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REVIEWER'S REPORT

Manuscript No.: IJAR-55582

Title: GLOBAL ATTRCTIVITY AND POSITIVITY SOLUTIONS FOR NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH MEASURES OF NONCOMPACTNESS

Recommendation:
Accept as it is

Rating	Excel.	Good	Fair	Poor
Originality		√		
Techn. Quality			√	
Clarity		√		
Significance		√		

Reviewer Name: Dr. Manju M

Date: 07-01-2026

Detailed Reviewer's Report

1. Main Objective of the Work

The main objective of this work is to establish existence, global attractivity, and asymptotic positivity of solutions for a broad class of nonlinear functional differential equations defined on unbounded intervals. The study aims to employ measures of noncompactness and Dhage-type fixed point theorems to overcome the lack of compactness in infinite-dimensional Banach spaces. By developing suitable nonlinear set-contraction conditions, the work seeks to derive global and uniform attractivity results under weaker assumptions than those commonly used in the literature. Additionally, the work introduces and rigorously analyzes new concepts of local and global asymptotic positivity, thereby extending and generalizing existing stability and attractivity theories for functional differential and integral equations.

2. Background of Nonlinear Functional Differential Equations

Nonlinear functional differential equations (FDEs) arise naturally in models involving memory, delay, or hereditary effects. Classical studies mainly focus on bounded intervals, where compactness tools are easier to apply. However, unbounded intervals introduce new analytical challenges, particularly related to asymptotic behavior. Understanding attractivity and positivity on $\mathbb{R}^+$ is crucial for long-term system dynamics. This motivates the need for advanced functional analytic tools. Measures of noncompactness provide a robust framework for such analysis.

3. Importance of Attractivity and Positivity Concepts

Attractivity describes whether solutions converge to each other or to an equilibrium as time progresses. Positivity ensures solutions remain non-negative, which is essential in physical, biological, and economic models. These properties guarantee realistic and stable system behavior. Traditional fixed point methods often fail to capture these features on unbounded domains. Hence, a generalized framework is required. The paper addresses this gap systematically.

4. Limitations of Classical Fixed Point Techniques

Classical fixed point theorems rely heavily on compactness assumptions. On unbounded intervals, compactness is generally lost, limiting their applicability. Moreover, ensuring global attractivity and positivity simultaneously is difficult with classical tools. These limitations necessitate alternative

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approaches. Measures of noncompactness relax compactness requirements. This makes them suitable for functional differential equations on $\mathbb{R}^+$.

5. Role of Measures of Noncompactness

Measures of noncompactness quantify how far a bounded set is from being compact. They provide a numerical characterization rather than a binary compact/noncompact classification. This concept is particularly useful in infinite-dimensional Banach spaces. It allows fixed point arguments without strict compactness. The paper adopts Dhage's axiomatic definition. This choice ensures flexibility and applicability to nonlinear operators.

6. Banach Space Framework

The analysis is conducted in the Banach space $BC(\mathbb{R}^+,\mathbb{R})$, consisting of bounded and continuous real-valued functions. This space naturally accommodates solutions defined on unbounded intervals. The supremum norm provides completeness and analytical convenience. Boundedness ensures control over solution growth. Continuity supports integral and differential formulations. This setting is well-suited for attractivity analysis.

7. Hausdorff–Pompeiu Metric and Hyperspaces

The Hausdorff–Pompeiu metric measures distances between bounded subsets of a Banach space. It turns the hyperspace of closed bounded sets into a complete metric space. This structure is essential for defining convergence of sets. It supports continuity properties of measures of noncompactness. The paper relies on this metric for defining limiting sets. This forms a foundation for later fixed point arguments.

8. Definition of Measure of Noncompactness

A measure of noncompactness satisfies axioms such as monotonicity, invariance under closure and convex hull, and vanishing on relatively compact sets. These properties ensure mathematical consistency. The kernel of the measure consists of relatively compact subsets. Completeness of the measure strengthens convergence results. Sublinearity further enhances operator estimates. These properties are crucial for nonlinear analysis.

9. Sublinear Measures and Their Advantages

Sublinear measures of noncompactness satisfy homogeneity and subadditivity. These features simplify estimates involving nonlinear operators. They allow decomposition of operator actions into manageable components. Sublinearity plays a key role in contraction-type inequalities. It also supports comparison functions used in fixed point theorems. The paper exploits these advantages extensively.

10. Dhage-Type Fixed Point Theorem

The Dhage fixed point theorem applies to nonlinear D-set contractions. It replaces classical compactness with measure-based contraction conditions. This theorem guarantees existence of fixed points in closed, bounded, convex sets. It is particularly effective in Banach spaces. The paper uses this theorem as its main analytical tool. It underpins both existence and attractivity results.

11. D-Set Lipschitz and Nonlinear Contractions

A D-set Lipschitz operator satisfies a measure-based Lipschitz condition. When the associated function strictly reduces measures, the operator becomes a nonlinear D-set contraction. This concept generalizes classical contractions. It accommodates nonlinear and noncompact operators. Such operators still admit fixed points. This framework is essential for functional differential equations.

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12. Specialized Measures in BC($\mathbb{R}^+$,$\mathbb{R}$)

The paper introduces tailored measures (μ_a, μ_b, μ_c) based on modulus of continuity and asymptotic diameter. These measures capture both local equicontinuity and asymptotic behavior. They are sensitive to solution oscillations and long-term trends. Extensions (μ_ad, μ_bd, μ_cd) incorporate asymptotic deviations. These constructions are novel and effective. They allow precise attractivity analysis.

13. Local and Global Attractivity Definitions

Local attractivity concerns convergence within a neighborhood of solutions. Global attractivity extends this convergence to the entire solution set. Uniform versions ensure convergence independent of initial conditions. These distinctions are important for stability classification. The paper rigorously formalizes these concepts. This provides clarity and analytical precision. Such definitions are essential for long-term behavior studies.

14. Asymptotic Attractors and Constant Solutions

An asymptotic attractor is defined as a constant line y(t)=c approached by solutions. This concept links functional equations to equilibrium analysis. It generalizes stability notions beyond fixed points. Asymptotic behavior is central to applications. The paper integrates this idea with measure-based methods. This leads to stronger convergence conclusions.

15. Introduction of Asymptotic Positivity

Positivity ensures that solutions eventually become and remain non-negative. The paper introduces local and global asymptotic positivity concepts. These notions are new in the context of functional differential equations. They are especially relevant for population and control models. Positivity complements attractivity results. Together, they ensure physically meaningful solutions.

16. Functional Differential Equation Model

The considered FDE includes delayed arguments and nonlinear terms. It generalizes many known functional and integral equations. The model captures hereditary and integral effects. Reduction to a functional integral equation simplifies analysis. This formulation allows operator-based methods. It broadens applicability to various mathematical models.

17. Assumptions on Delay and Kernel Functions

Continuity and boundedness assumptions ensure well-defined operators. Growth restrictions control nonlinear effects. Delay functions satisfy causality conditions. Integral kernels vanish asymptotically to ensure stability. These assumptions are mild yet sufficient. They allow general nonlinearities. This balance enhances the strength of results.

18. Construction of the Associated Operator

The FDE is transformed into an operator equation Qx = x. This operator acts on BC($\mathbb{R}^+$,$\mathbb{R}$). Its structure incorporates delay and integral terms. Bounding arguments show Q maps a closed ball into itself. Continuity of Q is established carefully. This sets the stage for fixed point application.

19. Boundedness and Global Existence of Solutions

Uniform bounds on the operator imply boundedness of solutions. This ensures global existence on $\mathbb{R}^+$. Unlike local existence results, this is a strong conclusion. It avoids blow-up in finite time. The approach relies on operator estimates rather than differential inequalities. This highlights the power of the method.

20. Measure-Based Contractive Estimates

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Key inequalities show that Q reduces the measure of noncompactness. These estimates involve both local oscillations and asymptotic diameters. The contraction condition is verified using assumptions on nonlinear terms. This ensures applicability of Dhage's theorem. Such estimates replace classical compactness arguments. They are central to the proof.

21. Existence of Fixed Points and Solutions

Applying the fixed point theorem yields existence of solutions. Fixed points of Q correspond to solutions of the FDE. The fixed point set lies in the kernel of the measure. This implies relative compactness and equicontinuity. Existence results are global in nature. This strengthens classical local existence results.

22. Global Uniform Attractivity of Solutions

Belonging to the kernel of the measure implies convergence of solutions. All solutions approach each other uniformly as $t \to \infty$. This establishes global uniform attractivity. The result holds without strong Lipschitz conditions. It generalizes earlier stability results. This is a major contribution of the paper.

23. Additional Conditions for Positivity

Extra assumptions ensure asymptotic positivity of solutions. These conditions control behavior of nonlinear terms at infinity. They ensure differences between positive and negative parts vanish. Measure μ_ad captures this asymptotic positivity. The analysis remains within the same framework. This shows flexibility of the approach.

24. Uniform Global Attractivity and Positivity

Under strengthened hypotheses, solutions are both uniformly globally attractive and ultimately positive. This dual property is rare in the literature. It ensures stability and physical relevance simultaneously. The result is robust under weaker conditions than previous works. It extends several known theorems. This highlights the novelty of the study.

25. Contributions and Research Significance

The paper generalizes and improves multiple existing results. It introduces new positivity concepts for FDEs. The use of novel measures of noncompactness is a key innovation. Results hold under weaker assumptions on nonlinearities. The framework is applicable to many functional equations. Overall, it significantly advances stability theory on unbounded domains.