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**RESEARCH ARTICLE**

# CERTAIN TRANSFORMATION FORMULAE INVOLVING BASIC ANALOGUES OF HYPERGEOMETRIC FUNCTION OF TWO VARIABLES

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## *Abstract*

In the present paper, we have derived transformation formulae of basic analogues of hypergeometric functions of two variables. Results derived in this survey are believed to be new.



## Introduction

The basic analogue of Appell's hypergeometric functions of two variables were first defined and studied by F.H. Jackson[8], R.P. Agarwal[1,2] also studied these functions and gave some general identities involving these functions. G.E. Andrews [3] also studied these functions and showed that the first of the Appell series $\phi^{(1)}$ can be reduced to a series ${}_3\phi_2$ series.

**q- Analogues of Appell functions-** The q- analogues of Appell [4] functions of two variables are defined as –

$$\phi_1\begin{bmatrix}\alpha,\beta,\beta^{'} & ;q;x,y.\\ \gamma; & \end{bmatrix}=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha:q)_{m+n}(\beta:q)_m(\beta^{'}:q)_n}{(\gamma:q)_{m+n}(q:q)_m(q:q)_n}x^m\ y^n$$

It can be verified that on taking the limit q→1, the q-Appell functions reduce to corresponding Appell function.

## 2. Main results

In this section we derive certain transformation formulae for the q- analogues of hypergeometric functions of two variables.

**Theorem 2.1**

$$\phi_1(\alpha;\beta,\beta';\gamma;q;x,y)=(1-y)^{-\beta'}\phi_3\left(\alpha;\gamma-\alpha;\beta,\beta';\gamma;q;x,-\frac{y}{1-y}\right)$$

**Proof:** We have

$$\phi_1(\alpha;\beta,\beta';\gamma;q;x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha;q)_m(\alpha+m;q)_n(\beta;q)_m(\beta';q)_n}{(\gamma;q)_m(\gamma+m;q)_n(q;q)_m(q;q)_n}x^m y^n.$$

$$=\sum_{m=0}^{\infty}\frac{(\alpha;q)_m(\beta;q)_m}{(\gamma;q)_m(q;q)_m}x^m\ \phi(\beta',\alpha+m;\gamma+m;q;y)$$

$$=\sum_{m=0}^{\infty} \frac{(\alpha;q)_m \binom{\beta:q}{}_m}{(\gamma;q)_m (q;q)_m} x^m \left(1-y\right)^{-\beta'} \phi(\beta',\gamma-\alpha;\gamma+m;q;-y/1-y).$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(1-y)^{-\beta'}(\alpha;q)_m(\beta;q)_m x^m}{(\gamma;q)_m(q;q)_m} \frac{(\beta';q)_n(\gamma-\alpha;q)_n}{(\gamma+m;q)_n(q;q)_n}\left(\frac{-y}{1-y}\right)^n$$

$$=(1-y)^{-\beta'}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{(\alpha;q)_m(\beta;q)_m(\gamma-\alpha;q)_n(\beta';q)_n}{(\gamma;q)_{m+n}(q;q)_m(q;q)_n} x^m\left(\frac{-y}{1-y}\right)^n$$

$$=(1-y)^{-\beta'}\ \phi_3(\alpha,\gamma-\alpha;\beta,\beta';\gamma;q;x,-y/1-y)$$

**Theorem 2.2**

$$\phi_1(\alpha;\beta;\beta';\gamma;q:x,y)=(1-x)^{-\beta}(1-y)^{-\beta'}\phi_3\left(\gamma-\alpha,\gamma-\alpha;\beta,\beta';\gamma;q;\frac{-x}{1-x};\frac{y}{1-y}\right)$$

**Proof**: $\phi_1(\alpha;\beta;\beta';\gamma;q:x,y)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\alpha;q)_m(\alpha+m;q)_n(\beta;q)_m(\beta';q)_n}{(\gamma;q)_m(\gamma+m;q)_n(q;q)_m(q;q)_n}x^m y^n.$

$$=\phi\begin{bmatrix}\beta,\alpha\\ \gamma\end{bmatrix};q;x\Bigg]\phi\begin{bmatrix}\beta',\alpha+m\\ \gamma+m\end{bmatrix};q;y\Bigg]$$

$$=(1-x)^{-\beta}\phi\left[\begin{matrix}\beta,\gamma-\alpha\\ \gamma\end{matrix};q;\frac{-x}{1-x}\right](1-y)^{-\beta'}\phi\left[\begin{matrix}\beta',\gamma-\alpha\\ \gamma+m\end{matrix};q;\frac{-y}{1-y}\right]$$

$$=(1-x)^{-\beta}(1-y)^{-\beta'}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(\beta;q)_m(\gamma-\alpha;q)_m(\beta';q)_n(\gamma-\alpha;q)_n}{(\gamma;q)_m(q;q)_m(\gamma+m;q)_n(q;q)_n}\left(\frac{-x}{1-x}\right)^m\left(\frac{-y}{1-y}\right)^n$$

$$=(1-x)^{-\beta}(1-y)^{-\beta'}\phi_3\left(\gamma-\alpha;\gamma-\alpha,\beta,\beta';\gamma;q;\frac{-x}{1-x},\frac{-y}{1-y}\right)$$

## References


[1] Agarwal R.P., Generalized Hypergeometric series, Asia Publishing House, Bombay, (1963).

[2] Agarwal R.P., Some Basic Hypergeometric identities, Ann. Soc. Sci. Bruxelles 67, 1-21 (1953).
[3] Andrews, G.E., On Basic Hypergeometric series, Mock theta functions, and partition, Quart. J. Math. (Oxford) (2) 17, 64-80 (1966).

[4] Appell and Kampe de Feriet, Function Hypergeometric el Hyperspheriques (1926).

[5] Baily, W.N. , Generalized Hypergeometric Series, Cambridge Math. Tract No. 32, Cambridge Univ. Press, Cambridge (1935); Reprinted by Stechert, New York (1964).

[6] Bhaskar Srivastava, A note on certain bibasic q-Appell and Lauricella series, Glasnik Matematicki 30,29-36 (1955).

[7] Gasper, G. and Rahman M., Basic Hypergeometric Series, Cambridge University Press, Cambridge, (1990).

[8] Jackson F.H., On Basic double Hypergeometric functions, Quart, J. Math. 13: 69-82(1948).

[9] Rainville, E.D., Special Functions, Macmillan, New York (1960), Reprinted by Chelsa Publ. Co., Bronex, New York (1971).

[10] Saxena R.K., Rajendra kumar, A basic Analogue of the Generalized H- function Le Mathematiche Vol.L- Fase II 263-271(1995).

[11] Slater, L.J., Generalized Hypergeometric Functions, Cambridge University Press, Cambridge, London and New York (1966).

[12] Yadav, R.K., Purohit, S.D., on fractions q-derivatives and transformation of the generalized basic hypergeometric functions, Indian Acad. Math. 28(1) 321-326.