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RESEARCH ARTICLE

ESTIMATION OF POPULATION VARIANCE USING KNOWN COEFFICIENT OF KURTOSIS AND MEDIAN OF AN AUXILIARY VARIABLE

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Abstract

We have suggested an improved ratio type log estimator for population variance by using coefficient of kurtosis and median of an auxiliary variable x. The properties of proposed estimator have been derived up to first order of Taylor's series expansion. The efficiency conditions derived theoretically under which the proposed estimator performs better than existing estimators. The proposed estimator as illustrated by the empirical studies using real populations performs better than the existing estimators i.e. it has the smallest mean squared error and the highest Percentage Relative Efficiency.

Introduction:-

Consider a finite population of size N identifiable and non-overlapping units. From this population size, a sample of size n is drawn by simple random sampling. Let, the variable under study is denoted by y and the variable which contains the auxiliary information about the study variable is denoted by x. The population mean of study and auxiliary variable is denoted by $\overline{Y}$ and $\overline{X}$. Further, the population and sample variances are given as under

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i-\overline{Y})^2,\ S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i-\overline{X})^2 \text{ and}$$
$$s_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{y})^2,\ s_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2$$

In general, we define the following parameters

$\mu_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}(y_i-\bar{y})^r(x_i-\bar{x})^s$, $\lambda_{22} = \frac{\mu_{22}}{\mu_{02}\,\mu_{20}}$, C_y and C_x be the coefficient of variation of the study and auxiliary variable.

$K_x = \lambda_{04} = \frac{\mu_{04}}{{\mu_{02}}^2}$ and $K_y = \lambda_{40} = \frac{\mu_{40}}{{\mu_{20}}^2}$ be the coefficient of kurtosis of the study and auxiliary variable.

$\rho_{xy} = \frac{S_{xy}}{S_y S_x}$ be the coefficient of correlation between y and x.

M_x be the population median of auxiliary variable.

Many authors have come up with more precise estimators by employing prior knowledge of certain population parameter(s). [2] for example attempted use of the coefficient of variation of study variable but prove inadequate for in practice, this parameter is unknown. Motivated by [2] work, [3] [4] and [5] used the known coefficient of variation but now that of the auxiliary variable for estimating population mean of study variable. Reasoning along the same path [6] used the prior value of coefficient of kurtosis of an auxiliary variable in estimating the population variance of the study variable y. Kurtosis in most cases is not reported or used in many research articles, in spite of

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the fact that fundamentally speaking every statistical package provides a measure of kurtosis. This may be attributed to the likelihood that kurtosis is not well understood or its importance in various aspects of statistical analysis has not been explored fully. Kurtosis can simply be expressed as

$$K = \frac{E(x-\mu)^4}{(E(x-\mu)^2)^2}$$

where E denotes the expectation, μ is the mean. Moreover, one of the measure of central tendency which we use in this article is Median. Median is the middle term of the series of the distribution under study when the values are arranged in ascending or descending order. Inspite of many applications of median, the most important is advantage of median is that it is less affected by the outliers and skewed data, thus is preferred to the mean especially when the distribution is not symmetrical. We can therefore utilize the median and the coefficient of kurtosis of the auxiliary variable to derive a more precise ratio type log estimator for population variance.

Estimators Available in Literature:

In this section we have reviewed some finite population variance estimators existing in literature which will help in the construction and development of the proposed estimator. Notably, when auxiliary information is not available the usual unbiased estimator to the population variance is

$$t_1 = s_y^2 \tag{1}$$

The bias and MSE of t_1 is

$$\text{Bias}(t_1) = \frac{1-f}{n} S_y^2 \left\{(K_x-1)\Psi_1\left(\Psi_1 - \frac{\lambda_{22}-1}{K_x-1}\right)\right\} = 0 \tag{2}$$

$$\text{MSE}(t_1) = V(t_1) = \frac{1-f}{n} S_y^4 \left\{(K_y-1) + (K_x-1)\Psi_1\left(\Psi_1 - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\} \tag{3}$$

$$= \frac{1-f}{n} S_y^4 \left(K_y-1\right) \text{; where } \Psi_1 = 0$$

Population variance, estimation using auxiliary information was considered by [7], and proposed ratio type population variance estimator, given by

$$t_2 = \frac{S_x^2}{s_x^2} s_y^2 \tag{4}$$

The bias and Mean Squared Error of Isaki's estimator,

$$\text{Bias}(t_2) = \frac{1-f}{n} S_y^2 \left\{(K_x-1)\Psi_2\left(\Psi_2 - \frac{\lambda_{22}-1}{K_x-1}\right)\right\} = \frac{1-f}{n} S_y^2 \left\{(K_x-1) - (\lambda_{22}-1)\right\}$$

$$\text{MSE}(t_2) = \frac{1-f}{n} S_y^4 \left\{(K_y-1) + (K_x-1)\Psi_2\left(\Psi_2 - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\} \tag{5}$$

$$= \frac{1-f}{n} S_y^4 \left\{(K_y-1) + (K_x-1) - 2(\lambda_{22}-1)\right\} \text{; where } \Psi_2 = 1$$

[6] initiated the use of coefficient of kurtosis in estimating population variance of a study variable y. Later, the coefficient of kurtosis was used by [3] [5] [8] in the estimating the population mean. [9] using the known information on both S_x^2 and k_x suggested modified ratio type population variance estimator for S_y^2 as

$$t_3 = \left[\frac{S_x^2 + k_x}{s_x^2 + k_x}\right] s_y^2 \tag{6}$$

The estimator, bias and MSE obtained as

$$\text{Bias}(t_3) = \frac{1-f}{n} S_y^2 \left\{(K_x-1)\Psi_3\left(\Psi_3 - \frac{\lambda_{22}-1}{K_x-1}\right)\right\} \tag{7}$$

$$\text{MSE}(t_2) = \frac{1-f}{n} S_y^4 \left\{(K_y-1) + (K_x-1)\Psi_3\left(\Psi_3 - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\} \tag{8}$$

where $\Psi_3 = \left[\frac{S_x^2}{S_x^2 + k_x}\right]$

[10] suggested four modified ratio type variance estimators using known values of C_x and k_x,

$$t_4 = \left[\frac{S_x^2 - C_x}{s_x^2 - C_x}\right] s_y^2 \tag{9}$$

$$t_5 = \left[\frac{S_x^2 - k_x}{s_x^2 - k_x}\right] s_y^2 \tag{10}$$

$$t_6 = \left[\frac{S_x^2\, k_x - C_x}{s_x^2\, k_x - C_x}\right] s_y^2 \tag{11}$$

$$t_7 = \left[\frac{S_x^2 \ C_x - k_x}{s_x^2 \ C_x - k_x}\right] s_y^2 \tag{12}$$

The biases and MSE of their estimators,

$$\text{Bias}(t_4) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_4 \left(\Psi_4 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{13}$$

$$MSE(t_4) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_4 \left(\Psi_4 - 2 \left(\frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{14}$$

$$Bias(t_5) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_5 \left(\Psi_5 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{15}$$

$$MSE(t_5) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_5 \left(\Psi_5 - 2 \left(\frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{16}$$

$$Bias(t_6) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_6 \left(\Psi_6 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{17}$$

$$MSE(t_6) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_6 \left(\Psi_6 - 2 \left(\frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{18}$$

$$Bias(t_7) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_7 \left(\Psi_7 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{19}$$

$$MSE(t_7) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_7 \left(\Psi_7 - 2 \left(\frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{20}$$

Where

$$\Psi_4 = \left[\frac{S_x^2}{S_x^2 - C_x}\right], \Psi_5 = \left[\frac{S_x^2}{S_x^2 - K_x}\right], \Psi_6 = \left[\frac{S_x^2 \ K_x}{S_x^2 \ K_x - C_x}\right], \Psi_7 = \left[\frac{S_x^2 C_x}{S_x^2 C_x - K_x}\right]$$

[11] utilizing population median M_x came up with a modified ratio type population variance estimator as

$$t_8 = \left[\frac{S_x^2 - M_x}{s_x^2 \ - M_x}\right] s_y^2 \tag{21}$$

The bias and MSE of their estimator

$$Bias(t_8) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_8 \left(\Psi_8 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{22}$$

$$MSE(t_8) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_8 \left(\Psi_8 - 2 \left(\frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{23}$$

Where $\Psi_8 = \left[\frac{S_x^2}{S_x^2 + M_x}\right]$

[12] using the known quartiles (upper and lower quartile Q_3 and Q_1 respectively) of the auxiliary variable x suggested

$$t_9 = \left[\frac{S_x^2 + Q_1}{s_x^2 \ + Q_1}\right] s_y^2 \tag{24}$$

$$t_{10} = \left[\frac{S_x^2 + Q_3}{s_x^2 \ + Q_3}\right] s_y^2 \tag{25}$$

The biases and MSE of their estimators $t_9$and t_{10} as follows

$$Bias(t_9) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_9 \left(\Psi_9 - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{26}$$

$$MSE(t_9) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_9 \left(\Psi_9 - 2 \left(\frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{27}$$

$$Bias(t_{10}) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_{10} \left(\Psi_{10} - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{28}$$

$$MSE(t_{10}) = \frac{1-f}{n} S_y^4 \left\{ (K_y - 1) + (K_x - 1) \Psi_{10} \left(\Psi_{10} - 2 \left(\frac{\lambda_{22}-1}{K_x-1} \right) \right) \right\} \tag{29}$$

Where $\Psi_9 = \left[\frac{S_x^2}{S_x^2 + Q_1}\right]$ and $\Psi_{10} = \left[\frac{S_x^2}{S_x^2 + Q_3}\right]$

Motivated by [10] and [11] [13] considered the estimation of finite population variance using known coefficient of variation and median of an auxiliary variable, proposed an estimator.

$$t_{11} = \left[\frac{C_x \ S_x^2 + M_x}{C_x s_x^2 \ + M_x}\right] s_y^2 \tag{30}$$

The bias and MSE obtained to be,

$$Bias(t_{11}) = \frac{1-f}{n} S_y^2 (K_x - 1) \left\{ \Psi_{11} \left(\Psi_{11} - \frac{\lambda_{22}-1}{K_x-1} \right) \right\} \tag{31}$$

$$MSE(t_{11}) = \frac{1-f}{n}\ S_y^4\ \left\{\left(K_y - 1\right) + (K_x - 1)\,\Psi_{11}\ \left(\Psi_{11} - 2\ \left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\} \qquad (32)$$

where $\Psi_{11} = \left[\frac{C_x S_x^2}{C_x S_x^2 + M_x}\right]$

Motivated by [23] considered the estimation of finite population variance using known kurtosis and median of an auxiliary variable.

$$t_{12} = s_y^2 \left\{\frac{K_x\ S_x^2 + M_x^2}{K_x s_x^2\ + M_x^2}\right\} \qquad (33)$$

The bias and MSE obtained to be,

$$Bias(t_{12}) = \frac{1-f}{n}\ S_y^2\ (K_x - 1)\ \left\{\Psi_{12}\ \left(\Psi_{12} - \frac{\lambda_{22}-1}{K_x-1}\right)\right\} \qquad (34)$$

$$MSE(t_{12}) = \frac{1-f}{n}\ S_y^4\ \left\{\left(K_y - 1\right) + (K_x - 1)\,\Psi_{12}\ \left(\Psi_{12} - 2\ \left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\} \qquad (35)$$

where $\Psi_{12} = \left[\frac{K_x S_x^2}{C_x S_x^2 + M_x^2}\right]$

Proposed Estimator:

Motivated by the works of [23] [24] ,[25] and [26] in the improvement of the performance of the population variance estimator of the study variable using known population parameters of an auxiliary variable. We introduce the following improved ratio type log estimator for population variance using a known value of population coefficient of kurtosis k_x and median M_x of an auxiliary variable.

$$t_{13} = s_y^2 \left[1 + \ log\left\{\frac{K_x\ S_x^2 + M_x^2}{K_x s_x^2\ + M_x^2}\right\}\right]^a \qquad (36)$$

To calculate the bias and the MSE of t_{13};

We let,

$\epsilon_0 = \frac{(S_y^2 - s_y^2)}{S_y^2}$, $\epsilon_1 = \frac{(S_x^2 - s_x^2)}{S_x^2}$

$E(\epsilon_0) = E(\epsilon_1) = 0$, $E(\epsilon_0^2) = \frac{1-f}{n}\ (\lambda_{40} - 1)$, $E(\epsilon_1^2) = \frac{1-f}{n}\ (\lambda_{04} - 1)$,

$E(\epsilon_0 \epsilon_1) = \dfrac{1-f}{n}\ (\lambda_{22} - 1)$

Now expressing t_{13} in terms of epsilons ϵ we have,

$$t_{13} = S_y^2\ (1 + \epsilon_0)\left[1 + \ log\left\{\frac{K_x\ S_x^2 +\ M_x^2}{K_x S_x^2\ (1 + \epsilon_1)\ +\ M_x^2}\right\}\right]^a$$

$$= S_y^2\ (1 + \epsilon_0)\left[1 + \ log\left\{1 + \frac{K_x\ S_x^2\ \epsilon_1}{(K_x S_x^2\ \ +\ M_x^2)}\right\}^{-1}\right]^a$$

$$= S_y^2\ (1 + \epsilon_0)[1 + \ log\{1 + \ \eta_{13}\ \epsilon_1\}^{-1}]^a \qquad (37)$$

where $\eta_{13} = \left[\frac{K_x S_x^2}{K S_x^2 + M_x^2}\right]$, we assume that $|\eta_{13}\ \epsilon_1| < 1$, so that $\{1 + \ \eta_{13}\ \epsilon_1\}^{-1}$ is expandable.

Expanding the right hand side of (37) and multiplying out we have

$$= S_y^2\ (1 + \epsilon_0)[1 + \ log\{1 - \ \eta_{13}\ \epsilon_1 + (\eta_{13}\ \epsilon_1)^2\ \}]^a$$

$$t_{13} - S_y^2 = S_y^2\ \left[\epsilon_0 - a\,\eta_{13}\ \epsilon_0 \epsilon_1 - a\,\eta_{13}\ \epsilon_1 + \frac{a^2}{2}\ {\eta_{13}}^2\ {\epsilon_1}^2\right] \qquad (38)$$

Taking expectations on both the sides, we get

$$E\left(\ t_{13} - S_y^2\right) = S_y^2\ \left[E(\epsilon_0) - a\,\eta_{13}\,E(\epsilon_0 \epsilon_1) - a\,\eta_{13}\,E(\epsilon_1) + \frac{a^2}{2}{\eta_{13}}^2\ E\ ({\epsilon_1}^2)\right]$$

$$Bias(t_{13}) = \frac{1-f}{n}\ S_y^2\ (K_x - 1) a\,\eta_{13}\left(\frac{a}{2}\ \eta_{13} - \frac{\lambda_{22}-1}{K_x-1}\right) \qquad (39)$$

Squaring on both the sides of equation (38), we get MSE

$$MSE(t_{13}) = \frac{1-f}{n}\ S_y^4\ \left\{\left(K_y - 1\right) + (K_x - 1)\ a\,\eta_{13}\ \left(a\,\eta_{13} - 2\ \left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\} \qquad (40)$$

On differentiating $MSE(t_{13})$ with respect to a, we get

$$a_{\ opt} = \left[\frac{\lambda_{22}-1}{\eta_{13}(K_x - 1)}\right] \qquad (41)$$

$$[MSE(t_{13})]_{min} = \frac{1-f}{n} S_y^4 \left\{(K_y - 1) - \frac{(\lambda_{22}-1)^2}{K_x-1}\right\} \quad (42)$$

is the desired optimum mean squared estimator for proposed estimator.

Theoretical Comparisons:

The theoretical conditions under which the proposed modified ratio type estimators t_{12} is more efficient than the other existing estimator t_j, j=1, 2, … , 11, 12 from MSE of t_j, j=1, 2, … , 11, 12; given to the first degree of approximation in general as

$$MSE(t_j) = \frac{1-f}{n} S_y^4 \left\{(K_y - 1) + (K_x - 1)\,\Psi_j \left(\Psi_j - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\} \quad (43)$$

Using Equations (40) and (43), we have $MSE(t_{13}) < MSE(t_j)$, if

$$a\,\eta_{13} \left(a\,\eta_{13} - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right) < \Psi_j \left(\Psi_j - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right) \quad (44)$$

Empirical Studies:

Using the data from Population I (Source:[[21], 228]) and Population II (source: [22]). We perform the working of proposed and existing estimators to this real data set and the data statistics are given below:

Population I:

X = Fixed capital, Y = output of 80 factories, N = 80, n = 20, $\bar{X}$ = 11.265 , $\bar{Y}$ = 51.826, S_x^2 = 71.504 , S_y^2 = 336.979, S_{xy} = 146.068, $\lambda_{04} = K_x$ = 2.866, $\lambda_{40} = K_y$ = 2.267 , λ_{22} = 2.221, ρ_{xy}= 0.941, C_y= 0.354, C_x= 0.751, M_x= 10.300, Q_1= 5.150 , Q_3= 16.975

Population II:

X = acreage under wheat crop in 1973, Y = acreage under wheat crop in 1974, N = 70, n = 25, $\bar{X}$ = 175.2671 , $\bar{Y}$ = 96.700, S_x^2 = 19840.7508 , S_y^2 = 3686.1898, $\lambda_{04} = K_x$ = 7.0952, $\lambda_{40} = K_y$ = 4.7596 , λ_{22} = 4.6038, ρ_{xy}= 0.7293, C_y= 0.6254, C_x= 0.8037, M_x= 72.4375, Q_1= 80.1500 , Q_3= 225.0250

Using the above summary values we have the results in Table 1 below. From the table, Mean Squared Errors it is clear that our proposed ratio type log estimator t_{13} for population variance has the least Mean Squared Error (MSE). The efficiency of our proposed estimator t_{13} is examined numerically by its Percentage Relative Efficiency (PRE(s)) in comparison with those of existing

Table 1:- Bias and Mean Squared Errors (MSE).

Estimator	Population I		Population II	
	Bias	MSE	Bias	MSE
t_1	0	5395.289	0	1313625.261
t_2	8.151	3276.421	236.154	924946.481
t_3	6.956	2740.349	235.656	924324.375
t_4	8.512	3006.373	236.187	925017.011
t_5	9.518	3186.399	236.445	925569.577
t_6	8.279	2965.067	236.159	924956.421
t_7	10.002	3275.722	236.517	925721.916
t_8	4.530	2377.418	233.201	918641.426
t_9	6.126	2609.91	232.889	917976.121
t_{10}	2.934	2181.488	227.099	905689.896
t_{11}	3.656	2314.033	232.485	917116.922

t_{12}	0.708	1993.270	207.653	865134.030
t_{13}	**-5.0481**	**1993.1**	**-100.985**	**569127.7**

estimators using real populations from [[21], p.228] and [22].
We have computed the PRE(s) of the estimators t_j , j=1, 2, … , 11,12 using the formulae

$$PRE = \frac{MSE(s_y^2)}{MSE(t_j)} \text{ x } 100$$

$$PRE = \frac{\frac{1-f}{n} S_y^4 \ (K_y - 1)}{\frac{1-f}{n} S_y^4 \ \left\{(K_y-1)+(K_x-1)\,\Psi_j \left(\Psi_j - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\}} \text{ x } 100$$

$$= \frac{(K_y - 1)}{\left\{(K_y-1)+(K_x-1)\,\Psi_j \left(\Psi_j - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\}}$$

Then PRE for our proposed estimator is subsequently,

$$PRE = \frac{MSE(s_y^2)}{MSE(t_{13})} \text{ x } 100$$

$$= \frac{\frac{1-f}{n} S_y^4 \ (K_y - 1)}{\frac{1-f}{n} S_y^4 \ \left\{(K_y-1)+(K_x-1)\,a\,\eta_{12} \left(a\,\eta_{12} - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\}} \text{ x } 100$$

$$= \frac{(K_y - 1)}{\left\{(K_y-1)+(K_x-1)\,a\,\eta_{12} \left(a\,\eta_{12} - 2\left(\frac{\lambda_{22}-1}{K_x-1}\right)\right)\right\}} \text{ x } 100$$

Using formula (40) and (43) we compute the Percent Relative Efficiencies and tabulate the results in Table 2.
Percentage Relative efficiency being a robust statistical tool that is used to ascertain the efficiency of suggested estimator over conventional estimator. From the findings summarized in the table above it is clear that our proposed estimator t_{13} performed best, that is it has the highest PRE among all the other estimators. This therefore implies that we can apply our proposed estimator to appropriate practical situations and obtain better and more efficient results than the traditional and other existing population variance estimators.

Table 2:- Percent Relative Efficiencies (PRE).

Estimator	**PRE of Population I**	**PRE of Population II**
t_1	100	100
t_2	164.67	142.02
t_3	196.88	142.12
t_4	179.46	142.01
t_5	169.32	141.93
t_6	181.96	142.02
t_7	164.71	141.90
t_8	226.94	143.00
t_9	206.72	143.10
t_{10}	247.32	145.04
t_{11}	233.16	143.23
t_{12}	270.68	151.84
t_{13}	**270.70**	**230.81**

Conclusion:-

In this article, we have proposed an improved log type ratio estimator for population variance estimator using known coefficient of kurtosis and the median of the auxiliary variable x. We have analyzed the performance of our proposed estimator against the usual unbiased variance estimator and existing estimators using two natural populations by comparing their PRE(s). Based on the results of our studies, it is proved that our proposed estimator works better than the other existing estimators having the highest Percentage Relative Efficiency. Hence can be applied to practical applications, where knowledge of population parameters of auxiliary variable is available. We also recommend that our proposed estimator can be further improved by extending the number of Taylor's series terms to be more than two.

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