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RESEARCH ARTICLE

ASURVEY AND COMPARISON ON SYNCHRONIZATION METHODS OF CHAOTIC SYSTEMS

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Abstract

In this paper, several methods forsynchronization of chaotic systems are explained and compared. The idea is based on drive-response systems synchronization. The methods include: active Control, recursive control, adaptive control, and partial linearizationmethod which are implemented and applied to a Lorenz chaotic system. The partial linearization method is used to synchronize a subset of states of the system to synchronize other states as well. Active control and rebound control methods are used when the system parameters are known while adaptive control method is used when some of the parameters of the system are unknown. Inthesemethods,synchronization is based on Lyapunov stability theory. Three methods, namely, adaptive, active and recursive and are implemented on a T system successfully. A newmatrix method has been presented for synchronization based on the theory of Lyapanovkrakfskytheory and linear matrix inequality (LMI).This method has been implemented to a Rösslersystem with delay. Comparingto classical methods used to synchronize chaotic system the matrix method seems the best because of easy design of input, suitable for synchronization of chaotic systems with delay, simple calculations, no need to find aLyapunov function for stability.

Introduction:-

Over the years, scientists have tried to invent with different methods, in order to make complex control systems stable and efficient. In this regard, one of the best methods has been proposed by scientists and Russian mathematician, Lyapunov.Lyapunovintroduced a quasi-energy function model for nonlinear systems and then concluded that, if the system energy is constantly reducing, energy finally will finish and the system will be stable.
In this paper, we have investigated the synchronization of chaotic systems. Several methods for synchronization of chaotic systems, namely active control, recursive control, and adaptive control arereviewed and a special matrix method is proposed for designof chaotic system synchronization observer.

The remaining parts of this paper consist of four sections as follows: Inspection 2, an overview of the phenomenon of chaos in the systems is presented with some practical examples of chaotic systems. In section 3, the synchronization of chaotic systems is expressed. Conclusions are expressed in section 4.

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Chaos phenomenon:-

Chaos literally means anarchy, confusion and disorder. In chaotic systems, long-term behavior is not possible to predict. This characteristic is the result of high sensitivity of these systems to their initial condition. Chaotic behavior is not random behavior; it is deterministic.[1] But from the view of an observer unaware of the structure and function of chaotic system, the signal cannot be distinguished from a random signal using statistical tests. Since the signal can be measured only with limited precision, even if the chaotic signal generator is completely known, because of uncertainty in initial conditions, the output precision is constantly decreased[1].Therefore, the future of signal, especially in the long-term cannot be predicted and thus, the behavior of signal seems random. Due to sensitivity to initial conditions, a small uncertainty in initial conditions will lead to huge amounts of uncertainty in long term, therefore the system is unpredictable [2].Examples of the chaotic systems include: Burning a neon lamp, the point of incidence and magnitude of earthquakes, the trajectory of the fluid in a tube, population growth organisms, lightning in the sky, and the chain of random numbers generated by computer. [1, 2, 3].The chaotic systems are studied in several fields of science such as mathematics, astronomy, physics, medicine, meteorology, engineering, mechanics, construction, pharmaceutical and aerospace and even in the fields of psychology, sociology, and management [2, 3].

The study of chaos has a lot of applications including:

- Ability to explain many events and natural phenomena, ability to predict disasters and avoid risks of accidents and reduce injuries
- Control of behavior of the systems in the desired direction
- Understanding the process leading to chaos in certain areas and under controlled conditions to achieve optimal conditions[2]
- Developing algorithms and logic of measurement, computing, administrative and operational
- Diagnose the cause of some irregularities, the correction of the laws of science and discovery of new laws[2] Due to the complex and unstable dynamics control, chaos control seems impossible but chaotic systems are capable of self-control and various control objectives are introduced as below:
- Remove chaotic behavior and sustainability point of balance
- stabilization of unstable alternate routes (creation of sustainable Limit Cycle)
- synchronization of two chaotic systems
- iPod control chaos (chaos-chaotic anti-control off)
- Control of bifurcation

Synchronization:-

Is this paper synchronization of two chaotic systems is examined as follows. It is synchronization of changes in two systems so that they show the same behavior. For example, data transmission in telecommunication systems, both client and transmitter have access to a carrier signal forsynchronization. In 1990, Pecorrat, Carroll showed that in specific circumstances two chaotic systems can become synchronized by applying error signal to systems [1].Assume two identical copies of the dynamical system[2]

$$\dot{X} = f(x), \quad x \epsilon R^n \tag{1}$$

For example, two identical oscillators with different initial conditions. One of these systems is called drive and another one is called response.

$$\text{Drive System: } \begin{cases} \dot{Z}_d = Z(Z_d, y_d) \\ \dot{Y}_d = Z(Z_d, y_d) \end{cases} \tag{2}$$

$$\text{Response System: } \begin{cases} \dot{Z}_r = Z(Z_r, y_r) \\ \dot{Y}_r = Z(Z_r, y_r) \end{cases} \tag{3}$$

In equations (2) and (3), $Z_d$and Y_d are drive system's state variables andZ_r ‘ y_r are state variables is the answer system.

Synchronization condition is defined as follows:

$$\lim_{t\to 0} |Z_d(t)\text{-}Z_r(t)| \to 0 \tag{4}$$

$$\lim_{t\to 0} |y_d(t)\text{-}y_r(t)| \to 0 \tag{5}$$

Dynamic equations of error signal will be:-

$$\begin{cases} \dot{e_z} = f_o(e_z, t) \\ \dot{e_y} = f_1(e_z, e_y, t) \end{cases} \quad (6)$$

Where$e_z = z_d - z_r$and$e_y = y_d - y_r$.

Theorem [4]: Suppose for every initial conditions $z_d(0)$and$z_r(0)$and$y_d(0)$, responses $z_d(t)$and$y_d(t)$in the interval are extremely large and equilibrium point $e_z = 0$in$\dot{e_z} = f_o(e_z, t)$is asymptotically stable in general and uniform.[3] $\dot{e_y} = f_1(e_z, e_y, t)$is stable. The purpose of synchronization is met then for any initial condition. There are other methods for synchronization, including synchronization with the passive-based, ActiveX control, recursion, adaptive control, partial linearization, and particular matrix methods.[1]

ActiveX control Methods:-

This method is used to synchronize two systems which are identical (T) in parameters. A system (T) in general is as follows:[5]

$$\text{T system: } \begin{cases} \dot{x} = a(y - x) \\ \dot{y} = (c - a)x - axz \\ \dot{z} = -bz + xy \end{cases} \quad (7)$$

When the coefficients a = 2.1 and b = o.6 and chaotic system is c = 30 is selected. Synchronization process is as follows. Consider the drive and response systems:

$$\text{Drive system: } \begin{cases} \dot{x}_1 = a(y_1 - x_1) \\ \dot{y}_1 = (c - a)x_1 - ax_1z_1 \\ \dot{z}_1 = -bz_1 + x_1y_1 \end{cases} \quad (8)$$

$$\text{Response system: } \begin{cases} \dot{x}_2 = a(y_2 - x_2) + u_1(t) \\ \dot{y}_2 = (c - a)x_2 - ax_2z_2 + u_2(t) \\ \dot{z}_2 = -bz_2 + x_2y_2 + u_3(t) \end{cases} \quad (9)$$

The error for the systems is define as:

$$e_1 = x_2 - x_1 و e_2 = y_2 - y_1 و e_3 = z_2 - z_1 \quad (10)$$

The error dynamic equations are obtained as:-

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + u_1(t) \\ \dot{e}_2 = (c - a)e_1 - a(x_2z_2 - x_1z_1) + u_2(t) \\ \dot{e}_3 = -be_3 + x_2y_2 - x_1y_1 + u_3(t) \end{cases} \quad (11)$$

Then$u_1$and$u_2$andu_3 are defined control functions for active control like below, error in the dynamic, non-linear segments are eliminated and will be only a function of e.

$$\begin{cases} u_1(t) = v_1(t) \\ u_2(t) = a(x_2z_2 - x_1z_1) + v_2(t) \\ u_3(t) = -x_2y_2 + x_1y_1 + v_3(t) \end{cases} \quad (12)$$

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + v_1(t) \\ \dot{e}_2 = (c - a)e_1 + v_2(t) \\ \dot{e}_3 = -be_3 + v_3(t) \end{cases} \rightarrow \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} -a & a & 0 \\ c - a & 0 & 0 \\ 0 & 0 & -b \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad (13)$$

Now, it is sufficient to calculate the control vector **v**, depending on the state variables so that the error is diminished. Thatis$\dot{e} = -e$the response of the equation is e^{-t}. That is theerrors approach zero when $t \to \infty$[16]

The matrix A is chosen so that the eigenvalues of matrix A are -1.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = A \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \rightarrow A = \begin{bmatrix} a - 1 & -a & 0 \\ a - c & -1 & 0 \\ 0 & 0 & b - 1 \end{bmatrix} \quad (14)$$

$$\begin{cases} \dot{e}_1 = -e_1 \\ \dot{e}_2 = -e_2 \\ \dot{e}_3 = -e_3 \end{cases} \longrightarrow \begin{matrix} e_1(t) = c_1e^{-t} \\ e_2(t) = c_2e^{-t} \\ e_3(t) = c_3e^{-t} \end{matrix} \quad (15)$$

The initial values for the simulation are:-

$x_1(0) = 0.1$ ، $x_2(0) = 2.4$،$y_1(0) = -0.3$ ، $y_2(0) = -3.3$، $z_1(0) = o.2$ و $z_2(0) = 14.5$

Figure (1) shows the changes of x_1 and x_2 where Figures (2) and (3) show y_1 and y_2 and z_1 and z_2 respectively. Figure (4) presents e_1,e_2 and e_3 versus time [7].

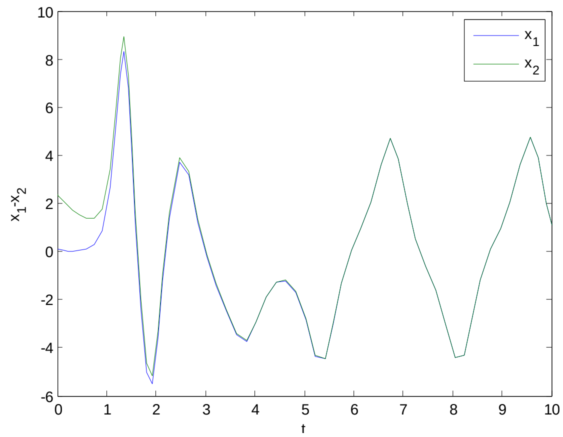

Figure 1:-signals x_2 and x_1.

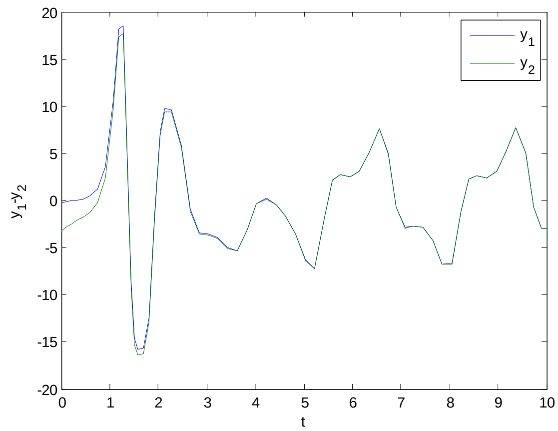

Figure 2:-.Signals y_1 and y_2

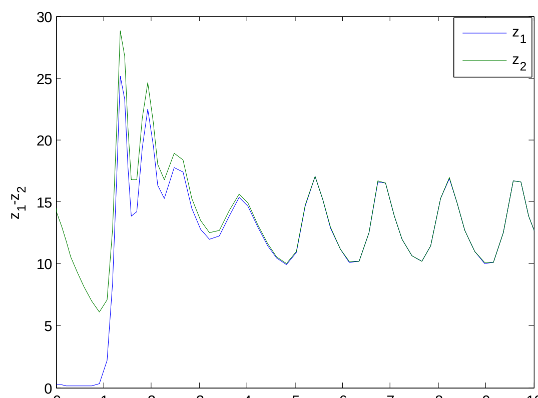

Figure 3:- Signals z_1 and z_2

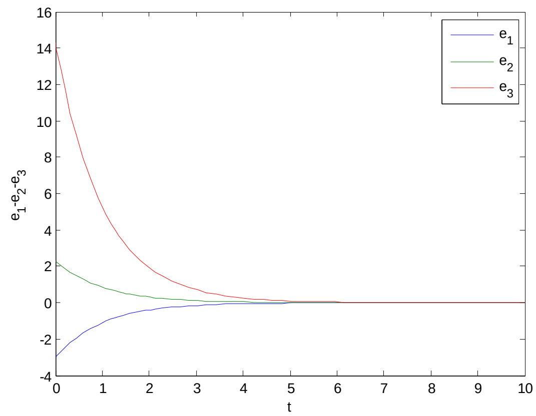

Figure 4:- signalse_1,e_2 and e_3

Synchronization by adaptive control

In particular situations, some parameters of the system are unknown. [9]
Once again there where the dynamic equations error as follows: [10]

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + u_1(t) \\ \dot{e}_2 = (c-a)e_1 - a(x_2 z_2 - x_1 z_1) + u_2(t) \\ \dot{e}_3 = -be_3 + x_2 y_2 - x_1 y_1 + u_3(t) \end{cases} \quad (16)$$

A positive definite Lyapunov function is defined as:-

$$v(e_1, e_2, e_3, \tilde{a}, \tilde{b}, \tilde{c}) = (\tfrac{1}{2})(e_1^2 + e_2^2 + e_3^2 + \tilde{a}^2 + \tilde{b}^2 + \tilde{c}^2) \quad (17)$$

Where$\tilde{a} = a - a_1$ ،$\tilde{b} = b - b_1$ ،$\tilde{c} = c - c_1 and a_1, b_1, c_1$are the estimated values of the unknown parameters a, b, c. we assume control functions as:

$$\begin{cases} u_1(t) = -a_1(e_2 - e_1) \\ u_2(t) = -(c_1 - a_1)e_1 + a_1(x_2 z_2 - x_1 z_1) \\ u_3(t) = (b_1 - 1)e_3 - x_2 y_2 - x_1 y_1 \end{cases} \quad (18)$$

The updating rules Are:-

$$\begin{cases} \dot{a}_1 = -e_1^2 - e_2(x_2 z_2 - x_1 z_1) \\ \dot{b}_1 = -e_3^2 \\ \dot{c}_1 = e_1 e_2 \end{cases} \quad (19)$$

Then, the derivative of the Function v:-

$$v(e_1, e_2, e_3, \tilde{a}, \tilde{b}, \tilde{c}) = e_1^2 - e_1^2 - e_1^2 < 0 \quad (20)$$

The signals$x_2$and$y_2$and$z_2$are from response system, while $x_1$and$y_1$andz_1 are synchronized drive system variables.[10]

The initial values for the simulation are as follows:-

$x_1(0) = 0.1$ ، $x_2(0) = 2.4$ ، $y_1(0) = -0.3$ ، $y_2(0) = -3.3$ ، $z_1(0) = o.2$ ، $z_2(0) = 14.5$ و $a_1(0) = b_1(0) = c_1(0) = 0.1$
Figure (5) showsx_1- x_2, Figure (6) shows$y_1 - y_2$, Figure (7) showsz_1- z_2, and Figure (8) shows$e_1$and$e_2$and $e_3$versus time.

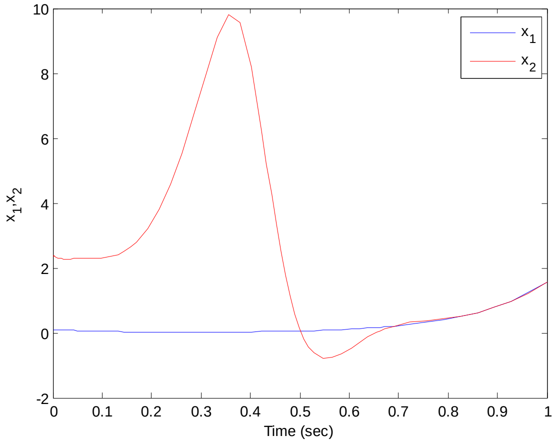

Figure 5:- .forx_1- x_2

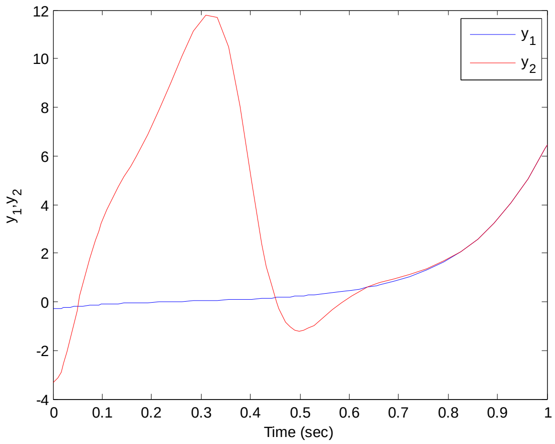

Figure 6:- .$y_1 - y_2$

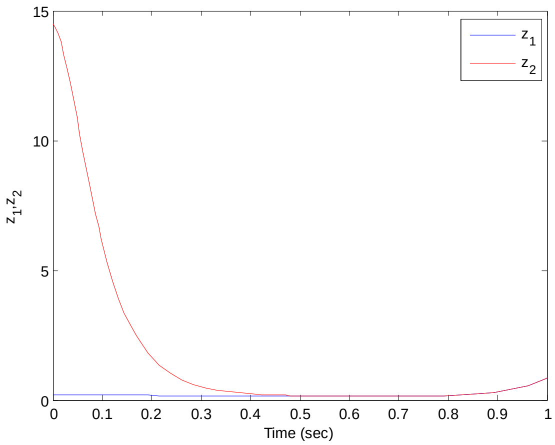

Figure 7:-.$z_1 - z_2$

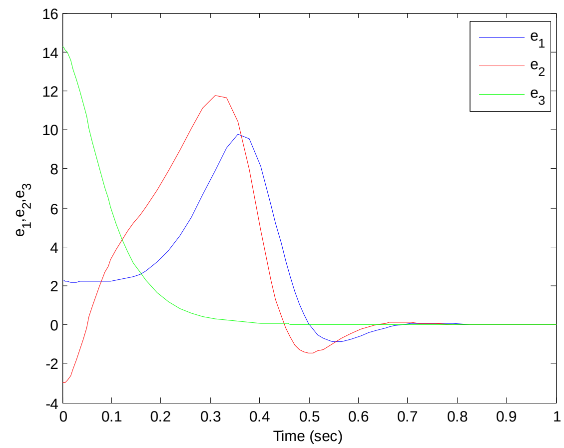

Figure 8:- .e_1,$e_2$and e_3

Synchronization by backstopping:-

This method is efficient for up to two parameters in the design of the system (T) [11]. The dynamical equation of system error is:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + u_1(t) \\ \dot{e}_2 = (c - a)e_1 - a(e_1 e_3 + x_1 e_3 + e_1 z_1) + u_2(t) \\ \dot{e}_3 = -be_3 + e_1 e_2 + e_1 y_1 + e_2 x_1 + u_3(t) \end{cases} \quad (21)$$

There are three stages in backstopping, in the first step we define$z_1 = e_1$, then:

$$\dot{z}_1 = ae_2 - az_1 + u_1(t) \quad (22)$$

Considering $e_2 = \alpha_1(z_1)$ as a virtual controller. where$\alpha_1(z_1)$is designed to stabilize the $z_1$in (22), Then$v_1$is chosen as aLyapunov function:

$$v_1 = \frac{z_1^2}{2} \quad (23)$$

$$\dot{v}_1 = z_1 \dot{z}_1 = -az_1^2 + az_1 \alpha_1 + u_1(t) z_1 \quad (24)$$

If $u_1(t) = 0$and$\alpha_1 = z_1 - \frac{z_1}{a}$ then$\dot{v}_1$will be negative.

$\alpha_1(z_1)$is a virtual control function.$z_2$is defined as:

$$z_2 = e_2 - \alpha_1 \quad (25)$$

$$\begin{cases} \dot{z}_1 = az_2 - z_1 \\ \dot{z}_2 = (c - a)z_1 - a(z_1 e_3 + x_1 e_3 + z_1^2) - \left(1 - \frac{1}{a}\right)(az_2 - z_1) + u_2(t) \end{cases} \quad (26)$$

In the next step, thesecondLyapunov function is defined:

$$v_2 = v_1 + \frac{z_2^2}{2} \quad (27)$$

The derivation to of v_2 will be:

$$\dot{v}_2 = -z_1^2 - z_2^2 < 0 \quad (28)$$

The functions are chosen as$\alpha_2(z_1, z_2) = 0$ and$u_2 = -z_2 - cz_1 + az_1^2 + \left(1 - \frac{1}{a}\right)(az_2 - z_1)$ then:

$$\begin{cases} \dot{z}_1 = az_2 - z_1 \\ \dot{z}_2 = -az_1 - z_2 - a(z_1 + x_1)z_3 \\ \dot{z}_3 = -bz_3 + z_1 y_1 + (z_1 + x_1)\left(z_2 + z_1 - \frac{z_1}{a}\right) + u_3(t) \end{cases} \quad (29)$$

To thirdLyapunovfunction isv_3:

$$v_3 = v_2 + \frac{z_3^2}{2} \quad (30)$$

The derivative ofv_3 will be:

$\dot{v}_3 = -z_1^2 - z_2^2 - a(z_1 + x_1)z_2z_3 + z_3m$ (31)
Where
$m = \left[-bz_3 + z_1y_1 + (z_1 + x_1)\left(z_2 + z_1 + \frac{z_1}{a}\right) + u_3(t)\right]$ (32)

If $u_3(t) = (b - a)z_3 - z_1y_1 + (z_1 + x_1)\left((a + 1)z_2 + z_1 - \frac{z_1}{a}\right) and \alpha_2 = 0$, then $\dot{v}_3 < 0$ hence the equilibrium point (0,0,0) is asymptotically stable. By taking $z_1 = e$ ، $z_2 = e_2 - \alpha_1$ و $z_3 = e_3 - \alpha_2(z_1, z_2)$ it is observed that e_1 ، e_2 and e_3 asymptotically approach zero, that means the synchronization between the systems.The initial values for the simulation are as follows:
$x_1(0) = 0.1$ ، $x_2(0) = 2.4$ ، $y_1(0) = -0.3$ ، $y_2(0) = -3.3$ ، $z_1(0) = o.2$ ، $z_2(0) = 14.5$
Figures (9) to (12) show the variables x_1- x_2, $y_1 - y_2$, z_1–z_2 and errors e_1, e_2 and e_3 respectively [6,7,11].

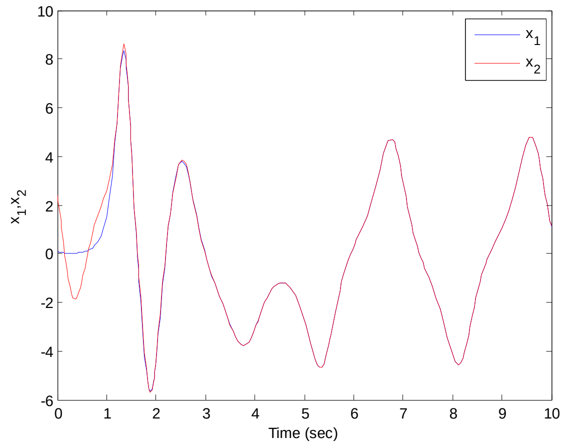

Figure 9:- x_1- x_2

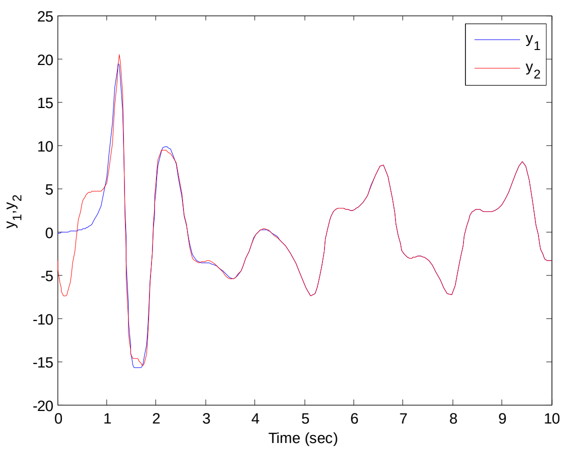

Figure 10:- $y_1 - y_2$

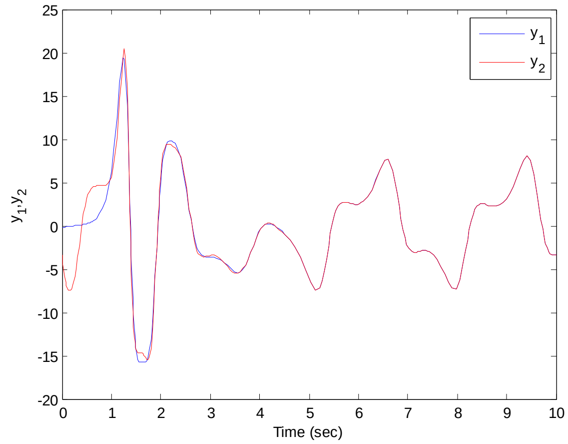

Figure 11:- z_1–z_2

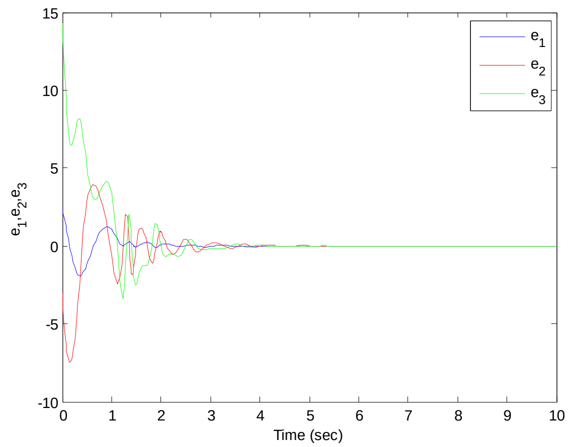

Figure 12:- errors e_1 and e_2 and e_3

Synchronization using the particular matrix:-

This method assumes unknown delay [14]. The equations are as follows for drive and response systems:

$$\begin{cases} \dot{x}_1(t) = -x_1(t) - x_2(t) + a_1[x_1(t-\tau_1)] + a_2[x_2(t-\tau_2)] \\ \dot{x}_2(t) = x_1(t) + \beta_1 x_2(t) \\ \dot{x}_3(t) = (x_1(t) - \gamma)x_3(t) + x_1(t)x_3(t) + \beta_2 \end{cases} \quad (33)$$

$$\begin{cases} \dot{y}_1(t) = -y_1(t) - y_2(t) + a_1[y_1(t-\tau_1)] + a_2[y_2(t-\tau_2)] - u_1(t) \\ \dot{y}_2(t) = y_1(t) + \beta_1 y_2(t) - u_2(t) \\ \dot{y}_3(t) = (y_1(t) - \gamma)y_3(t) + y_1(t)x_3(t) - \beta_2 u_3(t) \end{cases} \quad (34)$$

The equations in the form of a matrix can be written as follows:-

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \beta_1 & 0 \\ x_3 & 0 & x_1(t) - \gamma \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} a_1 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t-\tau_1) \\ x_2(t-\tau_2) \\ x_3(t-\tau_2) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \beta_2 \end{bmatrix} \quad (35)$$

$$\begin{bmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \\ \dot{y}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & \beta_1 & 0 \\ y_3(t) & 0 & y_1(t) - \gamma \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} + \begin{bmatrix} a_1 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_1(t-\tau_1) \\ y_2(t-\tau_2) \\ y_3(t-\tau_2) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \beta_2 \end{bmatrix} - \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} \quad (36)$$

The values τ_1 ، τ_2 and τ_3 are delay parameters $of the$ system.

According to data provided by the design of the control input Theorem 1 in [13], obtained control values as follows:-

$$u(t) = \begin{bmatrix} \zeta_1 & 0 & -1 + x_3(t) \\ 0 & \zeta_2 & 0 \\ 0 & 0 & \zeta_3 \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} \tag{37}$$

The system error equation is obtained as follows:-

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \\ \dot{e}_3(t) \end{bmatrix} = \begin{bmatrix} \zeta_1 & -1 & -x_3(t) \\ 1 & \beta_1 - \zeta_2 & 0 \\ x_3(t) & 0 & y_1(t) - \zeta_3 - \gamma \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \end{bmatrix} + \begin{bmatrix} a_1 & a_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1(t-\tau_1) \\ e_2(t-\tau_2) \\ e_3(t-\tau_2) \end{bmatrix} \tag{38}$$

According to Theorem 1 in [14] system error dynamic equations for asymptotic stability will be making the following conditions.

$$\begin{cases} -\zeta_1 + 2|a_1| + |a_2| < 0 \\ \beta_1 - \zeta_2 + |a_2| < 0 \\ y_1(t) - \gamma - \zeta_3 < 0 \end{cases} \tag{39}$$

The initial values for the simulation are as follows:-

$a_1 = 0.2$ ، $a_2 = 0.5$ ، $\beta_1 = o.2$ ، $\beta_2 = 0.2$ ، $\gamma = 5.7$ ، $\tau_1 = 0.2$ ، $\tau_2 = 0.1$ ، $\zeta_1 = 1$ ، $\zeta_2 = 1$ ، $\zeta_3 = y_1(t)$ ، $[x_1(0) \quad x_2(0) \quad x_3(0)] = [-2 \quad -4 \quad 10]$ ، $[y_1(0) \quad y_2(0) \quad y_3(0)] = [1 \quad 2 \quad 1]$

Figures (13) to (16) show x_1- y_1, x_2- y_2, x_3- y_3 and errors e_1, e_2 and e_3 respectively [15].

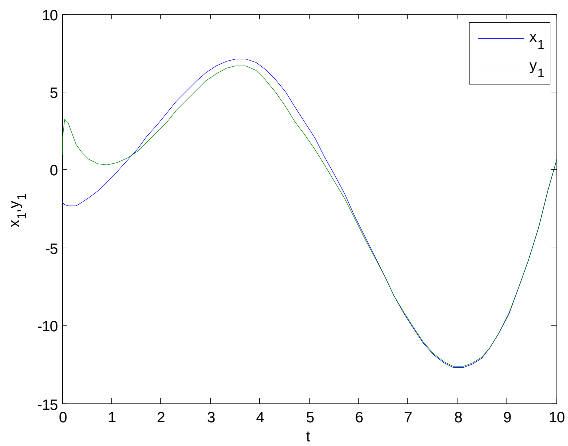

Figure 13:- x_1- y_1

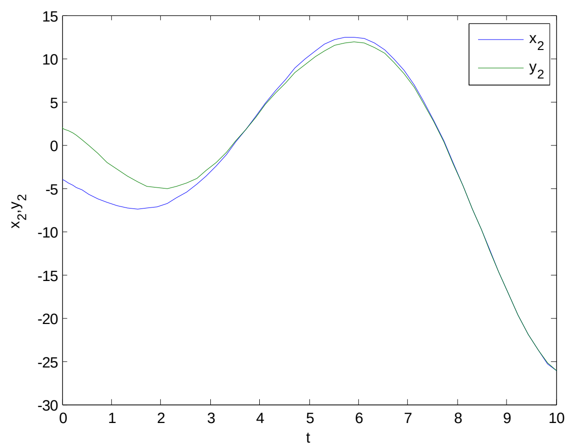

Figure 14:- x_2- y_2

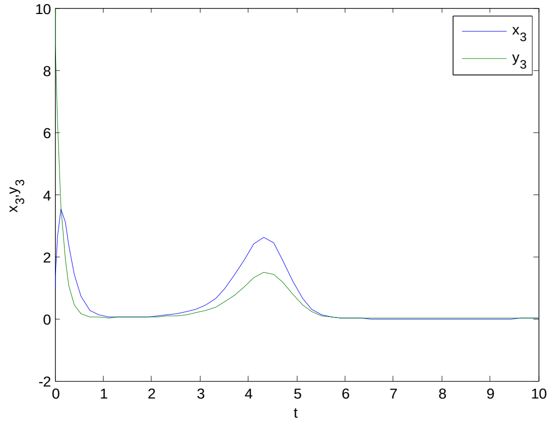

Figure 15:-x_3- y_3

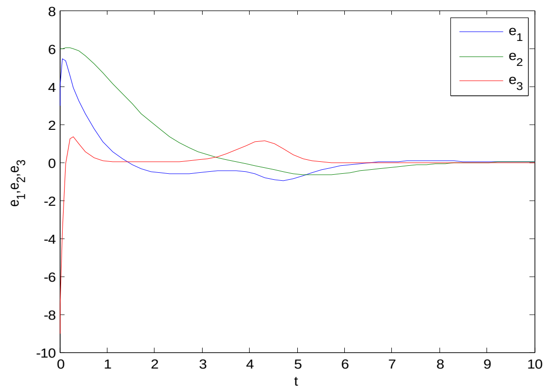

Figure 16:- errors $e_1$and$e_2$and e_3

linear method Details:-

In this method, the synchronization of one stateconcludes synchronization of all other states. Integrated system is considered with the following equations:

$$\begin{cases} \dot{x}_1 = (25\theta + 10)(y_1 - x_1) \\ \dot{y}_1 = (28 - 35\theta)x_1 - x_1 z_1 + (29\theta - 1)y_1 \\ \dot{z}_1 = x_1 y_1 - \frac{8+\theta}{3} z_1 \end{cases} \tag{40}$$

The system will be up for θ = 0, θ = 0.8 and θ = 1, respectively Lorenz chaotic system, Lu and Chen.
Considering the equation (40) as the actuator and the equation (41) as the system's response:

$$\begin{cases} \dot{x}_2 = (25\theta + 10)(y_2 - x_2) \\ \dot{y}_2 = (28 - 35\theta)x_2 - x_2 z_2 + (29\theta - 1)y_2 + u \\ \dot{z}_2 = x_2 y_2 - \frac{8+\theta}{3} z_2 \end{cases} \tag{41}$$

The error for the system is defined:

$$e_1 = x_2 - x_1 ؛ e_2 = y_2 - y_1 ؛ e_3 = z_2 - z_1 \tag{42}$$

Error dynamic equations will be obtained as follows:

$$\begin{cases} \dot{e}_1 = (25\theta + 10)(e_2 - e_1) \\ \dot{e}_2 = (28 - 35\theta)e_1 + (29\theta - 1)e_2 - e_1 e_3 - z_1 e_1 - x_1 e_3 + u \\ \dot{e}_3 = e_1 e_2 + x_1 e_2 + y_1 e_1 - \frac{8+\theta}{3} e_3 \end{cases} \tag{43}$$

So that the control vector u we is chosen as a linear Relationship:-

$$u = (35\theta - 28)e_1 - 29\theta e_2 + e_1 e_3 + z_1 e_1 + x_1 e_3 \quad (44)$$
$$\dot{e}_2 = e_2 \quad (45)$$

According to Theorem 1 in [13]. Implementation wishes to simulate this technique to the Lorenz system as follows:-

$$\begin{cases} \dot{e}_1 = 10(e_2 - e_1) \\ \dot{e}_2 = 28e_1 - 1\, e_2 - e_1 e_3 - z_1 e_1 - x_1 e_3 + u \\ \dot{e}_3 = e_1 e_2 + x_1 e_2 + y_1 e_1 - \frac{8}{3} e_3 \end{cases} \quad (46)$$

$$u = -28e_1 + e_1 e_3 + z_1 e_1 + x_1 e_3 \quad (47)$$

The initial values for the simulation are as Follows:-

$$[x_1(0) \quad y_1(0) \quad z_1(0)] = [10 \quad 10 \quad 10]$$
$$[e_1(0) \quad e_2(0) \quad e_3(0)] = [-5 \quad -10 \quad 10]$$

Figure (17) to (20) showx_1- x_2, $y_1 - y_2$, z_1–$z_2$and errors e_1, $e_2$and $e_3$respectively[15].

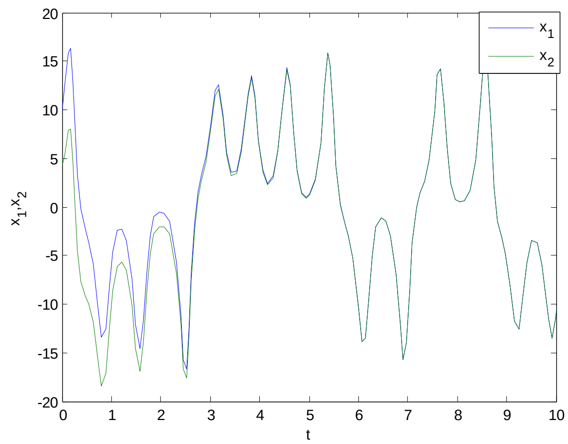

Figure 17:- forx_1- x_2

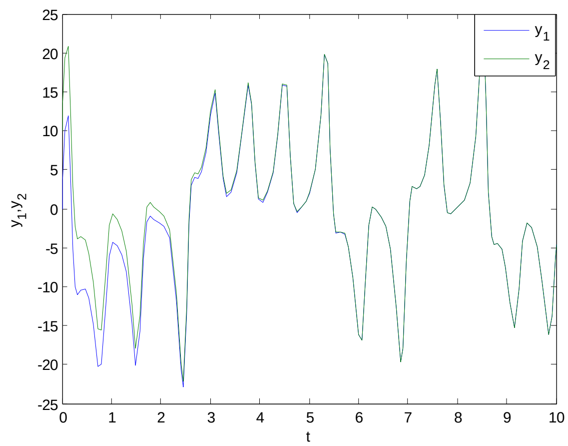

Figure 18:- $y_1 - y_2$

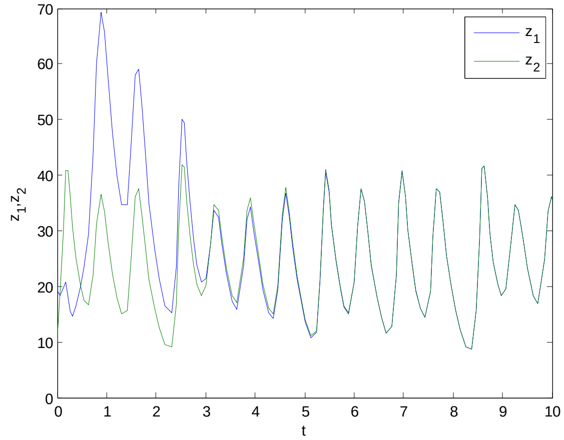

Figure 19:-$z_1 - z_2$

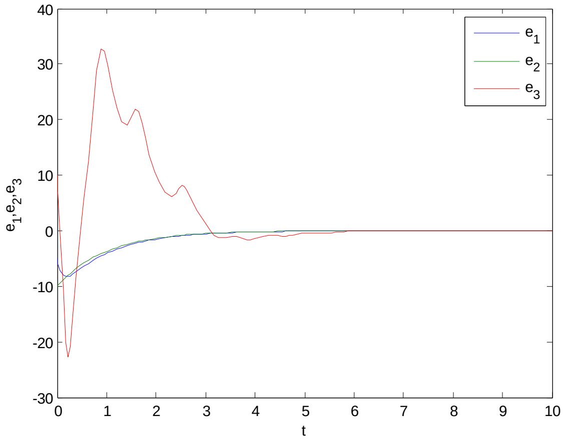

Figure 20:- errors e_1 ande_2 and e_3

Conclusion:-

In this paper we dealt with the history of chaos, chaos theory and synchronization of chaotic systems. Synchronization methods used in this paper areActive control, backstopping control, adaptive control, partial linearization matrix method, which is based on synchronization systems with a delay. All these methods are based on Lyapunov theory.

SIMULINK environment of MATLAB software has been utilized for simulation of synchronization of chaos systems. The methods designed for easy synchronization of chaotic systems introduce suitable methods for synchronization of delayed chaotic systems, with simple calculations, and no need to calculations of Lyapunov function for stability in contrast to the classical methods.

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