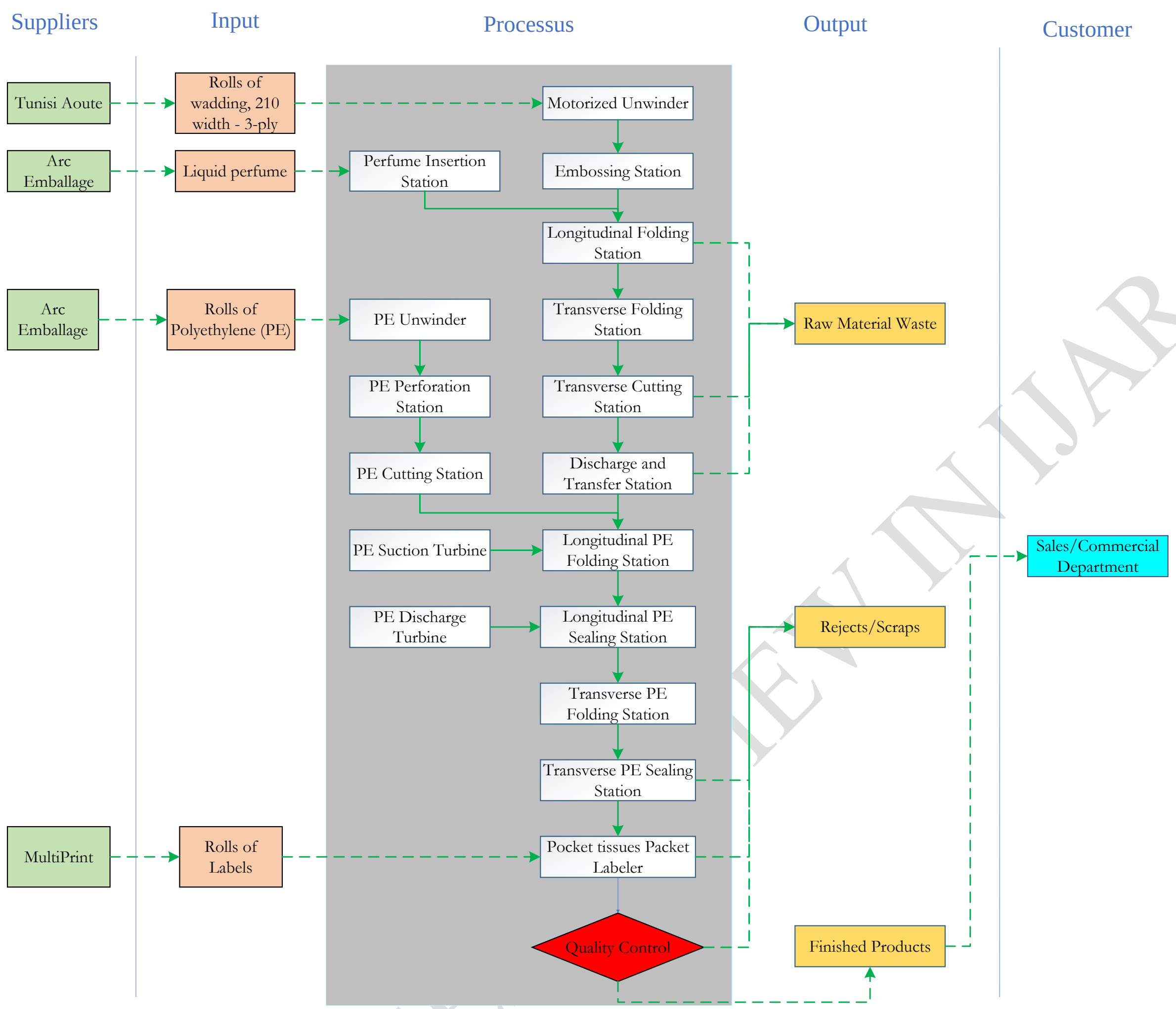


Figure 4 : SIPOC Physical Flows of Pocket Tissues (RN-04) Production Process

c) 5 Whys Diagram

The 5 Whys diagram of the chosen production system is presented in Table 6.

Tableau 6 : 5 Why Diagram of RN-04

Problem	**The company is unable to deliver products from these lines on time**.
Why	These production lines are not producing at full capacity
Why	Machine downtime.
Why	Due to corrective maintenance
Why	Some parts of these lines are successively breaking down
Why	Some parts of these lines are defective
Problem	**Why is the conformity rate of finished products less than 4.5σ ?**

Why	Some parts of these lines are defective.
Why	These parts are not well-maintained or are at the end of their lifespan
Why	Absence of order fulfillment
Why	Lack of a detailed planning and control of products
Why	Lack of motivation.

d) Project Charter

The charter for this case study is represented in Table 7, it refers to the precise definition of the problem and its impacts, as well as the clear delimitation of the project objectives.

Tableau 7 : Project Charter

Project Title	**Improving the quality of finished products**				
ProblemStatement	Low conformity rate during production on the RN-04 machine at the Faytex factory of SITRACEL in Yaoundé.				
WHO?	**WHAT?**	**WHERE ?**	**WHEN?**	**HOW?**	**WHY?**
Central OperationDepartment	Pocket Tissues	FAYTEX factory of SITRACEL	July 2024	Using Lean-Six Sigma	To reduce process variability and waste, and increase gains.
Final Customers :	**SITRACEL Commercial Department**				
Project Description	**CTQ Diagram**				
	Customer Needs	**Requirement**	**Characteristics**	**Specification**	
	Satisfaction du service commerciale	Quality Product	Conformity Rate	$\geq 4.5\sigma$	
		Products delivered on time	Rate of Delay	≥ 95%	
		Products delivered in the requested Quantity	Syntheticyield rate	≥ 95%	
	Current Conformity Rate		**Desired Conformity Rate**		
	RN-04 $\geq \mathbf{3.727\sigma}$		RN-04 $\geq \mathbf{4.5\sigma}$		
	Costs		Time spent by work group memebers ;		
			Data collection on the field ;		
			Slowdown and production stoppage due to testing or process modification;		
Project Description	MeasurableBenefits		Non- MeasurableBenefits		
	Reduction of scrap and rework rates		**Improvedcompany image**		
	Improvement of Synthetic Yield Rate (SYR)				
	Reduction of FinanacialLosses				
	Increase in Revenue due to improved Quality				
	Process Mapping		Black box process of SIPOC diagram		

2.3.2 Step 2: Development of the Set of Non-Quality Causes

We presented in Figure 5 the causes related to the observed non-quality and various machine stops that we grouped into five categories: the environment or milieu, the machine, the operator, energy, and raw materials. The data collected for each category are presented in Table 4.

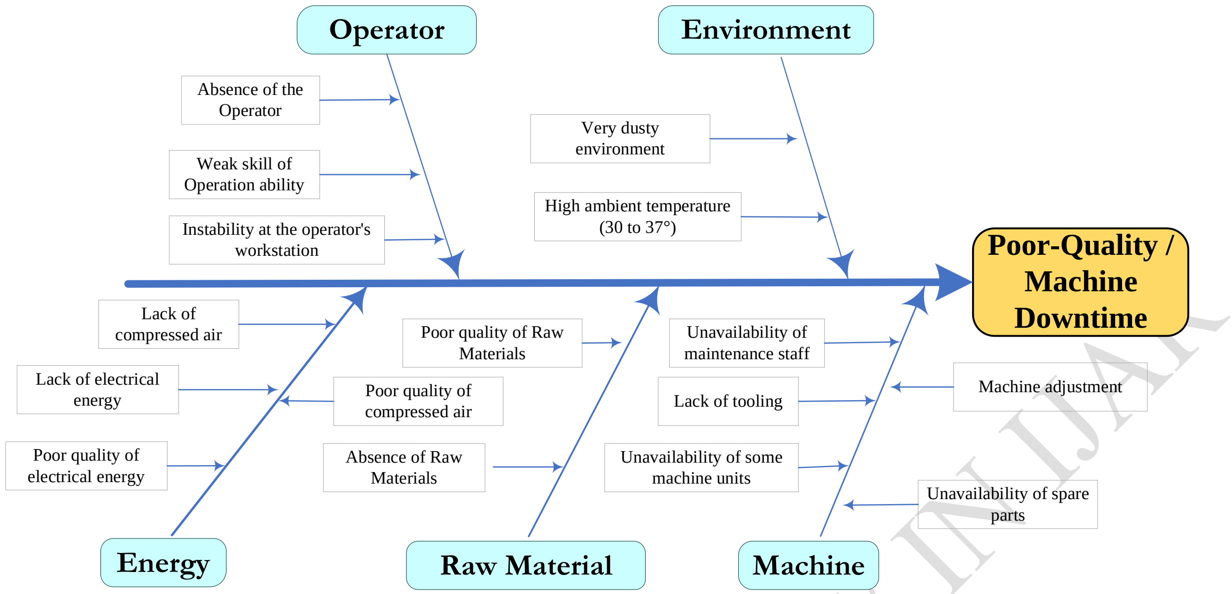


Figure 5 : Ishikawa Diagram of Non-Quality and Machine Downtime Causes

2.3.3 Step 3: Diagnosis of the Industrial Production System

a) Measurement of CTQs

These measurements were made through a survey conducted among 10 employees of the commercial department of SITRACEL. The summary of the results obtained during the survey on commercial service satisfaction and the average values of Delivery Time (DL), Quantity of products delivered (Q), and Quality of products delivered (QL) are presented in Table 8. With c being the Center of each class of the distribution and m being the supposed Mean. The results presented in Table 8 show that the actual customer satisfaction for Pocket Tissues does not correspond to the specifications listed in Table 5, which requires an improvement in the manufacturing process of PT.

Tableau 8 : CUSTOMER SATISFACTION RATE FOR PT

Satisfaction Rate	$\sum f$	**m=75(en %)**	$\sum f.(c-m)$	$\frac{\sum f.(c-m)}{\sum f}+m$
Delivery Time $\overline{\mathbf{DL}}$(%)	10	92.5	-125	80
Quantity of Products Delivered $\overline{\mathbf{Q}}$ (%)	10	85	17.5	86.75
Quality of Products Delivered $\overline{\mathbf{QL}}$ (%)	10	**75**	95	84.5

a) Measurement of Machine Conformity Rates

In this section, we will present in Table 9 the measurements related to machine conformity rates and the production of Pocket Tissues in the case of the RN-04 machine during the entire data collection period. Here, the data is cumulative over a frequency of two weeks.

Tableau 9 :Production Data Collected from RN-04

N°	**Total Quantity / 10x10 Packs**	**Non-Conforming Quantity / 10x10 Packs**	**Conforming Quantity / 10x10 Packs**	**Machine Conformity Rate**
1	20 410	410	20 000	98%
2	7 276	476	68 00	93%
3	8 503	703	7 800	92%

4	1 119	619	500	45%
5	8 777	377	8 400	96%
6	5 920	520	5 400	91%
7	16 504	504	16 000	97%
8	18 422	922	17 500	95%
9	14 661	661	14 000	95%
10	17 853	853	17 000	95%
11	3 187	1 187	2 000	63%
12	9 698	1 398	8 300	86%
Total	**132 330**	**8 630**	**123 700**	**93%**

b) Measurement of Sigma Performance Level

The defect opportunities per unit in a sample of pocket tissues at the output of the RN-04 machine are listed in Table 10.

Tableau 10 : Probable Quality Defects at RN 04

Quality Defects	**REFERENCE**
Perforation defect of 10 PT packets	APP
Poor sorting of PT in packets of 10	MCP
Lateral and/or longitudinal sealing defect of 10 PT packets	ASL
Lateral and/or longitudinal sealing defect of 10x10 PT packets	ASP
Labeling defect on packets of 10 PT	AET

According to Table 10, the number of defect opportunities is 5. During the measurement phase, we observed an average of 1 defect per unit of PT, and according to Table 9, the total number of non-conforming PT packets is 8630, and the sample size is 132,330.

The calculation of DPO [19] gives:

$$DPO_{PT} = \frac{8630 \times 1}{132330 \times 5} \qquad DPO_{PT} = 0.0130431497$$

The defect rate per million opportunities is: $DPMO_{PT} = 13043$

By referring to the Six Sigma table [20], we can determine the sigma performance level from the DPMO:

$$\frac{12224 - 13043}{12224 - 16793} = \frac{3{,}75 - x}{3.75 - 3/625} \Rightarrow x = 3.727$$

Therefore, the manufacturing process of PT by the RN04 machine has a sigma performance level of 3.727σ, and our conformity rate is 98.67%. Since the performance level is below 4.5σ, it needs to be improved.

c) Pareto analysis of non-quality on the RN04 machine

By exploiting Table 3, we created graphical representations following Pareto diagrams, as illustrated in Figures 6, 7, 8, and 9. These figures represent, respectively, the causes of rejects, the causes of waste, the frequencies of quality defects, and the frequencies of machine stops. Analyzing these diagrams allows us to note that the machine alone accounts for more than 80% of the causes of rejects and waste produced during production activity, and the machine is also responsible for 80% of the number of quality defects observed and machine stoppage frequencies. Thus, according to the 80/20 rule, we deduce that the machine is the primary lever by which we must reduce or eliminate the rate of waste and rejects during pocket tissue production.

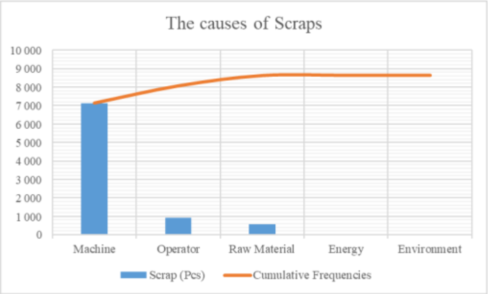


Figure 6 : Pareto representation of the causes of Scraps

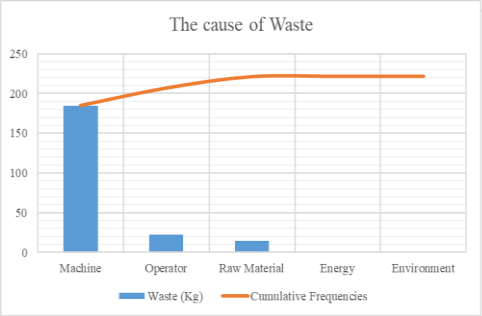


Figure 7 : Pareto representation of the cause of Waste

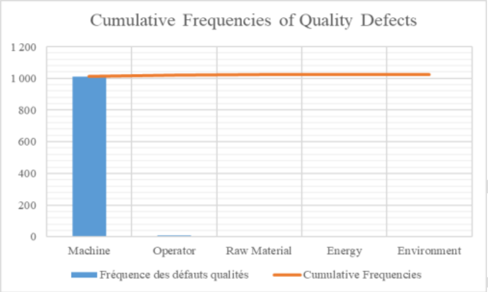


Figure 8 : Pareto representation of the cumulative frequencies of Qualities Defects

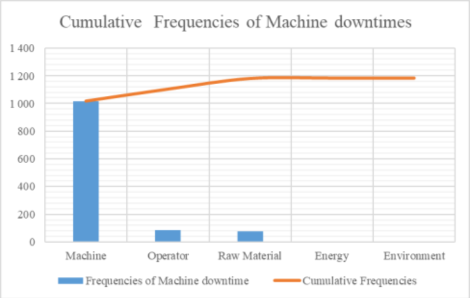


Figure 9 : Pareto representation of the cumulative frequencies of Machine Downtimes

d) Causes of non-quality

Table 11 presents the grouping of different possible causes of quality defects coming from the machine and production factors, and the coding we used to represent each quality defect and the grouping of different possible causes, which are the stations whose malfunction generates these quality defects.

Tableau 11 : Causes of quality defects from the RN04 machine and production factors

Quality defects	Reference	N°	Causes of quality defects		
			Machine		Production Factors
Defect in perforation of 10 PT packets	APP	D1	SDP	A5	M1, M2, M4
Poor sorting of PT in 10 packets	MCP	D2	SDG, SDT, DMO	A1, A2, A4	M1, M2, M4
Defective lateral and/or longitudinal sealing of 10 PT packets	ASL	D3	MCP, SDP, SPC, SEP	A3, A5, A6, D2	M1, M2, M3, M4,
Defective lateral and/or longitudinal sealing of 10x10 PT packet	ASP	D4	ASL, STE, SPP, SPS	A8, A9, A10, D3	M1, M2, M3, M4,
Labeling defect on 10 PT packets	AET	D5	SET	A7	M2, M3, M4,

2.3.4 Step 4: Graphical and Analytical Modeling of the Industrial Production System

The graphical models of the Bayesian networks are represented in Figures 10 and 11, and the mathematical representations are given below:

* Bayesian Network BN1: G (V, E)

- **Nodes (V) :** {A1, A2, A3, A4, A5, A6, A7, A8, A9, A10, D1, D2, D3, D4, D5, Q} representing random variables
- **Arcs (E) :** Directed arcs represent direct dependencies. For example, the arc from A1 to D2 indicates that A1 directly influences D2.
- The joint probability distribution is factorized as follows:

P(A1, A2,... , A10, D1, D2, D3, D4, D5, Q) = P(A1) * P(A2) * ... * P(A10) *P(D1 | A3, A4, A5, A6) * P(D2 | A1, A2, D1) * P(D3 | A5, A6, A7, A8, D1) *P(D4 | A8, A9, A10, D3) * P(D5 | D3, D4) * P(Q | D2, D3, D4, D5)

* Bayesian Network BN2: G (V, E)

- **Nodes (V):** {M1, M2, M3, M4, D1, D2, D3, D4, D5, Q} representing random variables.
- **Arcs (E):** Directed arcs represent direct dependencies. For example, the arc from M1 to D1 indicates that M1 directly influences D1.
- The joint probability distribution is factorized as follows:

P(M1, M2, M3, M4, D1, D2, D3, D4, D5, Q) = P(M1) * P(M2) * P(M3) * P(M4) * P(D1 | M1, M2, M4) * P(D2 | M1, M2) * P(D3 | M1, M2, M3, M4) *P(D4 | M3, M4) * P(D5 | M3, M4) * P(Q | D1, D2, D3, D4, D5)

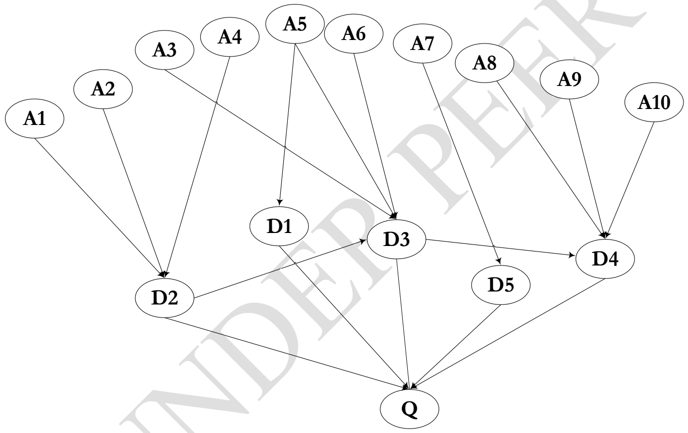


Figure 10 : Bayesian Network modeling of the causes of quality defects related to the machine only (BN1)

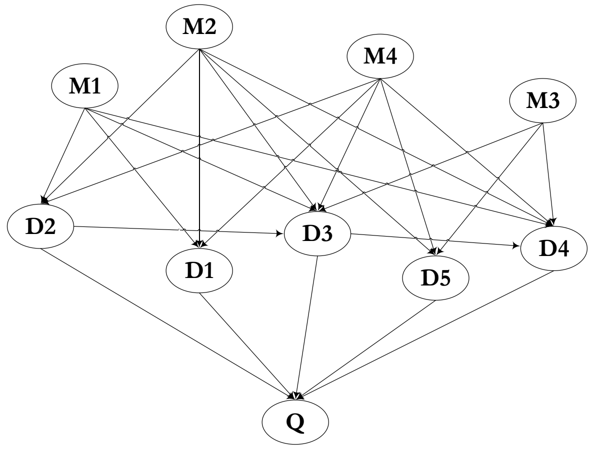


Figure 11 : Bayesian Network modeling of the causes of quality defects related to production factors (BN2)

2.3.5 Step 5 : Improvement and prediction of defects on the industrial production system

a) Diagnosis of quality defects based on causes related to production factors

The logical states of the nodes Di and Q were deduced based on the following assumptions:

- Node Q is in a quality defect state (Defa) if and only if at least one of its parent nodes Di presents a quality defect (Defa);

- A node Di is in a quality defect state if and only if at least one parent node Mi is unavailable (Indp) or at least one parent node Dj with i≠j is in a quality defect state (Defa) ;
- Nodes Mi have two states each: « Available » (Disp) and « Unavailable » (Indp) ;
- Nodes Di and Q have two states each: « Quality defect » (Defa) and « No quality defect » (Pasd).

Figure 12 represents the BN2 network, where the values of the root nodes Mi come from the data in Table 4. Note that in the node states of the network in Figure 12, we have not defined any observation hypotheses.

In this specific context of tissue production, P(Q=Defa) = 0.47 represents the prior probability that the production will have a quality defect, all things being equal.

Figure 13 simulates the Bayesian network when M1 is unavailable, and P(Q=Defa)=100%, so we are certain that there is a quality defect. We can conduct several simulations like this, which allows us to visualize the impact of the Mi on the Di and simultaneously the impact of the Di on the quality defects Qi. This is how improvement actions are oriented, which reduces investment risks.

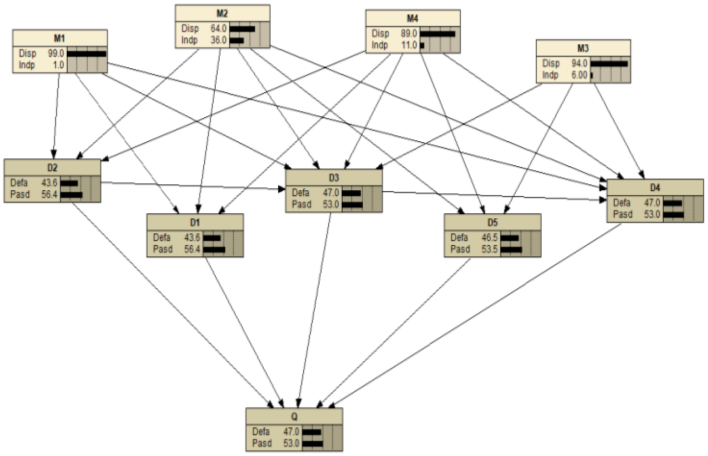


Figure 12 : Implementation of the Bayesian Network BN2

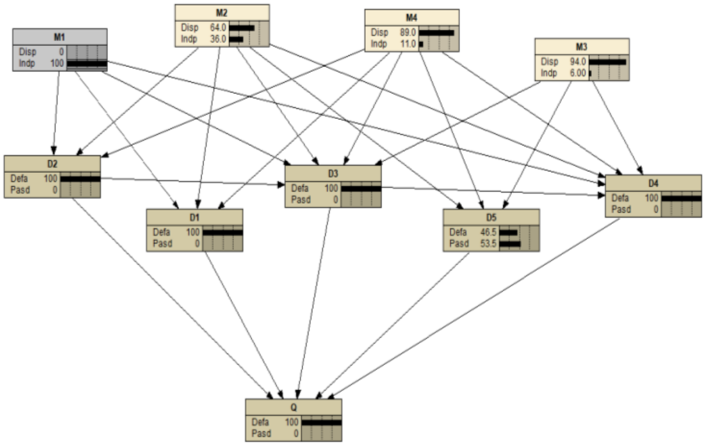


Figure 13 : Bayesian Network BN2 with M1 unavailable

b) Diagnosis of quality defects based on causes related only to the machine

In the implementation of the Bayesian network BN1, as illustrated in Figure 14, the assumptions are defined according to the same principle as in the previous section. The probabilities of the root nodes Ai are based on the data from Table 3.

Exploiting Figure 14, we can see that, all things being equal, the probability of having a quality defect is P(Q=Defa)=0.65.

Figures 15, 16, and 17 show a process of eradicating the quality defect D4 by improving the availability of station A6 from 82% to 100%. This reduces the probability of occurrence of a quality defect Q and D4 by 15% each.

Next, improving the availability of stations A3 and A4 from 95% to 100%, as illustrated in Figure 16, would reduce the probability of quality defect from 21% to 12%, a reduction of 9%, and D4 by 10%.

Finally, by improving stations A8, A9, and A10 to 100% each, D4 would directly go to 0%, and Q would go to 1%.

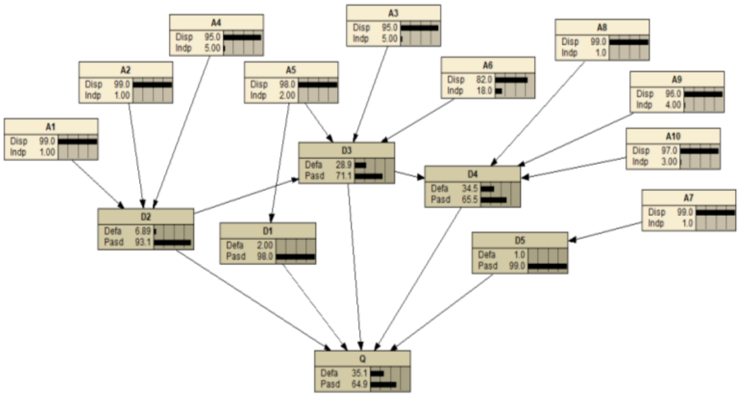


Figure 14 : Implementation of Bayesian Network BN1

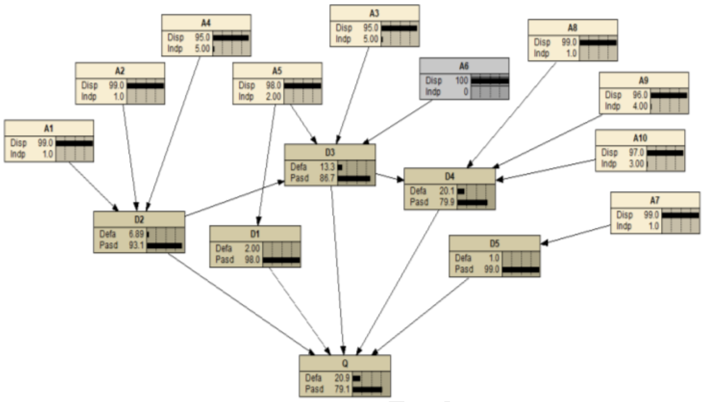


Figure 15 : Bayesian Network BN1 with A6 Fully Available

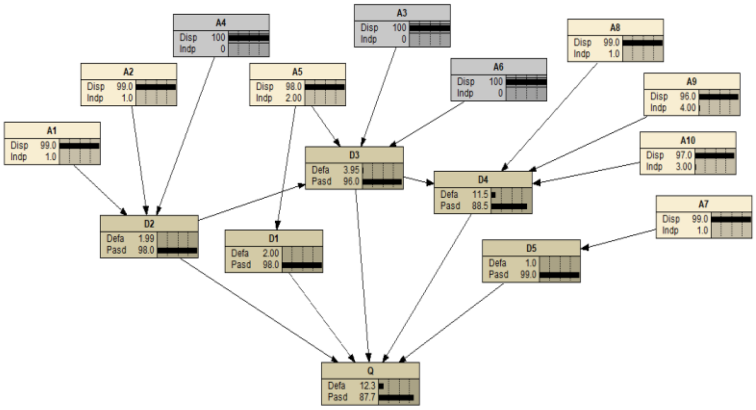


Figure 16 : Bayesian Network BN1 with A6, A4, and A3 Fully Available

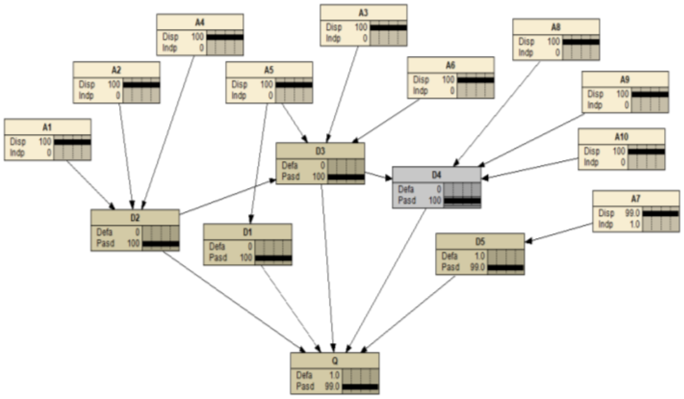
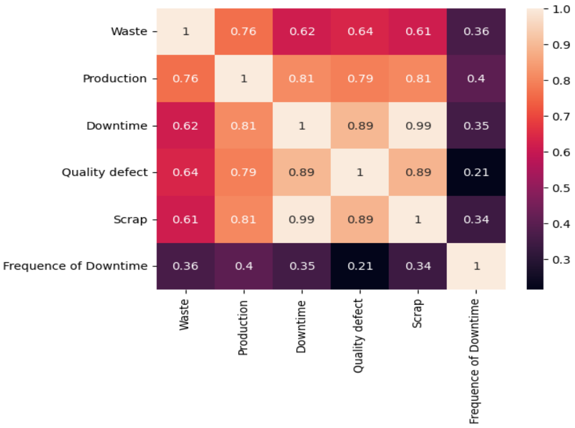


Figure 17 : Bayesian Network BN1 with D4 Having No Quality Defects

2.3.6 *Step 6 : Monitoring and Control of Operational Quality Control Process*

Quality control of finished products is tracked through two indicators: Scrap rate and Rejection rate. These indicators depend on five other variables (Xi), as shown in Table 12. The matrix representing the correlation between all variables is shown in Figure 18.



	Waste	Production	Downtime	Quality defect	Scrap	Frequence of Downtime
Waste	1	0.76	0.62	0.64	0.61	0.36
Production	0.76	1	0.81	0.79	0.81	0.4
Downtime	0.62	0.81	1	0.89	0.99	0.35
Quality defect	0.64	0.79	0.89	1	0.89	0.21
Scrap	0.61	0.81	0.99	0.89	1	0.34
Frequence of Downtime	0.36	0.4	0.35	0.21	0.34	1

Figure 18 : Correlation Matrix between All Variables.

The analysis of the values in this matrix reveals that we generally have a strong multicollinearity between our variables. We corrected this problem of strong multicollinearity by applying the principal component analysis (PCA) approach to all the data of the independent variables.

Tableau 12 : Variables Observed to Predict Rejects

Dependent Variable	**Independent Variables**				
Rejects	Frequency of quality defects	Frequency of breakdowns	Cumulative downtime	Cumulative Waste	Cumulative production
Ys	X_1	X_2	X_3	X_4	X_5
Dependent Variable	**Independent Variables**				
Waste	Frequency of quality defects	Frequency of breakdowns	Cumulative downtime	Cumulative rejects	Cumulative production
Y	X_1	X_2	X_3	X_4	X_5

By applying PCA to the independent variables of reject rate and waste rate, we obtained principal components Fi, eigenvalues, eigenvectors, and new correlation matrices of Fi, which are represented in Table 15 below. Thus, we obtained low multicollinearity between the variables.

Tableau 13 : Eigenvalue, Eigenvector, and Correlation Matrix of Fi

	Rejects (Y)					**Waste (Y)**				
Principal Component of Fi	**F1**	**F2**	**F3**	**F4**	**F5**	**F1**	**F2**	**F3**	**F4**	**F5**
	Eigenvalue									
Eigenvalue	3.504	0.883	0.485	0.123	0.006	3.751	0.874	0.244	0.125	0.006
Variability (%)	70.077	17.653	9.695	2.459	0.116	75.022	17.485	4.877	2.499	0.117
% cummulative	**70.077**	**87.729**	**97.424**	99.884	100.0	**75.022**	**92.507**	**97.384**	99.883	100.0
	Eigenvector									
X1	0.513	-0.147	-0.270	-0.400	-0.695	0.500	-0.116	0.347	-0.366	-0.694
X2	0.515	-0.137	-0.274	-0.353	0.719	0.502	-0.105	0.336	-0.325	0.719
X3	0.245	0.929	-0.234	0.152	-0.013	0.232	0.951	0.128	0.158	-0.011
X4	0.413	0.137	0.891	-0.131	0.006	0.465	0.034	-0.864	-0.190	-0.002
X5	0.491	-0.281	-0.063	0.822	-0.026	0.476	-0.264	0.062	0.836	-0.022
	Correlation Matix (Pearson)									
F1	1	-0.224	-0.117	0.191	-0.096	1	-0.192	-0.266	0.078	-0.091
F2	-0.224	1	0.113	-0.189	0.070	-0.192	1	-0.041	-0.182	0.060
F3	-0.117	0.113	1	0.016	0.017	-0.266	-0.041	1	0.044	0.010
F4	0.191	-0.189	0.016	1	-0.020	0.078	-0.182	0.044	1	-0.010
F5	-0.096	0.070	0.017	-0.020	1	-0.091	0.060	0.010	-0.010	1

Table 14 presents the three models obtained for predicting the reject rate. The three selected models all pass the Student’s t-test and Fisher’s F-test, making them statistically significant. For predicting the reject rate, we chose Model 2 because it simultaneously provides a good representation of the Xi variables’ data (97.5%) and regression parameters very close to those of Model 1.

Tableau 14 : Reject Rate Prediction Models

Obtained Model	**Equation**	***Representation of Xi***	***Regression Statistics Data***	***ANOVA Data***
Model 1	$\hat{Y} = 10.18 + 249F_1$	70%	Multiple R 0.869 R Square **0.755** Adjusted R Square 0.753	*P-value* Intercept 0.000801 F1 3.9E-64

Model 2	$\boldsymbol{\widehat{sqrt(Y)} = 3.26 + 13.07F_1 + 4.35F_2 + 6.02F_3}$	**97.5%**	Multiple R 0.862 R Square **0.743** Adjusted R Square 0.739	*P-value* Intercept 3.41E-47 F1 3.18E-61 F2 0.001147 F3 0.000185
Model 3	$\widehat{log(Y)} = 0.99 + 1.51F_1 + 0.83F_2 + 0.69F_3$	97.5%	Multiple R 0.737 R Square **0.543** Adjusted R Square 0.536	*P-value* Intercept 8.38E-82 F1 4.6E-36 F2 0.000624 F3 0.016305

Following the previous process, we chose Model 2 in Table 15 for predicting the waste rate because it simultaneously combines the best performances in terms of representation of the Xi variables' data (97.4%) and the values of the regressive statistics data (R Square = 66%).

Tableau 15 : Waste Prediction Models

Obtained Model	Equation	Representation of Xi	Regression Statistics Data	ANOVA Data
Model 1	$\hat{Y} = 0.13 + 3.17F_1 + 2.49F_2$	92.5%	Multiple R 0.712 R Square **0.506** Adjusted R Square 0.501	*P-value* Intercept 0.368958 F1 6.96E-33 F2 0.045797
Model 2	$\boldsymbol{\widehat{sqrt(Y)} = 0.45 + 0.87F_1 + 0.69F_2 - 4.56F_3}$	**97.4%**	Multiple R 0.812 R Square **0.659** Adjusted R Square 0.654	*P-value* Intercept 9.861E-28 F1 1.788E-38 F2 0.018277 F3 2.106E-09
Model 3	$\widehat{log(Y)} = -0.72 + 0.66F_1 + 0.79F_2 - 3.14F_3$	97.4%	Multiple R 0.729 R Square **0.531** Adjusted R Square 0.524	*P-value* Intercept 8.58E-53 F1 2.33E-27 F2 0.006139 F3 1.59E-05

3 CONCLUSION

In this work, we started from the instability and variability of quality control processes in industrial production systems, which result in conformity and customer satisfaction rates below reference values. We proposed a combined approach based on the integration of Lean Six Sigma (LSS), Bayesian Networks (BNs), and multilinear regression analysis. This combined approach is based on a methodology structured in six steps : Identify and define the industrial production system; Develop the set of non-quality causes; Diagnose the industrial production system; Graphically and analytically model the industrial production system; Improve and predict defects in the industrial production system and Monitor and control the operational quality control process. This combined approach was implemented in the industrial system of SITRACEL S.A. in Cameroon. The implementation revealed: An insufficient conformity rate of 3.727σ ; Customer dissatisfaction of 16.25% compared to reference values and Dominant quality defect causes coming directly from the machine

The approach also allowed modeling quality control indicators: Scrap rate $\hat{Y} = (3.26 + 13.07F_1 + 4.35F_2 + 6.02F_3)^2$ and Waste rate $\hat{Y} = (0.45 + 0.87F_1 + 0.69F_2 - 4.56F_3)^2$ Where Fi are the principal components $F_i = f(X_i)_{i=1 \text{ à } 5}$, and Xi are the observed variables in the production system. The obtained equations represent models that allow tracking variability in scrap and waste rates to implement improvement actions.

Future research could refine the analysis of influential points and extend the approach to other industrial production systems. Collecting additional data and refining Bayesian models will improve prediction accuracy and corrective action efficiency, contributing to sustainable improvement in product quality in Cameroon's industrial sector and other Central African industries.

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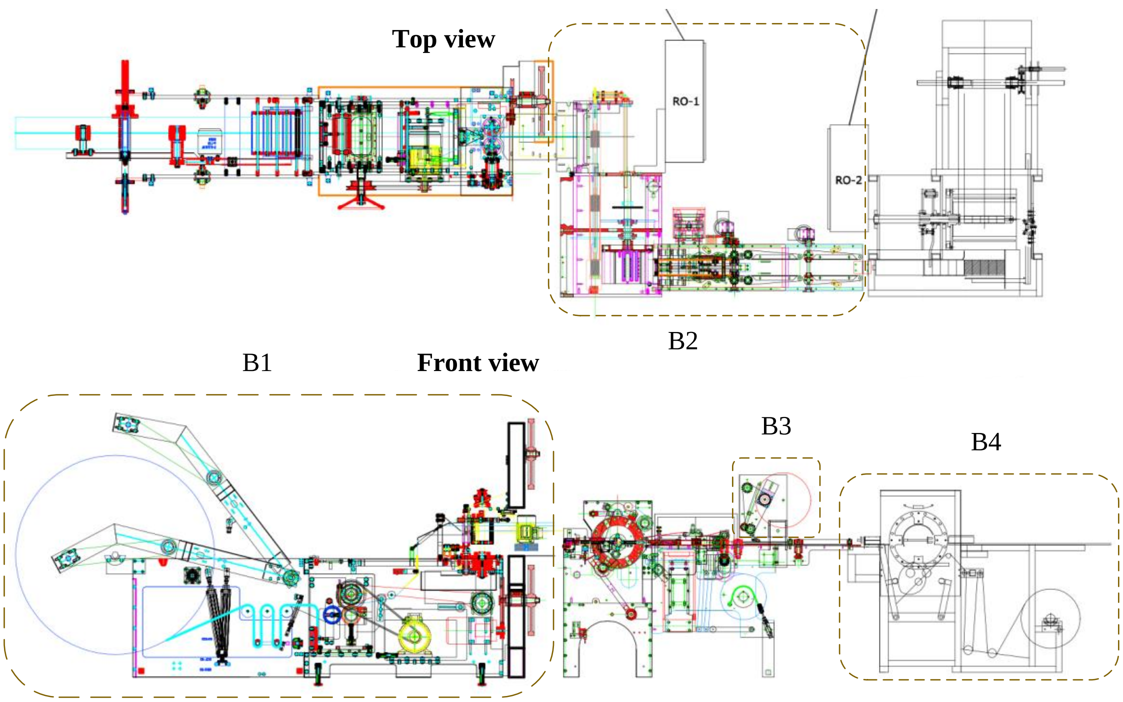
ACRONYMS

PCA	Principal Component Analysis
FMEA	Failure Mode and Effects Analysis
CTQ	Critical to Quality

DMAIC	Define, Measure, Analyze, Improve, Control
DPMO	Defects Per Million Opportunities
HAZOP	Hazard and Operability Study
LSS	Lean Six-Sigma
BN	Bayesian Networks
SIPOC	Suppliers, Inputs, Process, Outputs, Customers
OEE	Overall Equipment Effectiveness

APPENDICES

Appendix 1: Structural and Functional Diagram of the Industrial Production System for Pocket Tissues RN04



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