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RESEARCH ARTICLE

Performance Analysis of Rayleigh Fading Channels in the Presence and Absence of Transmit Beamforming in MIMO Systems

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Abstract

When multiple antennas are used to obtain beamforming or diversity gain instead of capacity gain, the same symbol which is weighted by a complex scale factor, is sent over each transmit antenna, thereby leading to an input covariance matrix with unit rank. This is known as Multiple Input Multiple Output beamforming or transmit beamforming. For wireless communications, the envelope of the received carrier signal is Rayleigh distributed and such a type of fading is called Rayleigh fading. This can be caused by multipath with or without the Doppler effect. In the multipath case, when the dominant signal becomes weaker, such as in the non-Line of Sight case, the received signal is the sum of many components that are reflected from the surroundings. In this paper the performance analysis of Rayleigh Fading Channels in the presence and absence of Transmit Beamforming techniques is done and the results are provided.

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1. INTRODUCTION TO BEAMFORMING

The MIMO technology can be used for spatial multiplexing, diversity and/or beamforming, but all these goals at full scale cannot be achieved at the same time. These goals are contradictory to one another. When a MIMO system is used in a Line of Sight environment, the achievable beamforming gain is Nt/Nr, with Nt at the transmitter and Nr at the receiver, however, there is no diversity gain since there is no fading. In a heavy scattering environment, the diversity order is NtNr, but the maximum beamforming gain is upper-limited by $(\sqrt{N_t}+\sqrt{N_r})^2$ [1]. In addition, the transmit diversity and beamforming are two complementary techniques. The first provides diversity with no array gain, while the second provides array gain with no diversity.
The gain of spatial multiplexing can be defined as follows [2]

$$G_{sm} = \lim(\bar{\gamma} \to \infty)\frac{Cout(\bar{\gamma})}{\log_2 \bar{\gamma}}$$

where Cout is the outage capacity in bits/s/Hz. Similarly, the diversity gain G_d can be obtained by approximating P_e or P_{out} at high SNR and it is expressed as follows

$$P_e \approx \gamma^{-G_d} \text{ or } P_{out} \approx \gamma^{-G_d}$$

1.1 SCALAR RAYLEIGH CHANNEL

For the scalar slow fading Rayleigh channel, the diversity-multiplexing tradeoff for the PAM, QAM, and the channel itself are given by [3] and it is expressed as follows

$$G_d = 1-2G_{sm}, G_{sm} \in \left[0, \frac{1}{2}\right] \quad \text{(PAM)}$$

$$G_d = 1 - G_{sm,} G_{sm} \in [0,1] \qquad \text{(QAM)}$$

$$G_d = 1 - G_{sm,} G_{sm} \in [0,1] \qquad \text{(channel)}$$

Thus, the uncoded QAM scheme trades off diversity and multiplexing gains optimally. For the slow Rayleigh channel, the outage probability at $R = C_{out} = G_{sm} \log_2 \gamma$ is expressed at high SNR as follows

$$P_{out} = P_r(\log_2(1+|h|^2\gamma) < G_{sm}\log_2\gamma) \approx \gamma^{-(1-G_{sm})}$$

The last approximation is obtained by using $P_r(|h|^2 < \in) \approx \in$ for small $\in$ in Rayleigh fading. Thus, the diversity-multiplexing trade off is obtained.

1.2 ISI CHANNEL

From a diversity perspective, ISI can be exploited by averaging the fluctuations in the channel gains across the different signal paths. The optimal diversity-multiplexing tradeoff curve for the ISI channel is [4] expressed as

$$G_d^* = L(1 - G_{sm})$$

where L is the number of taps corresponding to the ISI Channel.

Analysis shows that the MLSE and linear DFE equalizers achieve the optimal diversity-multiplexing tradeoff without coding, but the ZF and MMSE equalizers do not [4]. However, if each transmission block is ended with a period of silence lasting the coherence time of the channel, both the ZF and MMSE equalizers achieve the optimal diversity-multiplexing tradeoff ; thus, at high SNR, a simple precoding strategy can facilitate reliable communication with low complexity[4]. This avoids the use of the complex MLSE or linear DFE receiver. Besides, the OFDM system utilizes a prefix to obtain the optimal diversity-multiplexing tradeoff as the block length tends to infinity, and it also requires an additional linear encoding architecture at the transmitter. The diversity-multiplexing tradeoff for a V-BLAST OSIC receiver with ZF or MMSE processing at each stage is analyzed in [5]. It is verified that under general settings the optimal ordering rule for a V-BLAST SIC receiver will not improve its performance regarding diversity-multiplexing trade off in point-to-point channels. Particularly, when the rates of data streams are fixed, the diversity order is not improved by user ordering.

2. PARALLEL AND MISO RAYLEIGH CHANNEL

The optimal diversity-multiplexing tradeoff for the parallel channel with L diversity branches or the MISO channel with Nt=L can be derived as [3] follows

$$G_d = L(1 - G_{sm}), G_{sm} \in [0,1] \qquad \text{(channel)}$$

When transmitting the same QAM symbol over the L sub channels, the tradeoff is given by [3] as follows

$$G_d = L(1 - LG_{sm}), G_{sm} \in \left[0, \frac{1}{L}\right] \qquad \text{(QAM,repetition)}$$

2.1 MIMO RAYLEIGH CHANNEL

Based on the definitions of G_{sm} and G_d, a tradeoff between G_{sm} and G_d has been proposed in [2]. For a MIMO system with i.i.d. Rayleigh fading channels, if the space-time code matrix has a time span Nx>Nt+Nr-1, the optimal diversity achieved over all schemes for a given G_{sm} is given by [2]

$$G_d(G_{sm}) = (N_t - G_{sm})(N_r - G_{sm}) \qquad \text{(channel)}$$

for G_{sm}=0,1……min(Nt, Nr). The maximum diversity order NtNr and the maximum rate min (Nt, Nr) cannot be achieved simultaneously [2]. It is noted that at $G_{sm, max}$= min (Nt, Nr), and thus $G_{d,min}$=0. This is unreasonable since no channel provides zero diversity. This problem arises from the definition of the metric G_{sm}, which has no physical interpretation over finite SNR. At the other extreme, zero spatial multiplexing gain is achieved when G_d reaches it maximum Nt/Nr. The diversity-multiplexing tradeoff for many space-time coding/spatial multiplexing structures with different receivers (e.g. ZF, MMSE, ML, SIC) are described in [6]. The diversity-multiplexing tradeoff and outage performance for Ricean MIMO channels is analyzed in [7]. The diversity-multiplexing tradeoff characteristics of Rayleigh and Ricean channels are shown to be identical. In a high SNR regime, the outage probability versus SNR curve for a Ricean channel is a shifted version of that for the corresponding Rayleigh channel.

2.2 DIVERSITY OF THE SPACE-TIME FREQUENCY SELECTIVE FADING CHANNEL

The diversity of a space-time frequency selective fading channel is determined by the codeword dimensions and coherent parameters. Given a codeword duration T, the availability diversity can be up to T/Tc; for a bandwidth B, the independent frequency diversity branches can be up to B/Bc. For space diversity, the number of antennas Nt and Nr and their topology determine the diversity orders to be Nt/Pt and Nr/Pr, Pt and Pr being, respectively, the packing factors of the receive and transmit arrays, where the packing factor is the number of coherent distances occupied by at least one antenna. To achieve the maximum space diversity, the antenna arrays at both the receiver and the transmitter should be spaced Dc apart. The maximum available diversity is thus given by [8,9] as follows

$$G_d = \frac{T}{T_c}.\frac{B}{B_c}.\frac{N_t}{P_t}.\frac{N_r}{P_r}$$

3. MIMO BEAMFORMING

When multiple antennas are used to obtain beamforming or diversity gain instead of capacity gain, the same symbol, which is weighted by a complex scale factor, is sent over each transmit antenna, thereby leading to an input covariance matrix with unit rank. This is known as MIMO beamforming or transmit beamforming. By transmitting in the direction of the eigen vector corresponding to the largest eigen value of the channel, the output SNR after MRC at the receiver is maximized. The full diversity gain provided by a MIMO channel can be achieved by transmit beamforming and receive combining. The beamforming strategy corresponds to transmitter precoding and receiver shaping with V=v and U=u , where u and v are the normalized transmit and receive weight vectors, $\|u\| = \|v\| = 1$. The received signal is given by [10]

$$y = u^H Hvx + u^H n$$

When H is known, u and v can be selected as the first columns of U and V, which correspond to the maximum singular value of H, $\sigma_1 = \sigma_{2\max}$. In this case, the capacity corresponds to that of a scalar channel with channel power gain $\sigma_{\max}^2$ is expressed as follows

$$C = B\log_2(1+\sigma_{\max}^2 P/\sigma_n^2).$$

In this case, the array gain of beamforming is between max (Mt,Mr) and MrMt, and the diversity gain is MtMr[10]. The performance of BER and outage probability of transmit beamforming is analyzed in [11] for uncoded binary transmission over MIMO channels for flat Rayleigh fading channels. In [12], closed-form expressions have been derived for the outage probability and ergodic capacity of transmit-beamforming MIMO Maximal Ratio Combining (MRC) systems under Rayleigh fading. Implementation of transmit beamforming requires Channel State Information (CSI) at the transmitter. One solution is quantized beamforming. Instead of sending the quantized CSI to the transmitter, the receiver quantizes the beamforming vector using a fixed codebook available at both the transmitter and receiver. The quantized index is then sent to the transmitter. Systematic codebook design has been performed for the uncorrelated Rayleigh fading channels in [13,14] and for correlated Rayleigh fading channels in [15]. WCDMA contains explicit support for closed-loop transmit diversity. The CSI is estimated from the two common pilot channels (CPICHs), and is then fed back to the transmitter to control the beamforming weights at different transmitting antennas. Even a crude feedback signaling can be extremely useful in improving the downlink performance. When CSI is not available at the transmitter, the Alamouti or STBC scheme can be used to obtain full diversity gain and array gain.

4. PERFORMANCE ANALYSIS OF RAYLEIGH CHANNELS WITH/WITHOUT TRANSMIT BEAMFORMING USING BPSK MODULATION

For the transmit beamforming the channel assumed is flat fading and it is a Rayleigh multipath channel and the type of modulation engaged here is a Binary Phase Shift Keying (BPSK). The channel model is that it has one receive antenna and two transmit antennas. The channel is flat fading-in simple terms it means that the multipath channel has only one tap. So the convolution operation reduces to a simple multiplication. The channel experienced by each receive antenna is randomly varying in time. For the ith receive antenna, each transmitted symbol gets multiplied by a randomly varying complex number. As the channel under consideration is a Rayleigh channel, the real and imaginary parts are Gaussian distributed having a particular mean and variance. The channel experience by each transmit antenna to receive antenna is independent from the channel experienced by other transmit antennas. On each receive antenna, the noise has the Gaussian probability density function with $p(n)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(n-\mu)^2}{2\sigma^2}}$ with μ =0 at each transmit antenna, the channel is known. When transmit beamforming is applied, the multiplication of the symbol from each transmit antenna with a complex number corresponding to the inverse of the phase of the channel is carried out so as to ensure that two signals add constructively at the receiver. The simulation steps are that initially the random binary sequences of +1'S and -1'S are generated. Then the symbols are multiplied with the beam steering matrics which actually corresponding to the phase of the channel. After this process equalization is performed at the receiver. Then, the hard decision decoding is performed and the bit errors are counted. It is repeated for multiple values of Eb/No and then the simulation and theoretical results are plotted.

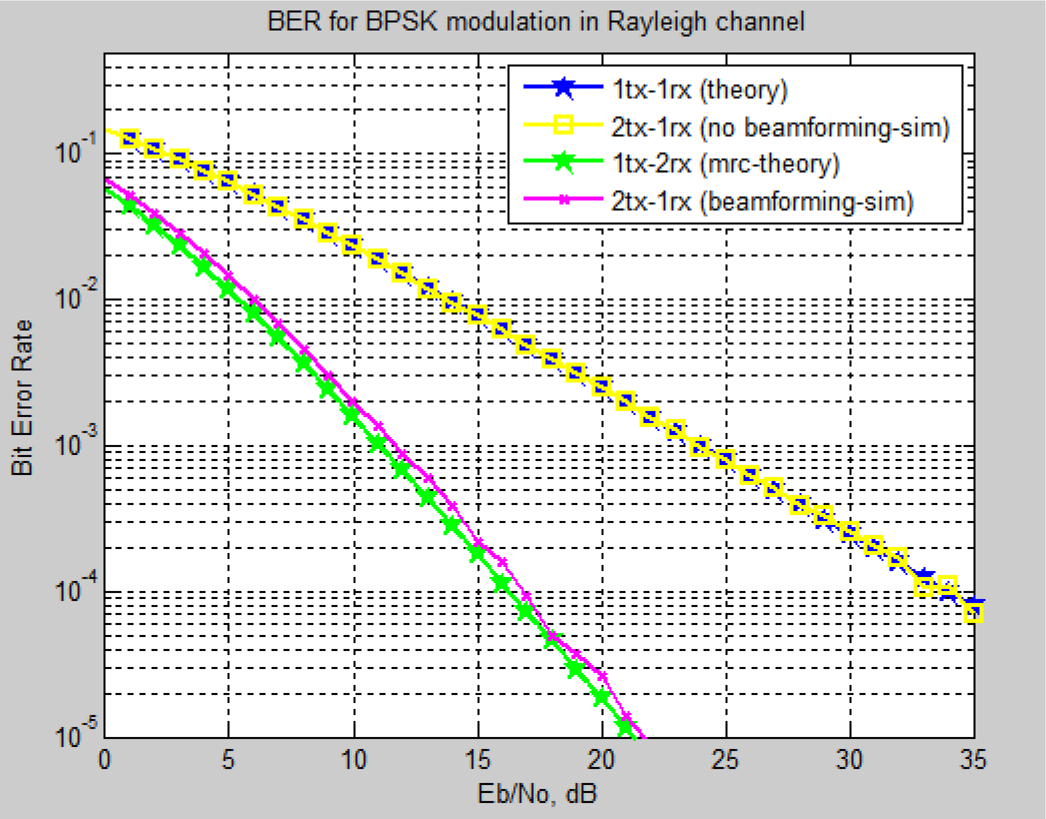

Fig. 1. Rayleigh Fading Channels in the Presence and Absence of Transmit Beamforming under BPSK Modulation

CONCLUSIONS

An introduction to the concept of beamforming is given followed by a review on parallel and MISO channels. From the figure 1 it is obvious that Rayleigh channels produce a very low Bit Error Rate in the presence of beamforming technique when compared to that of the Bit Error Rate in the absence of beamforming technique. Future works include the usage of different modulation techniques under different channel conditions to produce a low Bit Error rate in the presence of beamforming techniques.

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