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RESEARCH ARTICLE

On Hesitant Soft Relations

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Abstract

In this paper we have used measurable simple function as membership value in the soft set theory introduced by Molodtsov in 1999. The first part of paper contains some basic definition and the second part of this paper contains hesitant soft relation and anti reflexive kernel. The hesitant soft set is discussed to set framework for further work.

Introduction

There are the uncertainties of various types in economics, engineering, environmental science, sociology and computer engineering, environmental science, sociology and computer science. But classical mathematical approaches are often insufficient to derive effective or useful models because the uncertainties appearing in these domains are of various types, highly complicated and difficult. To avoid difficulties in dealing with uncertainties, many tools have been studied. These are fuzzy sets, rough sets and vague set. In 1999, Molodtsov introduced the concept of soft sets to solve complicated problems and various types of uncertainties. He introduced the concept that a soft set is an approximate description of an object precisely consisting of two parts, namely predicate and approximate value set. In, Maji et al. [6] introduced several operators for soft set theory: equality of two soft sets, subset and superset of a soft set, complement of a soft set, null soft set, and absolute soft set. Recently, soft set theory has been developed rapidly by some scholars in theory and practice. The latest development to this area would be the introduction of “Hesitant fuzzy sets” by Vicenc Torra. As the name suggests this allows scope for hesitancy. Hesitant fuzzy sets allow us to give room for imprecision in assigning the membership values by considering all the possible membership values.

This paper attempts at creating a theoretical framework in hesitant soft Relations. The first section discusses some preliminaries on hesitant soft sets and goes on to basic operations on hesitant soft sets. The second section introduces relation on hesitant soft sets and gives the conditions for it to be an anti-reflexive kernel. The paper then goes on to reflexive and anti-reflexive kernel and gives the formulae to find them.

Preliminaries

2.1 Basic Definitions

In this section we discuss some of the basic definitions regarding hesitant soft sets.

Definition 2.1 Let X be the reference set and F[0,1] by fuzzy power set then we define hesitant soft set by

h: $X \rightarrow F[0,1]$ such that $h(a) = f_a$ where f_a is measurable simple function on [0,1].

Empty set: $h_0(x) = 0(x) \quad \forall x \in X$

Full set : $h_X(x) = 1(x) \quad \forall x \in X$.

Now onwards hesitant soft set will be denoted as HSS.

Definition 2.2 The Score for a HSS is given by

$$s(h(a)) = \int f_a(x)dx, \quad \text{where a is a point of X.}$$

Definition 2.3 ***Subset***: Let h_1 and h_2 be two HSS's on X. Then we say that $h_1$is a subset of h_2 i.e.,

$$h_1 \subseteq h_2 \Leftrightarrow h_1(x) \subseteq h_2(x)$$

and $$h_1 = h_2 \quad iff \quad h_1 \subseteq h_2 \quad and\ h_2 \subseteq h_1.$$

Definition 2.4 ***Proper subset***: $h_1 \subset h_2$ if

$$h_1(x) \subseteq h_2(x) \forall x \in X \text{ and}$$
$$h_2(x) \neq h_1(x) \text{ for some } x \in X$$
$$i.e.,\ h_1(x) \subseteq h_2(x) \forall x \in X \quad \text{and}$$
$$h_2(x) \subset h_1(x) \text{ for some } x \in X.$$

Definition 2.5[2] ***Hesitant Equality*** : $h_A \approx h_B$ if $s(h_A(x)) = s(h_B(x)) \ \forall x \in X$.

Definition 2.6 ***Hesitant subset*** (h_A hesitant subset of h_B) : Let h_A and h_B be two hesitant soft set on X, then we say that h_A is a hesitant subset of h_B (denoted by$h_A \preceq h_B$) iff $s(h_A(x)) \leq s(h_B(x)) \ \forall x \in X$.

Definition2.7[2] ***Hesitant proper subset*** : $h_A \prec h_B$ if $s(h_A(x)) \leq s(h_B(x)) \ \forall x \in X$ and $s(h_A(x)) < s(h_B(x))$ for at least one $x \in X$.

Note: The usual or crisp subset notation defined above becomes a special case of the hesitant subset case, however

$$h_A \subseteq h_B \Rightarrow h_A \preceq h_B \text{ but } h_A \preceq h_B \not\Rightarrow h_A \subseteq h_B.$$

Definition 2.8 [2] ***Complement*** Let h be the hss in X and $x \in X$ be arbitrary point of X then $h^C(x) = \sum \Psi_{Ai}(1-\alpha_i)$ where $h(x) = \sum \Psi_{Ai}\alpha_i$ and Ψ_A is a characteristic function.

Definition 2.4 ***Union and Intersection*** let h_1 and h_2 are two hss, a is arbitrary point f_1 and f_2 are two values at a then $h_1 \cup h_2 = \sum \Psi_{A_i \cap B_j} C_{ij}$ and $h_1 \cap h_1 = \sum \Psi_{A_i \cap B_j} d_{ij}$, where C_{ij} and d_{ij} are maximum and minimum value on $A_i \cap B_j$.

2.2 Properties of Operations on Hesitant Soft Sets:

Let h be a HFS on X. h_X denotes the full set and h_0 denotes the empty set.

- $(h \cup h_X)(x) = h_X(x)$
- $(h \cup h_0)(x) = h(x)$
- $(h \cup h^c)(x) \neq h_X(x)$ and $(h \cap h^c)(x) \neq h_0(x)$

  Similarly,
  - $(h \cup h_0)(x) = h_0(x)$
  - ***Involution***: $(h^c)^c = h$

    Clearly $1-(1-\gamma) = \gamma \quad \forall \gamma \in h(x)$

- ***Commutativity*** :

$$h_1 \cup h_2 = h_2 \cup h_1$$
$$h_1 \cap h_2 = h_2 \cap h_1$$

- **Associativity** :

$$(h_1 \cup h_2) \cup h_3 = h_1 \cup (h_2 \cup h_3)$$
$$(h_1 \cap h_2) \cap h_3 = h_1 \cap (h_2 \cap h_3)$$

- ***Distributivity*** :

$$h_1 \cap (h_2 \cup h_3) = (h_1 \cap h_2) \cup (h_1 \cap h_3)$$

Proof: Suppose X be a reference set h_1,h_2,h_3 are hss on X. We have to show that

$$h_1 \cap (h_2 \cup h_3) = (h_1 \cap h_2) \cup (h_1 \cap h_3)$$

Let x be an arbitrary point. f_1,f_2,f_3 are simple function at x of h_1,h_2,h_3.So $f_1=\sum \Psi_{Ai}\,\alpha_i$, $f_2= \sum \Psi_{Bj}\,\beta_j$, $f_3= \sum \Psi_{Ck}\gamma_k$ consider $A_i \cap B_j \cap C_k$ for arbitrary i,j,k on $A_i \cap B_j \cap C_k$ there are six possibilities, viz. :

(1) $\alpha_i \leq \beta_j \leq \gamma k$ (2) $\beta_j \leq \alpha_i \leq \gamma_k$ (3) $\beta_j \leq \gamma_k \leq \alpha_i$

(4) $\gamma_k \le \beta j \le \alpha_i$ (5) $\alpha_i \le \gamma_k \le \beta_j$ (6) $\gamma_k \le \alpha_i \le \beta_j$.

Now on $B_j \cap C_k$ the max value is γ_k and on $A_i \cap B_j \cap C_k$ the min value is α_i. So in 1st case left hand side is α_i. Now take two set P and Q by $P = A_i \cap C_k$, $Q = A_i \cap, B_j$ on P minimum value is α_i and on Q min value is α_i so right hand side of theorem is α_i. Similarly in case 2 left hand side is β_j and right hand side is β_j and in case 3 left hand side is βj and right hand side is βj and in case 4 left hand side is γ_k and right hand side is γ_k. Similarly in case 5 left hand side is α_i , right hand side is α_i and in case 6 left hand side is γ_k and right hand side is γ_k.

Definition 3.1 ***Relation***: A Hesitant soft subset R of $X \times Y$ is called a hesitant soft relation R from X to Y.

i.e $$R: X \times Y \to F[0,1]$$

Note: HS (X,Y) denotes the family of all Hesitant soft relations from X to Y.

Definition 3.2 ***Identity Relation*** on X

$$I : X \times X \to P[0,1]$$

$$I(x,y) = \{ \begin{array}{l} \{1(x)\} \ if \ x = y \\ \{0(x)\} \ if \ x \ne y \end{array}$$

Definition 3.3 The **compliment** and **inverse** of R can be defined as

$$R \in HF(X,Y) \quad then \ R^{-1} \in HF(Y,X)$$

Such that $R^{-1}(x, y) = R(y, x)$ and $R^C \in HF(X,(X,Y)$

$$R^C(x, y) = [R(x,y)]^C = \bigcup_{\gamma \in R(x,y)} \{1-\gamma\}$$

Lemma 3.1 Let P, Q, R be three hesitant soft relations from X to Y then,

1) $P \le Q \Rightarrow P^{-1} \le Q^{-1}$ 2) $(P^{-1})^{-1} = P.$

Proof : 1) $P \le Q \Rightarrow s(P(x,y)) \le s(Q(x,y)) \quad \forall x, y \in X$

$$\Rightarrow s(P(y,x)) \le s(Q(y,x)) \quad \text{As } x, y \text{ are arbitrary}$$

$$\Rightarrow s(P^{-1}(x, y)) \le s(Q^{-1}(x, y)) \quad [\because P^{-1}(x, y) = P(y,x)]$$

$$\Rightarrow P^{-1} \le Q^{-1}$$

2) $(P^{-1})^{-1} = P$

$$P^{-1}(x,y) = P(y,x)$$

$$(P^{-1})^{-1}(x,y) = P^{-1}(y,x) = P(x,y)$$

$$\Rightarrow (P^{-1})^{-1} = P.$$

Definition 3.4 Let R be the hesitant soft relation on X then we say R is

(i) reflexive if $R(x,x) = 1(x)$

(ii) symmetric if $R(x,y) = R(y,x)$

(iii) anti reflexive $R(x,x) = 0(x)$ every $x \in X$

Definition 3.5 Let R be a Hesitant soft relation from X to X. The maximal anti-reflexive hesitant soft relation contained in R is called **anti-reflexive kernel** of R, denoted by ar(R).

Definition 3.6 Let R be a hesitant soft relation from X to X. If $R=R^{-1}$ then R is called a **symmetric** hesitant soft relation.

Theorem 3.1 : Let R be a Hesitant soft relation from X to X. Then $ar(R) = R \cap I^C$

Proof : For all $(a,b) \in X \times X$, suppose $z = (a,b)$ and $f = \sum \alpha_i \chi A_i$ at z of R.

Now take $I^C \cap R(a,a) = 0(x)$. So $I^C \cap R$ is anti reflexive.

So clearly $R\cap I^{C}$ is anti-reflexive ....(1)

Now

$$R\cap I^{C}(a,b) = \{ \begin{matrix} 0 \;\; if \;\; a=a \\ R(a,b) \; if \; a \neq b \end{matrix} \qquad \text{. ....(2)}$$

$$\therefore \; (R\cap I^{C}) \leq s(R) \qquad \forall \; x, y \in X \qquad \text{...........(3)}$$

Now Let T be an anti-reflexive relation and $T \leq R$.

To prove $s(T(a,b)) \leq s(R\cap I^{C}(a,b))$

When $x = a = b$ $\quad T(a,a) = 0 \;\; and \;\; (R\cap I^{C})(a,a) = 0$ ....(4)

When $x \neq y$ $\quad T(a,b) \leq R(a,b)$ (by assumption)

$$= R\cap I^{C}(a,b) \;\; by (2) \qquad \text{.................(5)}$$

$$\Rightarrow T \leq R\cap I^{C} \text{ by (4) \& (5)}$$

So $R\cap I^{C}$ is maximal. Hence $ar(R) = R\cap I^{C}$

4 . Conclusions

In this paper we have introduced the concept of hesitant soft set and hesitant soft relation. The introduction of hesitant softest will greatly help decision making problems and will further develop the areas in which uncertainty theory, fuzzy set theory and soft set have already been used with great success. The study of relations on hesitant soft sets will set a theoretical base for the further development of this area.

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