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RESEARCH ARTICLE

AN APPLICATION OF ESTIMATING CAPABILITY INDICES FOR FATIGUE LIFE DISTRIBUTION.

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Abstract

There are variety of statistical distributions which cover most of the problem in reliability and time to event analysis. Birnbaum Saunders's distribution popularly named as Fatigue Life model is commonly and extensively applied for those quality characteristics of measurements which are associated with high occurrences. In this paper an applicative example is illustrated for earlier presented data sets that exhibiting Fatigue measurements constituted by Fatigue failure time of 3034 aluminum coupons oscillated at 18 cycles per second under 3 different stress levels with the fact that varying stress level producing number of failures which make the process out of control. So an algorithm is defined to first transform fatigue measurements to standard normal, make the process statistically controlled and estimate capability indices for measurements exhibit Fatigue distribution. An algorithm is made in R-console.

Introduction:-

In statistics we have variety of distributions which cover time to failure analysis in different scenario but these distributions have lack to specify the failures due to fatigue and a family of distribution is needed which specify the monotonic failure rates which others do not. Birnbaum-Saunders's (1969) derived comparatively flexible distribution from a phenomenon of physical fatigue where failures are attributed by crack growth. This probabilistic model thereafter named as Fatigue Life Model is designed for failure occurrence as a result of shock or crack accumulation due to over stress and used to expect life time of failures.

Process capability indices allows quantifying how well a process can produce acceptable product with the prediction of model adequacy that meet certain specification(s) and quality requirement(s) preset by the product designer. The aim of process capability analysis is to estimate, monitor, and possibly reduce variability in production or manufacturing processes. Measuring process capability yields huge cost savings by eliminating non-value added activities, reducing scrap, rework and creating satisfied customers. The challenge in today's competitive market is to be on the leading edge of producing high quality product at minimum costs. Continuously monitoring the process quality through these process capability indices assure that specifications supplying information for product design and process quality improvement provide the basis for reducing the cost and product defectives see Pan and Wu (1997). The use of PCIs can be more constructive once the bias and variability of the process is determined and understood. A process may be in statistical control, but not capable of meeting specifications if; process is off-center

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from nominal value (Bias); process variability is too large relative to specifications (Variation) and process is both off-center and has large variation (Bias and Variation). Numerous work have been reported to estimate PCIs considering bias and variation for normal and non-normal processes see for details Juran (1974), Kane (1986), Chan et al. (1988), Boyles (1991), and Pearn et al. (1992) and for non-normal process see Gunter (1989), Boyles (1994), Zwick (1995) among many others. Ahmed and Safdar (2010), (2014), (2019) worked on estimating capability indices for non-normal process under varied distributional condition. Safdar et al. [14] also noted that no straightforward algorithm is reported to estimate capability indices for those processes whose measurements are due to stress and reveal a high skewed curve and proposed procedure to estimate PCIs based on 101 Fatigue measurements of an earlier presented data set by Birnbaum and Saunders's (1969)

In this paper capability indices are estimated for three data sets (Earlier presented by Birnbaum and Saunders's (1969) named as psi21, psi26 and psi31 constituted by Fatigue life (T) of N=304 6061-T6 aluminum coupons oscillated at 18 cycles per second (cps) under 3 levels and exposed to a pressure with maximum stress of 21,000 pounds per square inch (psi) on 101, 26,000psi on 102 and 31,000 psi on 101 specimens. psi pounds per square inch. For data sets see APPENDIX I.

Fatigue Life Model:
Birnbaum-Saunders's distribution (1969) is popularly known as fatigue life distribution whose density function is $f(t;\alpha,\beta)=\frac{1}{2\alpha\beta}\left(\frac{t}{\beta}\right)^{-1/2}\left[1+\left(\frac{t}{\beta}\right)^{-1}\right]\times\frac{1}{\sqrt{2\pi}}\operatorname{Exp}\left[-\frac{1}{2\alpha^2}\left[\left(\frac{t}{\beta}\right)^{-1/2}-\left(\frac{t}{\beta}\right)^{1/2}\right]^2\right];t>0,\alpha\beta>0$. Here α and β are shape and scale parameter respectively.

For details see Seeger (1990), Smith (1985) and Stephens et al. (2001) and Vilca-Labra and Leiva (2006).

Safdar et al.(2019) transformed Fatigue density function to standard normal density function using a well-known transformation for $Z=\frac{(t/\beta)^{\frac{1}{2}}-(t/\beta)^{-\frac{1}{2}}}{\alpha}$ such that, $f(t)\cong f(z)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}z^2}$ and estimate capability indices for transformed fatigue measurement using developed PCIs for Normal processes.

Process Capability Indices:
For four basic PCIs $C_p, C_{pk}, C_{pm}, C_{pmk}$ Vannman 1995 proposed a superstructure as under;

$$C_p(u,v)=\frac{d-u|\mu-m|}{3\sqrt{\sigma^2+v(\mu-T)^2}}\ u,v\geq 0 \tag{1}$$

Where
$C_p(0,0)=C_p, C_p(1,0)=C_{pk}, C_p(0,1)=C_{pm} C_p(1,1)=C_{pmk}$

$100(1-\alpha)\%$ Confidence interval for $C_p, C_{pk}, C_{pm}, C_{pmk}$ are

$$\left(\frac{\chi_{n-1,\alpha/2}}{(n-1)^{\frac{1}{2}}}\hat{C}_p,\frac{\chi_{n-1,1-\alpha/2}}{(n-1)^{\frac{1}{2}}}\hat{C}_p\right) \tag{2}$$

$$\hat{C}_{pk}\left[\frac{1\pm z_{1-\alpha/2}}{\sqrt{2(n-1)}}\right] \tag{3}$$

$$\left(\frac{\chi_{n,\alpha/2}}{\sqrt{n}}\tilde{C}_{pm},\frac{\chi_{n,1-\alpha/2}}{\sqrt{n}}\tilde{C}_{pm}\right) \tag{4}$$

$$\hat{\mathrm{C}}_{\mathrm{pmk}} \mp z_{\alpha/2} \frac{\hat{\sigma}_{pmk}}{\sqrt{n}} \tag{5}$$

For details see Kotz and Johnson (1992), Nagata and Nagahata (1992), (1993), Boyles (1991), Subbaiah (1991) Patnaik (1949) and Chen and Hsu (1995).

Method for Estimate PCIs for Fatigue Life Model:

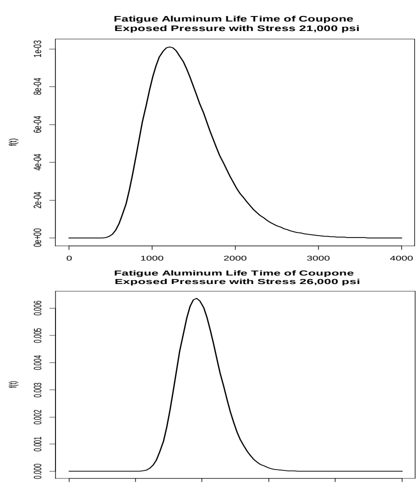
1. An algorithm is made to estimate PCIs using fatigue model in R-console and analysis is followed with packages *gbs* and *VGAM* in R-3.0.3.
2. Choose data set whose measurements come from two parameter Fatigue distribution and estimate $(\hat{\alpha}, \hat{\beta})$ based on maximum likelihood estimation MLE procedure.
3. From the estimated parameters simulate Fatigue samples "t" of sizes n=100, 200, 500 and 1000. Transformed Fatigue samples along with preset specification limits (LSL, USL) to standard normal varaite "Z".
4. Assess normality assumption by ShapiroWilk normality test for each simulated (t) and transformed sample (z) to check normality assumptions.
5. Construct $\overline{X} - R$ control chart for each transformed sample with subgroup size of 10 to check that the transformed process is in statistical control. The program is designed so that it exclude those samples which are beyond the control limits.
6. Estimate PCIs and construct 95% confidence intervals of each PCIs using Equation (1) to (5) for each transformed Fatigue Sample.

Data Sets for Illustration PCIs for Fatigue Life Model:

For the data sets (see APPENDIX I), the preset specification limits and MLE estimates of each data sets are summarized in Table 1. Figure 1 displays the density curves for three data sets.

Table 1:-Preset Limits & MLE Parameters of psi21, psi26, psi31

Data Sets	Size	LSL	USL	α	β
psi21	101	415	2417	0.3101	1336.377
psi26	102	240	560	0.1614	392.76
psi31	101	64	202	0.1704	131.82



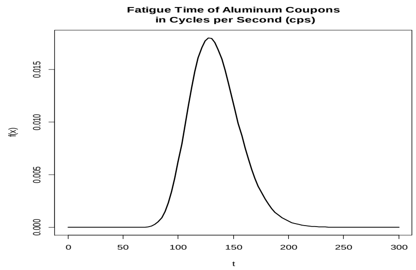


Fig 1:-Density Curves of Data Sets psi21, psi26, psi31

Before estimating capability indices the Shapiro-Wilk normality test is performed for each Fatigue and transformed samples for three data sets *psi21, psi26* and psi31. See Table 2.

Table 2:-Shapiro-Wilk Normality Test for *psi21*, *psi26*, *psi31*

n	**Data Sets**	***psi21***		***psi26***		**psi31**	
	Samples	**SW-Test**	**P-value**	**SW-Test**	**P-value**	**SW-Test**	**P-value**
100	t	0.990	0.669	0.960	0.004	0.980	0.133
	y	0.988	0.482	0.991	0.741	0.994	0.935
200	t	0.990	0.164	0.966	0.000	0.960	0.000
	y	0.995	0.808	0.989	0.146	0.991	0.230
500	t	0.995	0.119	0.959	0.000	0.981	0.000
	y	0.995	0.116	0.997	0.538	0.998	0.864
1000	t	0.992	0.000	0.961	0.000	0.982	0.000
	y	0.998	0.211	0.999	0.697	0.998	0.476

Table 2 shows that transformed Fatigue samples exhibiting Normal process and allow to obtain point and interval PCIs estimates using Equation (1) to (5) for each simulated transformed sample for three data sets psi21, psi26, psi31.

Table 3:-Point & Interval Estimates of PCIs for Transformed Samples psi21, psi26, psi31

Data Sets	**n**	**C_p**	**CI of C_p**	**C_{pk}**	**CI of C_{pk}**	**C_{pm}**	**CI of C_{pm}**	**C_{pmk}**	**CI of C_{pmk}**
psi21	100	0.986	(0.809, 1.169)	0.605	(0.465,0.745)	0.649	(0.533,0.769)	0.398	(0.339,0.458)
	190	1.035	(0.899, 1.173)	0.694	(0.583,0.805)	0.723	(0.629,0.819)	0.485	(0.436,0.533)
	500	1.017	(0.934,1.100)	0.631	(0.567,0.696)	0.665	(0.611,0.72)	0.413	(0.385,0.441)
	1000	1.007	(0.950,1.066)	0.64	(0.594,0.685)	0.677	(0.638,0.716)	0.43	(0.409,0.450)
	100	0.903	(0.777,1.028)	0.767	(0.642,0.892)	0.836	(0.720,0.952)	0.71	(0.642,0.778)
	190	0.876	(0.790,0.962)	0.75	(0.663,0.836)	0.819	(0.739,0.899)	0.701	(0.653,0.749)
	500	0.878	(0.823,0.932)	0.733	(0.679,0.787)	0.805	(0.755,0.855)	0.673	(0.644,0.702)
	1000	0.879	(0.841,0.972)	0.737	(0.699,0.776)	0.809	(0.774,0.844)	0.679	(0.658,0.699)
psi31	100	1.252	(1.078,1.426)	0.889	(0.749,1.029)	0.846	(0.729,0.963)	0.601	(0.540,0.662)
	200	1.212	(1.093,1.331)	0.895	(0.796,0.994)	0.879	(0.792,0.965)	0.649	(0.602,0.696)
	500	1.175	(1.102,1.203)	0.853	(0.793,0.914)	0.845	(0.793,0.898)	0.614	(0.586,0.642)
	1000	1.153	(1.203,0.801)	0.843	(0.801,0.885)	0.845	(0.808,0.882)	0.618	(0.598,0.638)

Table 3 summarized the results of point and interval PCIs estimates assuming normal process of transformed fatigue samples for each data set and for each sample.

Conclusion and Recommendation:-

A statistical procedure to estimate capability indices for BS distribution is implemented on three Birnbaum Saunders data sets exposed under three different stress levels. The PCIs along with their confidence intervals are estimated for transformed fatigue model and it is noted that each confidence interval contains the respective point estimate for each size of the sample.

This illustration allow quality practitioners and design producers who are working for Fatigue measurements to obtain capability indices for Fatigue processes along with making the process in statistical control.

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Psi21	370	706	716	746	785	797	844	855	858	886
	886	930	960	988	999	1115	1120	1134	1140	1199
	1115	1120	1134	1140	1199	1115	1120	1134	1140	1199
	1200	1200	1203	1222	1235	1238	1252	1258	1262	1269
	1270	1290	1293	1300	1310	1313	1315	1330	1355	1390
	1416	1419	1420	1420	1450	1452	1475	1478	1481	1485
	1502	1505	1513	1522	1522	1530	1540	1560	1567	1578
	1594	1602	1604	1608	1630	1642	1674	1730	1750	1750
	1763	1768	1781	1782	1792	1820	1868	1881	1890	1893
	1895	1910	1923	1924	1945	2023	2100	2130	2215	2268
	2440									
psi26	233	258	268	276	290	310	312	315	318	321
	321	329	335	336	338	338	342	342	342	344
	349	350	350	351	351	352	352	356	358	358
	360	362	363	366	367	370	370	372	372	374
	375	376	379	379	380	382	389	389	395	396
	400	400	400	403	404	406	408	408	410	412
	414	416	416	416	420	422	423	426	428	432
	432	433	433	437	438	439	439	443	445	445
	452	456	456	460	464	466	468	470	470	473
	474	476	476	486	488	489	490	491	503	517
	540	560								
psi31	70	90	96	97	99	100	103	104	104	105
	107	108	108	108	109	109	112	112	113	114
	114	114	116	119	120	120	120	121	121	123
	124	124	124	124	124	128	128	129	129	130
	130	130	131	131	131	131	131	132	132	132
	133	134	134	134	134	134	136	136	137	138
	138	138	139	139	141	141	142	142	142	142
	142	142	144	144	145	146	148	148	149	151
	151	152	155	156	157	157	157	157	158	159
	162	163	163	164	166	166	168	170	174	196
	212									